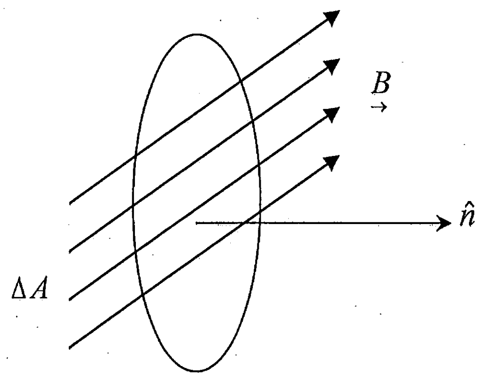
NON - COULOMB $\underset{\rightarrow}{E}$ FIELD
(INDUCTION)

We begin by considering a uniform $\underset{\rightarrow}{B}$-field represented by a set of parallel lines. Next, imagine an area $\Delta \underset{\rightarrow}{A}$ whose normal



is along $\hat{n}$. Then, as in the case of $\underset{\rightarrow}{E}$, we define flux of $\underset{\rightarrow}{B}$ through $\Delta \underset{\rightarrow}{A}$ to be

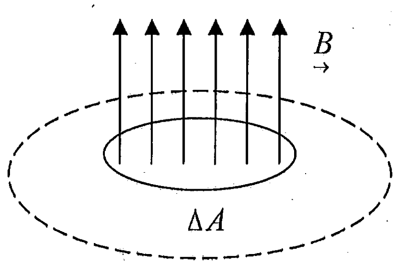
$$\Delta \Phi_B = \underset{\rightarrow}{B} \cdot \Delta \underset{\rightarrow}{A} = B \Delta A \, Cos(\underset{\rightarrow}{B}, \hat{n})$$

As before, flux is maximum when $\underset{\rightarrow}{B} \parallel \hat{n}$ ($\perp$ to area) and zero if $\underset{\rightarrow}{B} \perp \hat{n}$ ($\underset{\rightarrow}{B}$ lines lie in plane of ΔA).

Faraday's discovery was that if the flux of $\underset{\rightarrow}{B}$, $\Delta\Phi_B$, changes as a function of time, that is

$$\frac{\Delta\Phi_B}{\Delta t} \neq 0$$

There will be an $\underset{\rightarrow}{E}$-field induced in every loop surrounding the region where Φ_B is changing. For example, in the picture if the $\underset{\rightarrow}{B}$ field varies with time an $\underset{\rightarrow}{E}$-field will appear at every point of the dashed loop.



This new $\underset{\rightarrow}{E}$-field we call a Non-Coulomb $\underset{\rightarrow}{E}$-field. The following points are noteworthy:

1. Flux of $\underset{\rightarrow}{B}$ can vary with time in 3 ways

(i) Magnitude of $\vec{B}$ is a function of time

(ii) The area ΔA where $\vec{B}$ is non-zero varies with time

(iii) Angle between $\vec{B}$ and $\hat{n}$ varies with time

2. If there is an $\vec{E}$-field at every point on a closed loop, there must be an εmf in the loop given by

$$\varepsilon = \Sigma_C \vec{E}_{NC} \bullet \vec{\Delta l}$$

since εmf is the work done on a unit charge by an $\vec{E}$-field.

3. Since the change of potential on a closed loop is <u>NOT</u> zero, $\vec{E}_{NC}$ is NOT A CONSERVATIVE FIELD.

4. The $\vec{E}_{NC}$ field lines close on themselves, there is no beginning and no end. This is totally different from the Coulomb $\vec{E}$ which "started" at positive charges and "ended" at negative charges.

5. For $\vec{E}_{NC}$ Gauss' Law will always give

$$\Sigma_C \vec{E}_{NC} \bullet \vec{\Delta A} \equiv 0$$

Total flux of $\vec{E}_{NC}$ through any closed surface is always ZERO.

6. However, a charge q placed in $\vec{E}_{NC}$ experiences a force

$$\vec{F}_E = q\vec{E}_{NC}$$

exactly as for a Coulomb $\vec{E}$-field.

Although Faraday discovered the induced *emf*, it was left to Lenz to assert that the direction of $\vec{E}_{NC}$ must be such as to <u>oppose</u> the change in the flux of $\vec{B}$ that causes the $\vec{E}_{NC}$, namely

$$\varepsilon = \Sigma_C \vec{E}_{NC} \bullet \vec{\Delta l} = \frac{-\Delta\Phi_B}{\Delta t}$$

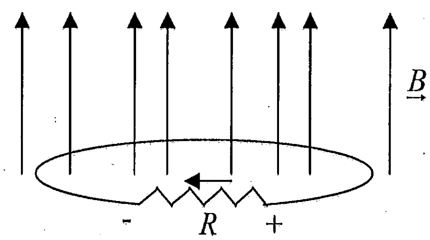
Eventually, Helmholtz showed that the minus sign on the right side is crucial because it arises from Conservation of Energy. Omitting the minus sign would lead to a "run away" situation causing an energy catastrophe.

To Summarize the above we say:
The circulation of non-coulomb $\vec{E}$ around a closed loop is determined by the time rate of change of the flux of $\vec{B}$ through the area inside the loop. The sense of $\vec{E}_{NC}$ is such as to oppose the change in the flux of $\vec{B}$

Applications

1. Consider the picture.

There is a uniform $\underset{\to}{B} = B\hat{y}$ everywhere. The radius of the loop is r. Let $\underset{\to}{B}$ increase with time $\frac{\Delta B}{\Delta t} > 0$. It will set up E_{NC} in the loop.



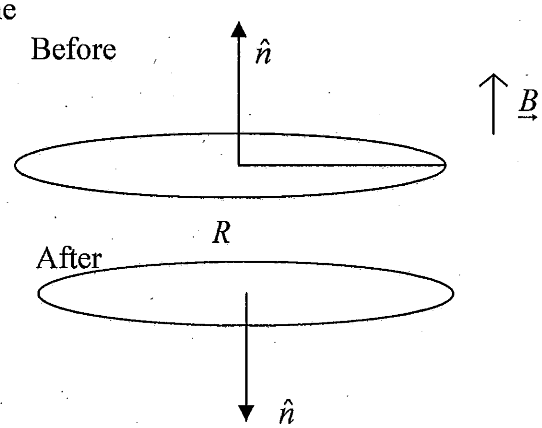
$$\varepsilon = E_{NC} 2\pi r = -\pi r^2 \frac{\Delta B}{\Delta t}$$

and the direction of E_{NC} in the loop must be clockwise. Current in R will flow from left from right.

$$I = \frac{\pi r^2}{R} \frac{\Delta B}{\Delta t}$$

[Notice: the loop acts like a "battery" and current flows from – to + inside the battery and from + to – in the load R.]

2. The coil of resistance R is in the xz-plane while $\underset{\to}{B} = B\hat{y}$



Flip the coil by 180°
Flux change will be

$$\Delta\Phi_B = -B\pi r^2 - B\pi r^2$$
$$= -2B\pi r^2$$

Supposing this flip takes t sec. Then

$$\frac{\Delta\Phi_B}{\Delta t} = \frac{-2B\pi r^2}{tR}$$

Since flux is reducing, *emf* in coil must be counter clockwise.

current in coil $\quad I = \frac{2B\pi r^2}{tR}$

current is flow of charge $I = \frac{\Delta Q}{t}$

so $\quad \Delta Q = \frac{2B\pi r^2}{R}$

that is, when coil is flipped ΔQ coulombs of charge circulate counter clockwise in it. If you keep flipping it, current will continue to flow, alternately cw and ccw.

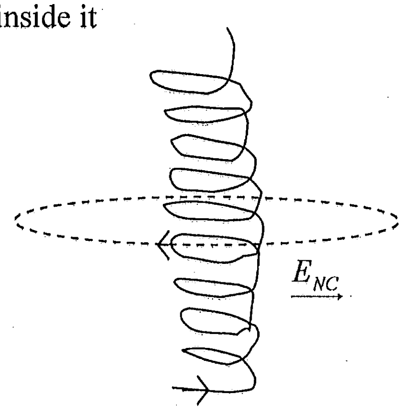
3. Calculate the $\underset{\rightarrow}{E_{NC}}$ for a solenoid at a distance r from its axis when the flux of $\underset{\rightarrow}{B}$ is varied by time variation of the current in the solenoid. That is, $\frac{\Delta i}{\Delta t} \neq 0 \Rightarrow \frac{\Delta B}{\Delta t} \neq 0 \Rightarrow \frac{\Delta \Phi_B}{\Delta t} \neq 0$.

Consider a solenoid wound on a tube of radius a. If there are n turns per meter and the current flow is as shown, there is a uniform $\underset{\rightarrow}{B}$ inside it

$$\underset{\rightarrow}{B} = \mu_0 n i \hat{y}$$

and hence

$$\frac{\Delta \underset{\rightarrow}{B}}{\Delta t} = \mu_0 n \frac{\Delta i}{\Delta t} \hat{y}$$



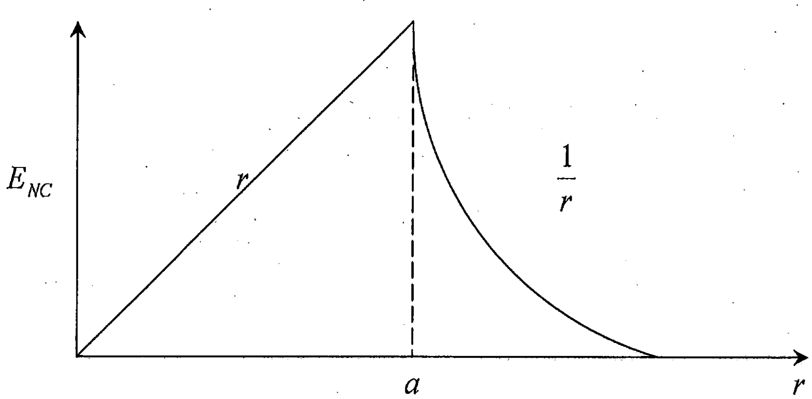
The problem has cylindrical symmetry about axis of solenoid, $\underset{\rightarrow}{E_{NC}}$ at r must be a function of r only and must be azimuthal. Let i increase with time. Then flux of $\underset{\rightarrow}{B}$ along $+\hat{y}$ is increasing with time. Take a circular loop. As shown direction of $\underset{\rightarrow}{E_{NC}}$ must be clockwise (as viewed from above) to oppose increase of Φ_B.

Next,

If $r<a$ $E_{NC} 2\pi r = -\mu_0 n \pi r^2 \frac{\Delta i}{\Delta t}$ *[LOOP INSIDE SOLENOID]*

If $r>a$ $E_{NC} 2\pi r = -\mu_0 n \pi a^2 \frac{\Delta i}{\Delta t}$ *[LOOP OUTSIDE SOLENOID]*

<u>*Magnitude*</u>



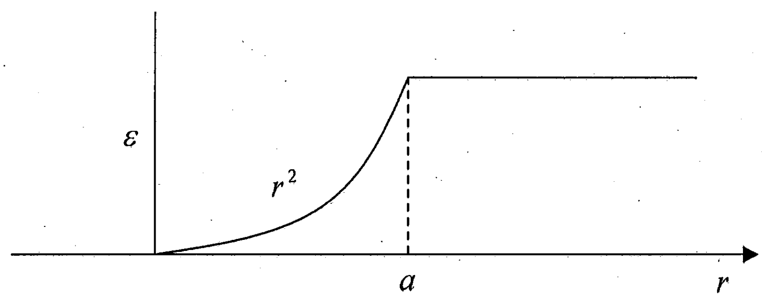
$$E_{NC} = \frac{-\mu_0 nr}{2}\frac{\Delta i}{\Delta t} \qquad r < a$$

$$E_{NC} = \mu_0 n \frac{a^2}{2r}\frac{\Delta i}{\Delta t} \qquad r > a$$

It is useful to ask what is the *emf* in these two loops $\varepsilon = \Sigma_C \underset{\rightarrow}{E_{NC}} \bullet \underset{\rightarrow}{\Delta l}$

$$r < a \qquad \varepsilon = E_{NC} \cdot 2\pi r = \frac{-\mu_o n\pi r^2 \Delta i}{\Delta t}$$

$$r > a \qquad \varepsilon = -\mu_o n\pi a^2 \frac{\Delta i}{\Delta t}$$



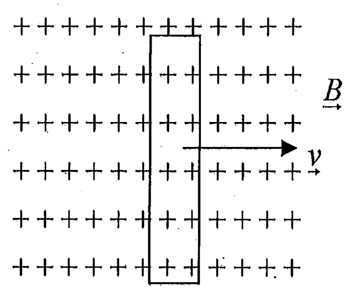
Inside ε increases as r^2
Outside it is constant for all r!

Non-Coulomb $\underset{\rightarrow}{E}$ [Motional *emf*]

It is far from easy for us to understand the origin of the $\underset{\rightarrow}{E_{NC}}$ and the consequent induced *emf* .

However, there is one analogous situation where we can use our present knowledge to see how motion of a conductor in a $\underset{\rightarrow}{B}$-field causes an *emf* to appear within the conductor.

Consider a copper bar of length l moving with velocity $v\hat{x}$ in the presence of a uniform $\underset{\rightarrow}{B}$ field $\underset{\rightarrow}{B} = -B\hat{z}$

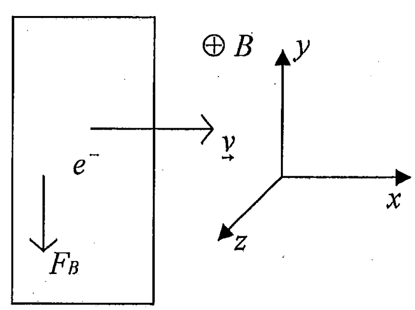


An electron inside the conductor feels the force

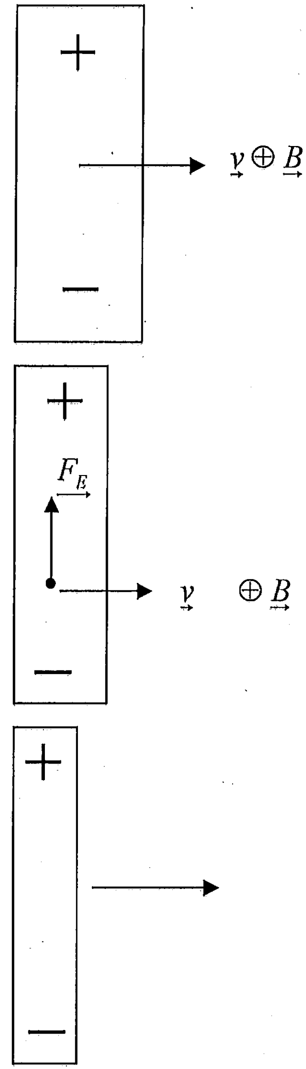
$$\underset{\rightarrow}{F_B} = q[\underset{\rightarrow}{v} \times \underset{\rightarrow}{B}]$$

which in the negative y-direction

$$\underset{\rightarrow}{F_B} = -evB\hat{y}$$



That will cause electrons to drift toward the bottom



leaving an excess positive charge at the top which will cause an $\vec{E}$ field

$$\vec{E} = -E\hat{y}$$

and a force

$$\vec{F}_E = +eE\hat{y}$$

on the electron.

Further segregation of charge will stop when

$$\vec{F}_E + \vec{F}_B = 0$$

that is

$$evB = eE$$
$$E = vB$$

and this will cause an εmf

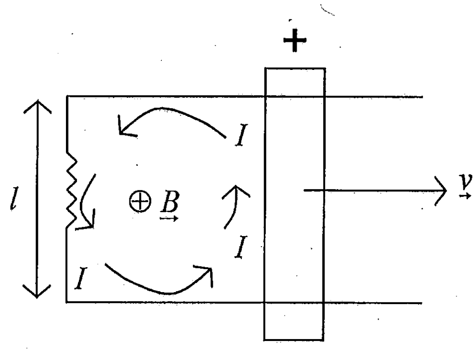
$$\varepsilon = VBl$$

to appear in the bar.

Question $\vec{B}$ field does no work on the electron, so where does the energy originate to set up E?

The εmf is exactly the answer you get if you consider the situation below and use $\varepsilon = \dfrac{-\Delta\Phi_B}{\Delta t}$

$$\frac{\Delta\Phi_B}{\Delta t} = B\frac{\Delta A}{\Delta t} = Bvl$$



and Lenz's Law tells us that the induced εmf must oppose the increase in flux of $\vec{B}$ into the page requiring negative at the bottom and plus at the top leading to a counterclockwise current.