GENERATION OF A $\underset{\rightarrow}{B}$ FIELD

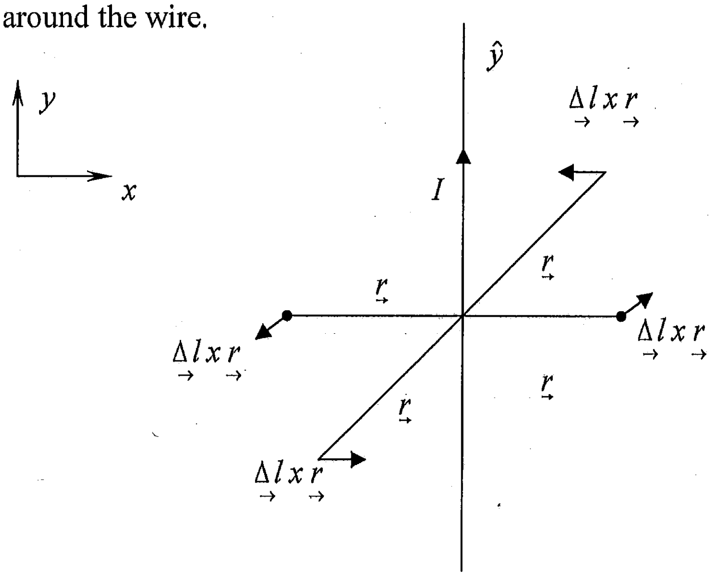
We have seen that a stationary charge experiences a force in an $\underset{\rightarrow}{E}$-field and a stationary charge generates a Coulomb $\underset{\rightarrow}{E}$-field. Now we know that a moving charge feels a force in a $\underset{\rightarrow}{B}$-field so it is natural to expect that a moving charge, such as a current in a conductor, will generate a $\underset{\rightarrow}{B}$-field. This is the content of the so-called Biot-Savart law. A current I in a conductor of length Δl will generate a $\underset{\rightarrow}{B}$-field at a point $\underset{\rightarrow}{r}$ away from the conductor given by the equation

$$\Delta \underset{\rightarrow}{B}(\underset{\rightarrow}{r}) = \frac{\mu_0}{4\pi} I \frac{\Delta \underset{\rightarrow}{l} x \underset{\rightarrow}{r}}{r^3}$$

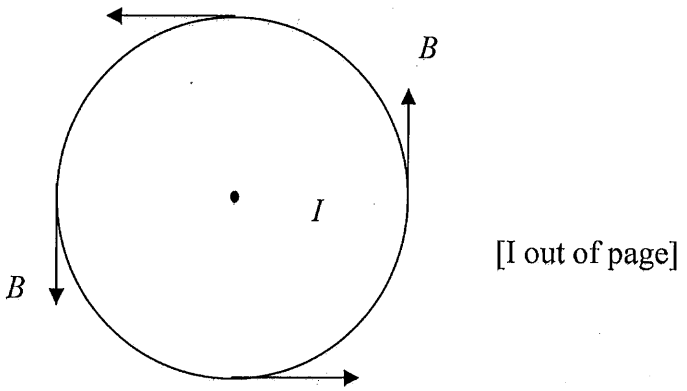
Where $\mu_0 = 4\pi \times 10^{-7} \frac{T-m}{A}$ is a universal constant. We will not use this equation in detail but, as shown next, it gives a very important cue as to the direction of $\Delta \underset{\rightarrow}{B}$.

CASES OF SPECIAL INTEREST.

Single current – I in a long wire: What can we say about $\underset{\rightarrow}{B}$-field at a distance r from the wire? Notice that $\Delta l \| + \hat{y}$. And the vector $\Delta \underset{\rightarrow}{l} x \underset{\rightarrow}{r}$ is perpendicular to both $\Delta \underset{\rightarrow}{l}$ and $\underset{\rightarrow}{r}$ so we can say that $\underset{\rightarrow}{B}$ must curl around the wire.



B

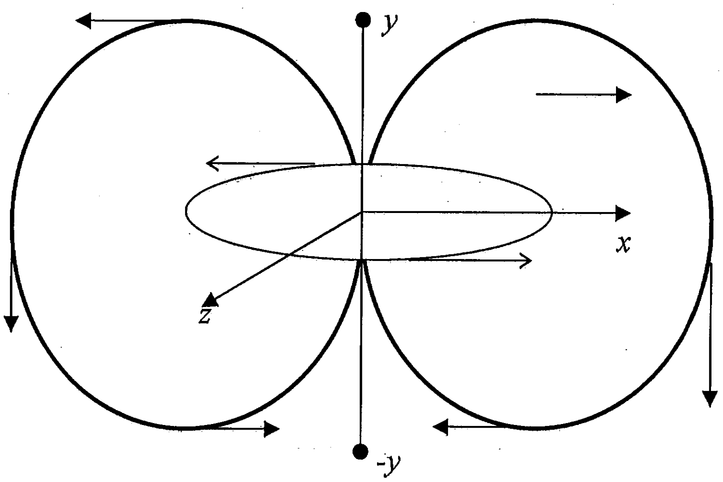


Looking end on (see picture) we have cylindrical symmetry so $\vec{B}$ can be a function of r only. It turns out that $B = \frac{\mu_0 I}{2\pi r}$

so, $B = \frac{\mu_0 I}{2\pi r}\hat{\varnothing}$

Thus, $\vec{B}$ is said to be Azimuthal, $\hat{\varnothing}$ is the direction which curls around I. [check with the sheet on right hand rules].

Next, take the wire and make a circular loop out of it, put it in *xz-pl.* with center at the origin. What is the $\vec{B}$-field at y or $-y$?

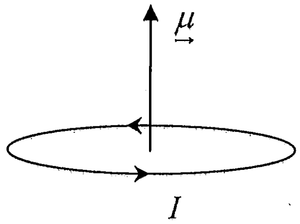


The $\vec{B}$-field lines are shown schematically, it turns out that

$$\vec{B}(y) = \vec{B}(-y) = \frac{\mu_0}{4\pi}\frac{2I\pi a^2}{(a^2+y^2)^{3/2}}\hat{y}$$

Once again, we encounter $IA\hat{n}$ so we can write using magnetic (dipole) moment

$$\vec{B}(y) = \frac{\mu_0}{4\pi}\frac{2\vec{\mu}}{(a^2+y^2)^{3/2}}$$

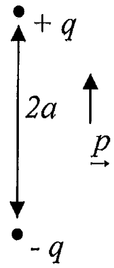


Far away from $\vec{\mu}, y >> a$

$$\vec{B}(y) = \frac{\mu_0}{4\pi}\cdot\frac{2\vec{\mu}}{y^3} \leftarrow \text{Magnetic Dipole}$$

Recall that for an Electric Dipole

$$\vec{p} = 2qa\hat{y}$$



and the $\vec{E}$-field at y is

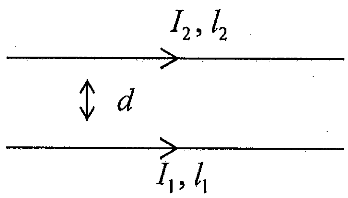
$$\vec{E}(y) = \frac{1}{4\pi\varepsilon_0}\frac{4qay}{\left(y^2-a^2\right)^2}\hat{y}$$

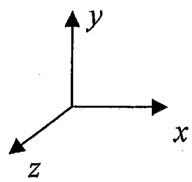
so that at $y >> a$

$$\vec{E}(y) = \frac{1}{4\pi\varepsilon_0}\frac{2\vec{p}}{y^3} \leftarrow \text{Electric Dipole}$$

Very Important! However, there is a major difference here: along y the magnetic dipole has no "size" while electric dipole has length ($2a$). You can split the latter but not the former. This has the extremely important consequence that whereas electric-field lines start at $+q$ and end at $-q$, magnetic field lines <u>close on themselves</u> there is no beginning and no end.

<u>Current-Current force</u>

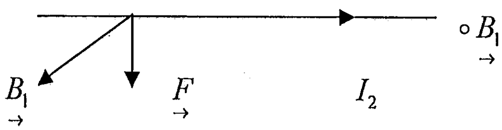




Two wires of lengths l_1, l_2 carry currents I_1, I_2. Separation d along y, wires parallel to x. Force on I_2 due to I_1. To calculate this first, write $\vec{B}_1$ at location of I_2

$$\vec{B}_I = \frac{\mu_0 I_1}{2\pi d}\hat{z}$$

So I_2 is located in $\vec{B}_1$: $\vec{F}_{I_2}$ looks like

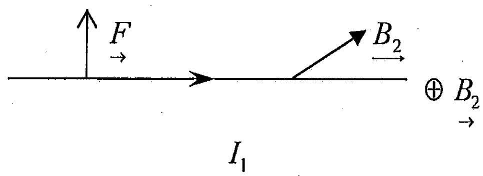


and is given by

$$\vec{F}_{l_1 l_2} = I_2 \Delta \vec{l}_2 \times \vec{B}_1$$
$$= -\frac{\mu_0 I_1 I_2 l_2}{2\pi d}\hat{y}$$

Force is attractive
Force on I_1 due to I_2

$$\vec{B}_2 = \frac{-\mu_0 I_2}{2\pi d}\hat{z}$$
$$\vec{F}_{l_1 l_2} = I_1 \Delta \vec{l}_1 \times \vec{B}_2$$
$$= \frac{\mu_0 I_1 I_2 l_1}{2\pi d}\hat{y}$$



Force is attractive. If $l_1 = l_2 = 1\,meter$ the forces/meter $\vec{F} = \frac{-\mu_0 I_1 I_2}{2\pi d}\hat{d}$ are an action-reaction pair. The –sign with $\hat{d}$ ensures force is attractive if I_1 I_2 parallel $\rightarrow$ $\rightarrow$ and repulsive when they are anti-parallel $\rightarrow$ $\leftarrow$ [You will do an experiment to check this equation]

Incidentally, this is a <u>very fundamental equation</u> as it is used to define the unit of current- The Ampere. That is,

If $I_1 = I_2 = 1 amp$
And $d = 1 meter$

Force per meter is $2 \times 10^{-7} N$ $\left(\frac{\mu_0}{2\pi}\right)$

And the claim is that I should be regarded as a "DIMENSION" in place of Q.
So we can write $L, T, M, \theta, [IT]$

rather than

L, T, M, θ, Q