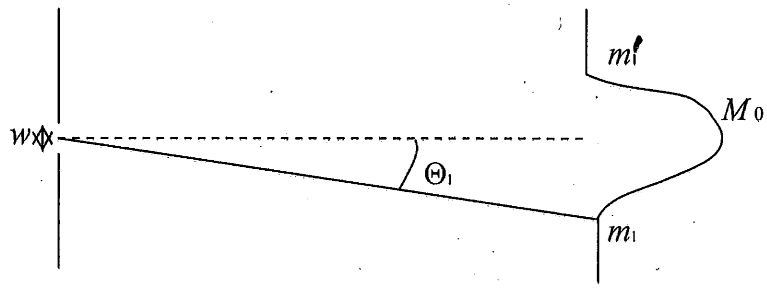
GEOMETRICAL OPTICS

Basis: We will learn that when light of wavelength λ passes through a slit of width w, diffraction causes it to spread by the angle Θ_1 given by

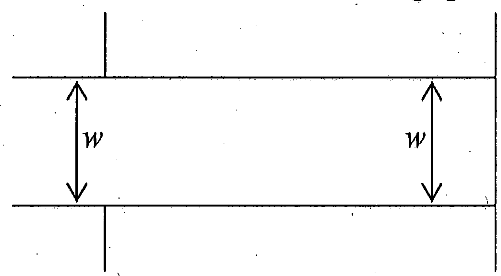
$$\sin\Theta_1 = \frac{\lambda}{w}$$



If w becomes much larger than λ, $\Theta_1 \to 0$ and this spread due to diffraction becomes negligible. In that case the propagation of light looks like the figure and on the screen we observe a patch of light of width w. Light appears to be propagating along a straight line and therefore its progress can be described by using geometry, hence

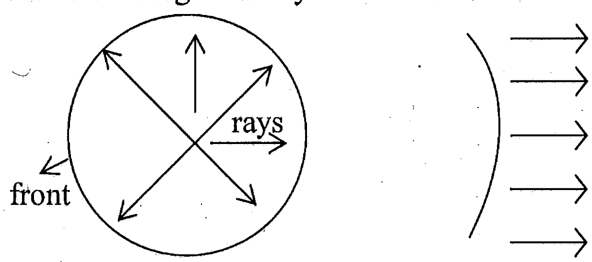
Geometrical optics

and we talk of the path of light by labeling a light RAY.



So far now we say that if openings and obstacles are much larger than λ, Geometrical optics prevails.

As one begins with a point source where light would emerge radially. Near the source the spheres are small but far away they begin to look like "planes" and the light "rays" essentially become parallel to one another. The basic principle which governs the propagation of light in Geometrical optics is due to Fermat.



→ Fermat's Principle: LIGHT INVARIABLY CHOOSES A PATH WHICH TAKES THE LEAST TIME OF TRAVEL.

Unobstructed light therefore travels in straight lines.

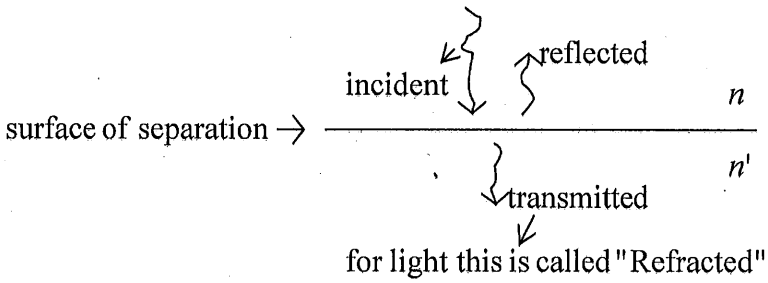
Next, it is notable that speed of light is not the same in all media. Indeed

$$v = \frac{c}{n} \qquad (c = \text{speed in vacuum})$$

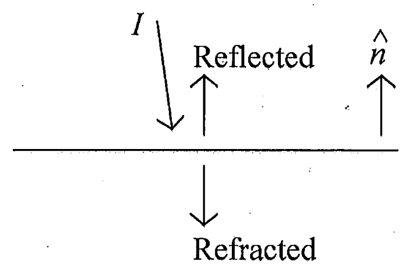
where n is the refractive index. Previously, we learnt that when a wave arrives at a point where velocity changes it gives rise to two waves – Reflected wave travels back in original medium

Transmitted wave travels forward in next medium

Light waves will do exactly the same

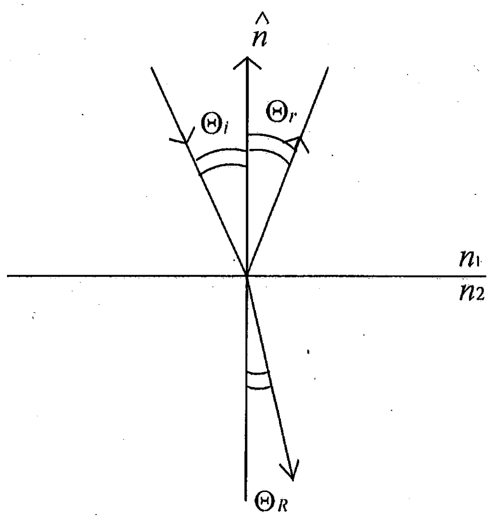


In the ray picture if light is travelling perpendicular to surface you get ⟶



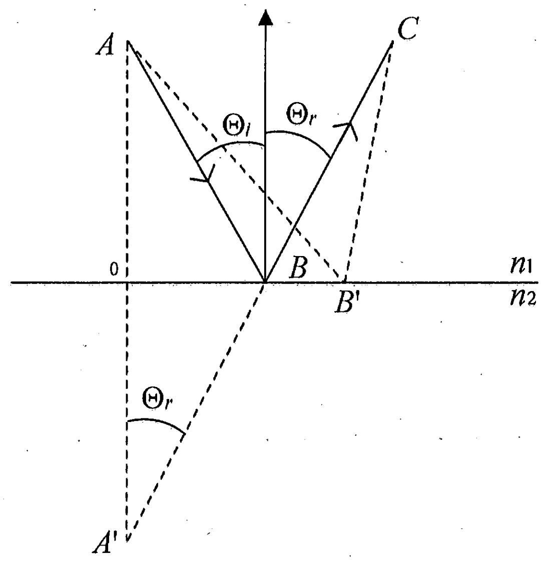
However, we are now interested in the more general case where path of light is not along $\hat{n}$ the vector normal to the surface,

Note: Henceforth, all angles are to be measured with respect to the NORMAL ($\hat{n}$)



So now light is arriving at angle Θ_i. Fermat's principle controls the angles Θ_r (reflected ray) and Θ_R (refracted ray).

The reflection case is easy to understand using Fermat's Principle.



Reflection at a plane surface

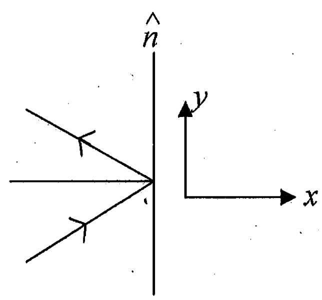
$OA' = OA$

Notice that $A'BC$ is the shortest distance light will travel in going from A to C. All other paths are longer and therefore will take more time.

Hence we have law of reflection: Angle of reflection = angle of incidence

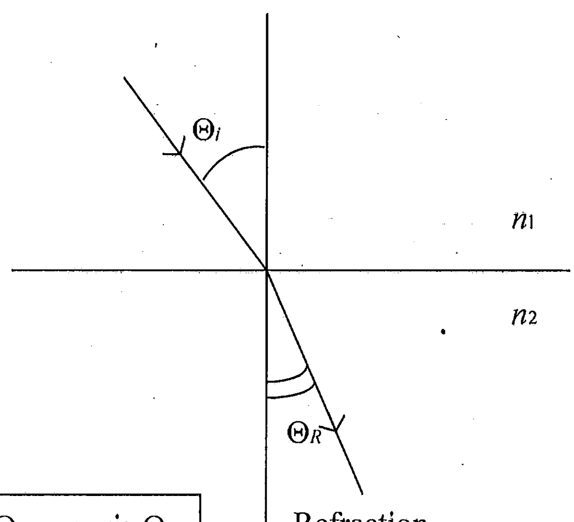
$$\Theta_r = \Theta_i$$

[Looks like the case of Elastic Collision with a wall that we analyzed in 121, x-component of $\vec{p}$ reversed y-component stayed the same]



The path of the refracted ray is also determined by Fermat's principle but the proof requires use of derivatives (which is a no-no for 122) so we just write the answer called

SNELL'S LAW



$$n_2 \sin\Theta_R = n_1 \sin\Theta_i$$

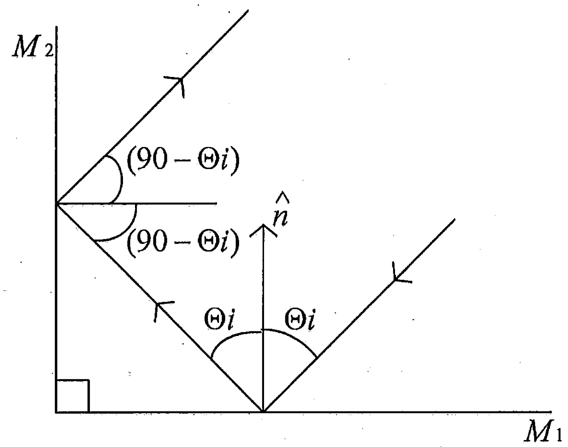
Refraction

Product of refractive index and sine of the angles with respect to normal is a constant.

Some Applications

"Corner" Reflector – uses two mirrors, the mirror being a device where only reflections occur – a polished piece of metal, glass with silver coating.

Two mirrors at right angles to one another



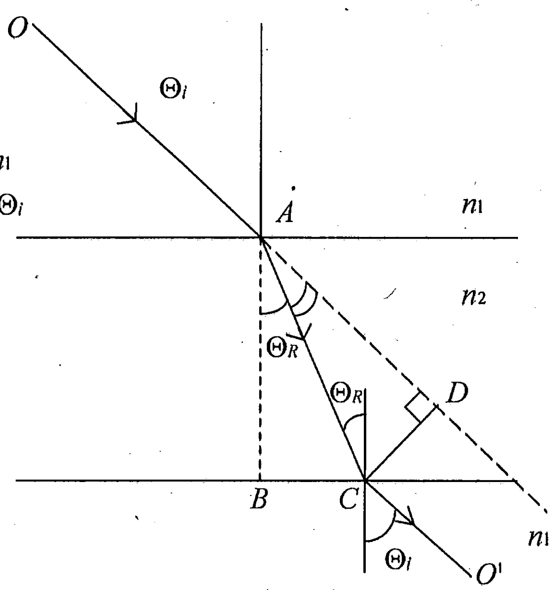
And from the figure above you can see that after the two reflections the light follows a path which is anti-parallel to the incident ray.

Refraction Through Parallel Plate – Lateral Shift

$n_2 > n_1$

$\Theta_R < \Theta_i$

$n_2 \sin \Theta_R = n_1 \sin \Theta_i$



Light Path

$OC \rightarrow$ Incident ray

$AC \rightarrow$ Refracted ray inside n_2

$CO' \rightarrow$ Transmitted Ray

Notice that OC' is parallel to OA, so the light is shifted laterally by the amount CD.

In ΔACD, angle $CAD = (\Theta_i - \Theta_R)$

hence

$$\frac{CD}{AC} = \sin(\Theta_i - \Theta_R)$$

In ΔABC, $AB = t$ [thickness of slab]

$$\frac{t}{AC} = \cos\Theta_R$$

Hence

$$CD = \frac{t}{\cos\Theta_R}[\sin\Theta_i \cos\Theta_R - \cos\Theta_i \sin\Theta_R]$$

$$= t\sin\Theta_i\left[1 - \frac{\cos\Theta_i}{\cos\Theta_R}\frac{\sin\Theta_R}{\sin\Theta_i}\right]$$

$$= t\sin\Theta_i\left[1 - \frac{n_1}{n_2}\frac{\cos\Theta_i}{\cos\Theta_R}\right]$$

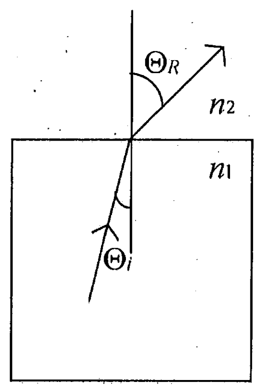
We will use this equation when we get to discussion of thin lenses.

Total Internal Reflection

Now $n_2 < n_1$

Since $n_2 \sin\Theta_R = n_1 \sin\Theta_i$

$\Theta_R > \Theta_i$

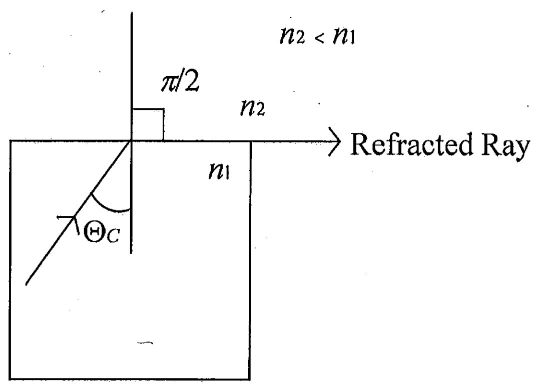


If you increase Θ_i, Θ_R increases until you get the critical case when refracted ray becomes parallel to the surface.

Now $\Theta_R = \frac{\pi}{2}$

$$n_2 \sin\frac{\pi}{2} = n_1 \sin\Theta_C$$

$$\sin\Theta_C = \frac{n_2}{n_1}$$

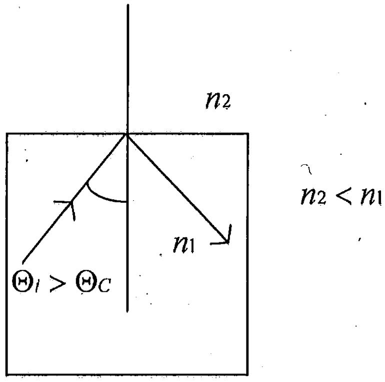


<u>Example</u> Glass $n_1 = 1.5$

Air $n_1 = 1$ $\sin \Theta_C = \frac{1}{1.5}$

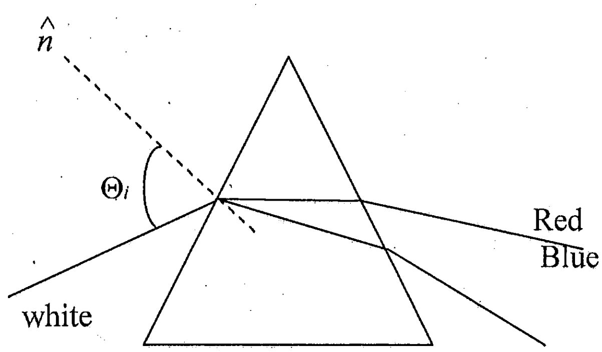
$\Theta_C \cong 41^\circ$

If you make Θ_i larger than Θ_C <u>no</u> light can go out – Total Internal Reflection



<u>Newton's Experiments</u>

He passed white light through a prism and found that it split into many colored rays. Technically we say light is dispersed into its component colors hence <u>Dispersion</u>.



What did we learn?

1. In vacuum/air speed of light is same for all colors

2. White light is a composite of many colors

V	I	B	G	Y	O	R
Violet	Indigo	Blue	Green	Yellow	Orange	Red

[Now we know that λ's in vacuum go from 400nm to 700nm]

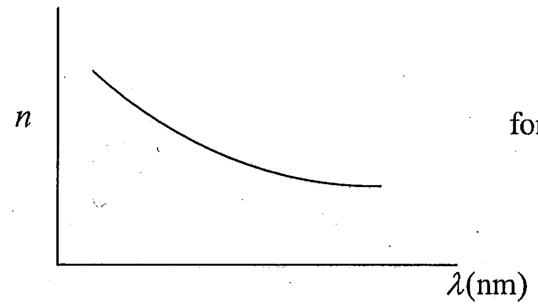
3. Speed of light in a medium is NOT THE SAME FOR ALL COLORS, which is why they split.

4. $\Theta_{Red} > \Theta_{Blue}$ but both satisfy Snell's Law

$$n_{Red} \sin\Theta_{Red} = n_{Blue} \sin\Theta_{Blue} = n_1 \sin\Theta_i$$

so
$$n_{Red} < n_{Blue}$$
$$\frac{c}{V_{Red}} < \frac{c}{V_{Blue}}$$
$$V_{Red} > V_{Blue}$$

5. Now that we know the wavelengths we can plot refractive index as a function of λ and find →



for glass

6. Our perception of color is controlled by frequency and not wavelength