# FORMATION OF IMAGE – MIRRORS

General Construct to locate image of a point object $O$ using the laws of reflection and refraction to locate the path of light

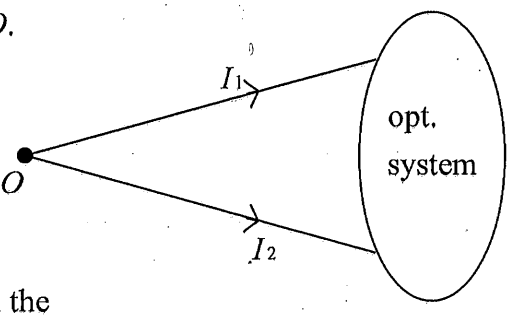
Step1: Take two incident rays $I_1$ and $I_2$ starting from $O$.

Step 2: Follow them trough the optical system using

$r = i$ for reflection

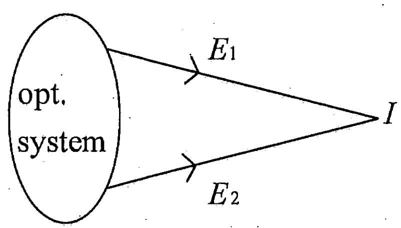
and

$n_2 \sin R = n_1 \sin i$ for refraction




Step 3: Locate the two rays $E_1$ and $E_2$ that emerge from the optical system. Two cases arise

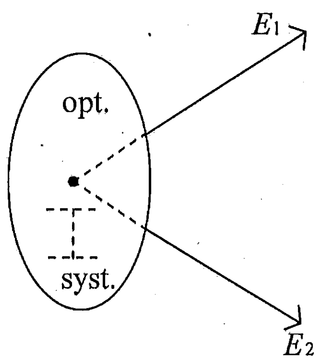
Case I




Point of intersection of $E_1$ and $E_2$ defines position of image $I$. Here, light actually goes through the point $I$ so it is called a REAL IMAGE

Note: A real image can be projected on a screen.

Case II



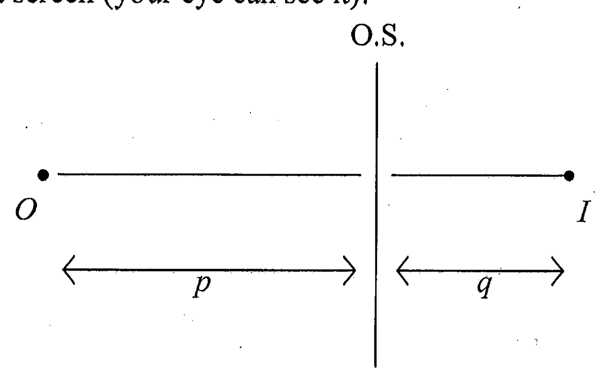

$E_1$, $E_2$ are diverging so they will not intersect. You will have to extrapolate the $E_1$, $E_2$ lines to locate the image $I$ as the point from which $E_1$, $E_2$ "appear" to be coming. No light actually goes through $I$ so it is called a VIRTUAL IMAGE

Note A virtual image cannot be projected on a screen (your eye can see it)!

## SOME DEFINITIONS

$p$ = distance of object from opt. system (O.S.)

$q$ = Distance of image from opt. system



Magnification $m = \frac{size\ of\ image}{size\ of\ object} = \frac{-q}{p}$

The minus sign on the right side ensures that for a single element O.S. all real images ($q$ positive) are inverted ($m$ negative) while all virtual images ($q$ negative) are upright ($m$ positive).

## MIRROR

A mirror is an optical system in which only reflections occur; so there is a light-side and a dark side. We use the "sign" convention that distances are "positive" on light side and "negative" on dark side.

1. Plane Mirror: Plane, silvered sheet of glass

Two Rays:

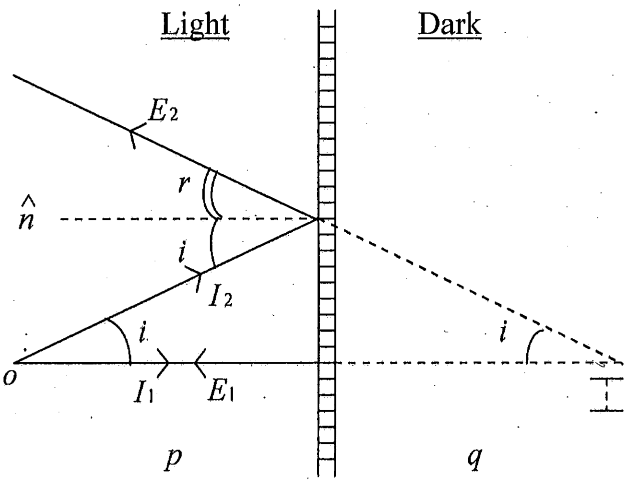
$I_1 \parallel \hat{n}$

$i = 0, \quad r = 0$

$E_1 \parallel \hat{n}$

$I_2: r = i$

Locates $E_2$




$E_1$ and $E_2$ diverge. Locate image by extrapolation to ⌶ .

$q$ is negative

Image is virtual, clearly $|q| = p$

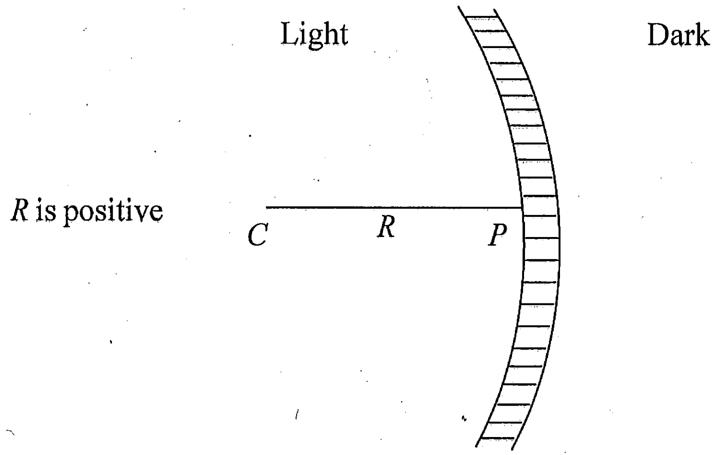
$$m = \frac{-q}{p} = +1$$

upright, virtual image is as far behind the mirror as the object is in front of it.

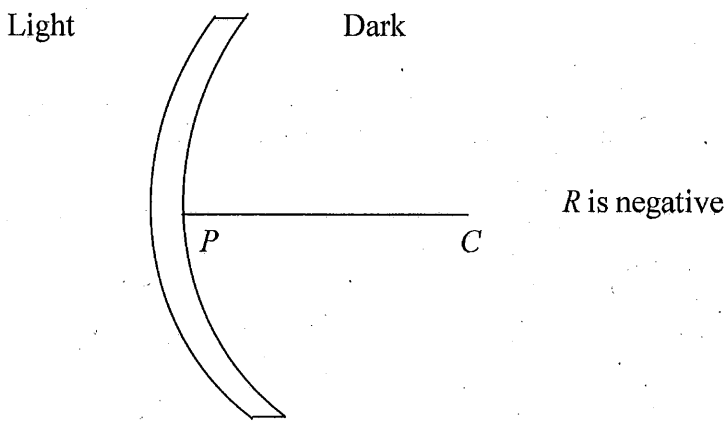
2. Spherical Mirrors: Mirror cut from a spherical shell of radius $R$.

Two Cases Arise

A. CONCAVE CENTER OF SPHERE IN FRONT OF MIRROR.

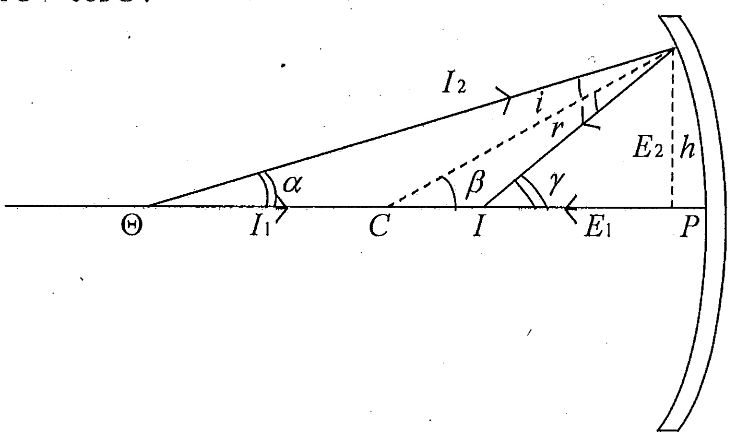



B. CONVEX CENTER OF SPHERE BEHIND MIRROR




A Images in Concave Mirror

We will consider only paraxial rays, that is all angles are taken to be very small so $\sin \approx \tan\Theta \approx \cos\Theta$.



Same construct. $I_1, I_2$ start from 0, after reflection we get $E_1, E_2$, intersection locate $I$.

Equations
$$\beta = \alpha + i$$
$$\gamma = \beta + r$$
$$i = r$$

Hence
$$\gamma = \beta + i = 2\beta - \alpha$$

Or
$$\gamma + \alpha = 2\beta$$

Angles are small, hence
$$\tan\gamma + \tan\alpha = 2\tan\beta$$
$$\frac{h}{IP} + \frac{h}{OP} = \frac{2h}{CP}$$

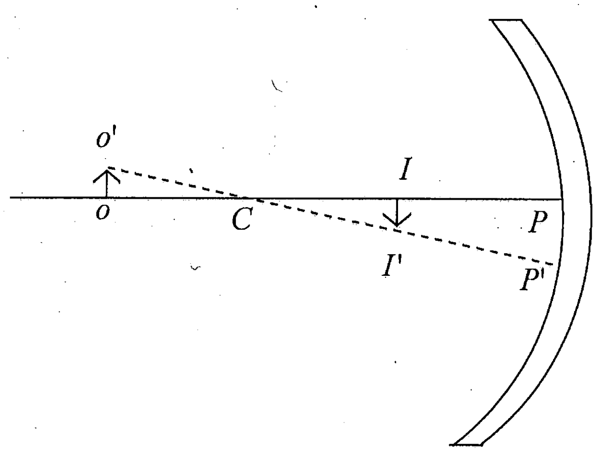
Or
$$\frac{1}{p} + \frac{1}{q} = \frac{2}{R} \qquad (1)$$

And combine with
$$m = \frac{-q}{p} \qquad (2)$$

These two equations describe all possible images, and their sizes, formed by a concave mirror.

OBJECT SIZE

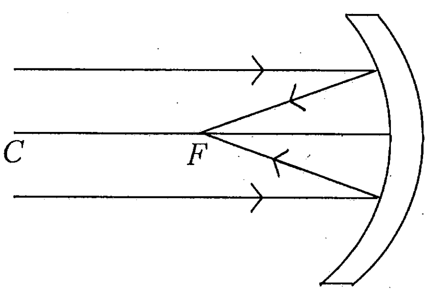
i) Small Object
Rotate picture about the center (of curvature) $C$. The above calculations apply. Our assumption of paraxial rays requires that all objects are small.




SPECIAL CASES [Eqs. (1) and (2)]

I $\quad p \to \infty, \; q = \frac{R}{2}$

$p \to \infty$ implies that incident light is a parallel beam, and we learn that when a parallel beam falls on the mirror, after



reflection it converges to a point. This defines the focus and the focal length

$$f = \frac{R}{2}$$

So $\quad p \to \infty, \; q \to f \quad m \to 0 \quad$ Real Image

Concave mirror makes parallel light converge to a point hence

Concave mirror = CONVERGENT MIRROR

NOTE NO REAL IMAGE CAN COME CLOSER TO THAN $f$!

II $\quad p > R$ object lies beyond $C$.

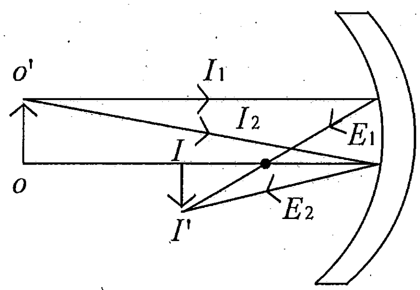
$$\frac{1}{q} = \frac{2}{R} - \frac{1}{p} = \frac{2p - R}{pR}$$

$$\frac{p}{q} = \frac{2p - R}{R} = \frac{> 2R - R}{R} > (2-1) > 1$$

So $\quad q < R$

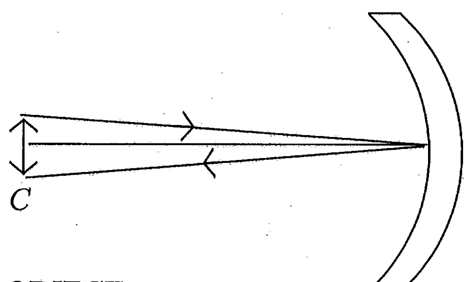
$$m = \frac{-q}{p}, \quad |m| < 1$$

Inverted, Real, Reduced Image




III $\quad p = R, \; q = R, \; m = -1$ [Lamp Expt.]

Object lies at $C$. All light falls on mirror at $i = 0$, so it leaves at $r = 0$ and goes right back to $C$.



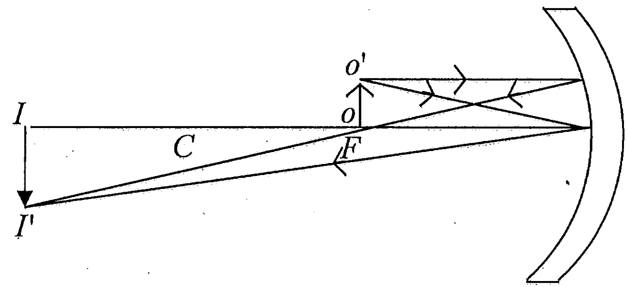

REAL, INVERTED IMAGE SAME SIZE AS OBJECT.

IV $p < R$: Object lies between C and F

$$\frac{p}{q} = \frac{2p - R}{R} = \frac{<(2R - R)}{R} =< 1$$

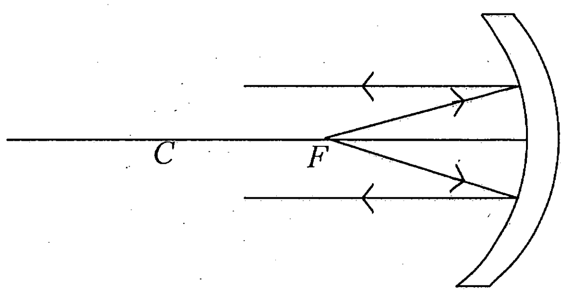
$q > R$, $m = \frac{-q}{p}$, $|m| > 1$

Invested, Real, Enlarged Image




V $p = f$, object at $F, q \to \infty$, $m \to \infty$

Point source at $F$ on reflection produces a parallel beam.



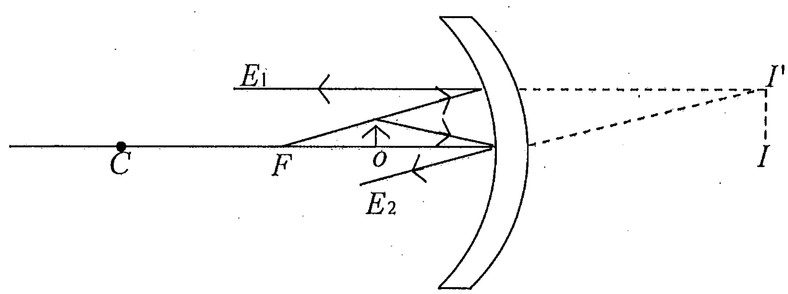

VI $p < f$, object closer to mirror than $F$,

Since

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$

This equation cannot be satisfied unless $q$ becomes <u>negative</u>: IMAGE IS BEHIND MIRROR, ON THE DARK SIDE.

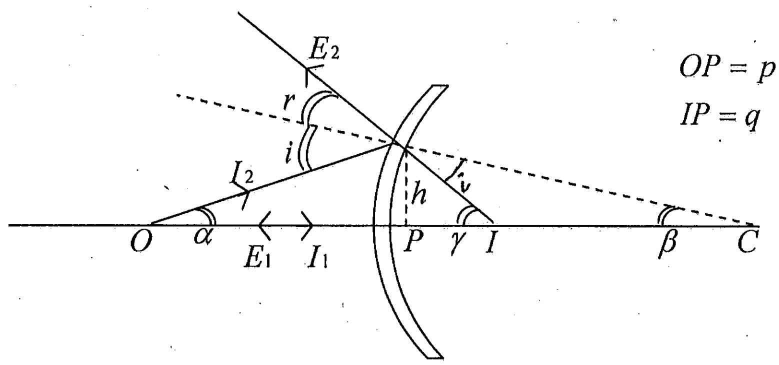
We get a VIRTUAL, ENLARGED IMAGE



<u>To Summarize</u>: Start $O$ far away, $I$ at $f$ (closest to mirror), bring $O$ closer, $I$ moves away from mirror, gets bigger but remains inverted real, $O$ at $C$, $I$ at $C$, $m = -1$, $O$ between $C$ and $F$, $I$ beyond $C$, enlarged, inverted, real, $O$ at $F$, $I \to \infty$, Parallel light, $O$ closer than $F$, $I$ goes <u>BEHIND</u> mirror becomes virtual and upright.

## B Images in Convex Mirror

<u>R is Negative</u>




$OP = p$

$IP = q$ [$q$ is negative]

<u>Equations</u>

$$\gamma = i + \beta$$
$$i = \alpha + \beta$$
$$\gamma = \alpha + 2\beta$$
$$\alpha - \gamma = -2\beta$$
$$\tan\alpha - \tan\gamma = -2\tan\beta$$
$$\frac{h}{p} - \frac{h}{q} = \frac{-2h}{R}$$

Or

$$\frac{1}{p} - \frac{1}{q} = \frac{-2}{R}$$

However, both $q$ and $R$ are negative so one should write

$$\frac{1}{p} + \frac{1}{q} = \frac{2}{R} \qquad (3)$$

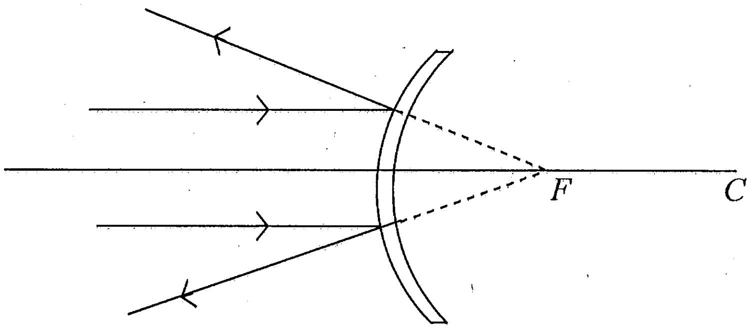
and supplement

$$m = \frac{-q}{p} \qquad (4)$$

# SPECIAL CASES

I $p \to \infty$, parallel light falls on mirror

$q = \frac{R}{2} = f$ but $f$ is negative, $f$ is behind mirror on dark side




A parallel beam of light incident on a convex mirror, on reflection appears to start from F and becomes divergent.
Hence

CONVEX MIRROR ⇒ DIVERGENT MIRROR

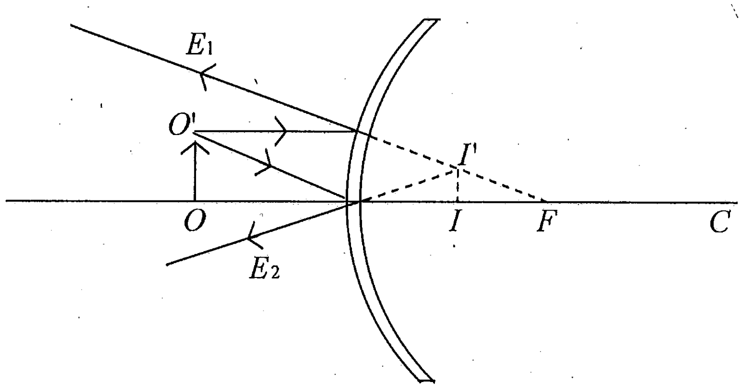
II Since $q$ is negative all images are VIRTUAL and upright

Also $$\frac{1}{p} - \frac{1}{q} = \frac{-2}{R}$$

Or $$\frac{1}{p} = \frac{1}{q} + \frac{2}{R} \qquad \left[\frac{1}{q} > \frac{1}{p}\right]$$

So $q$ is always less than $p$, and always less than $f$.

All images are virtual, upright and reduced.




Right rear view mirror of automobiles!!