FORMATION OF IMAGES – THIN LENSES

This case also involves refraction so we have the same sign convention:

distances measured along the path of light: positive

distances measured against the path of light: negative

Lens consists of a transparent material which has two curved surfaces. We will deal with surfaces which are spherical so only two radii are required.

Lens Makers' Formula Focal length f is given by

$$\frac{1}{f} = (n-1)\left[\frac{1}{R_F} - \frac{1}{R_B}\right]$$

Here $n =$ refractive index of material which is placed in air ($n = 1$)

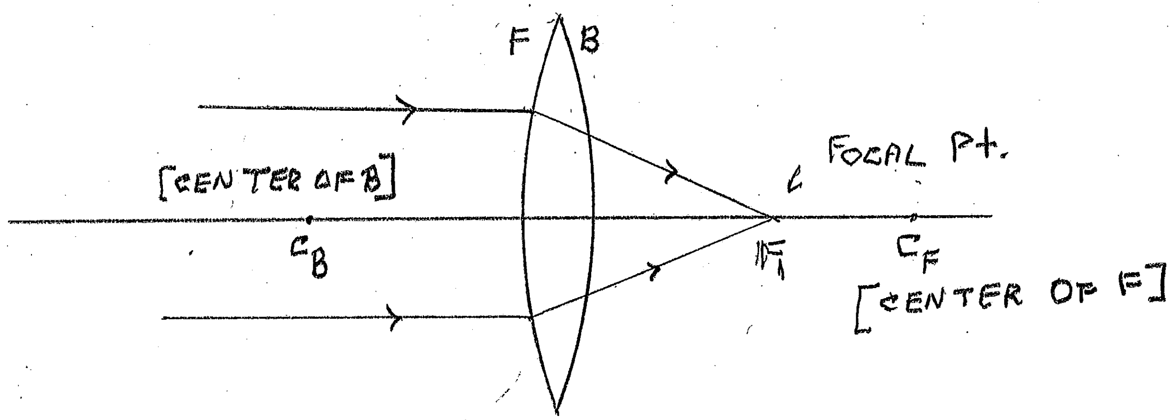
$R_F =$ radius of front surface (facing the incident light)

$R_B =$ radius of back surface

Thin Lenses the thickness of the lens is much smaller the R_F and R_B.

Two Cases Arise

I Convergent Lens



In this case R_F is positive

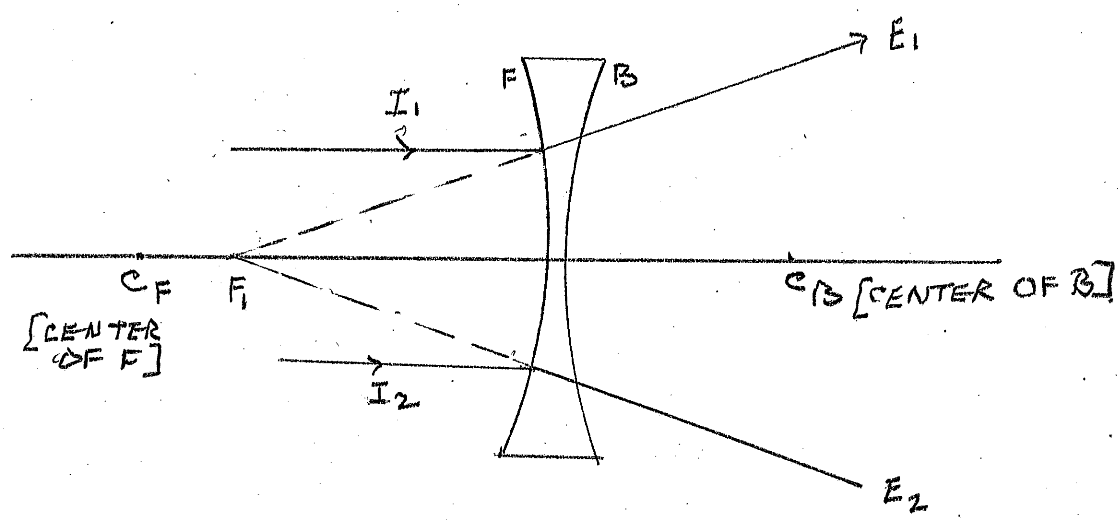
R_B is negative

$$\frac{1}{f} = (n-1)\left[\frac{1}{R_F} - \frac{1}{R_B}\right] \text{ is positive}$$

Because f is positive focal point is to the right of the lens hence parallel light falling on the lens will be made to converge to a point, as shown above.

Light actually goes though the focal point, you can project it on a screen.

II Divergent Lens



Here R_F is negative

R_B is positive

$$\frac{1}{f} = (n-1)\left[\frac{1}{R_F} - \frac{1}{R_B}\right] \text{ is negative}$$

Because f is negative focal point is to the left of the lens hence parallel light falling on the lens will appear to diverge from a point, as shown above.

Notice, no light actually goes through this focal point; it is a VIRTUAL point (negative distance). You CANNOT project it on a screen.

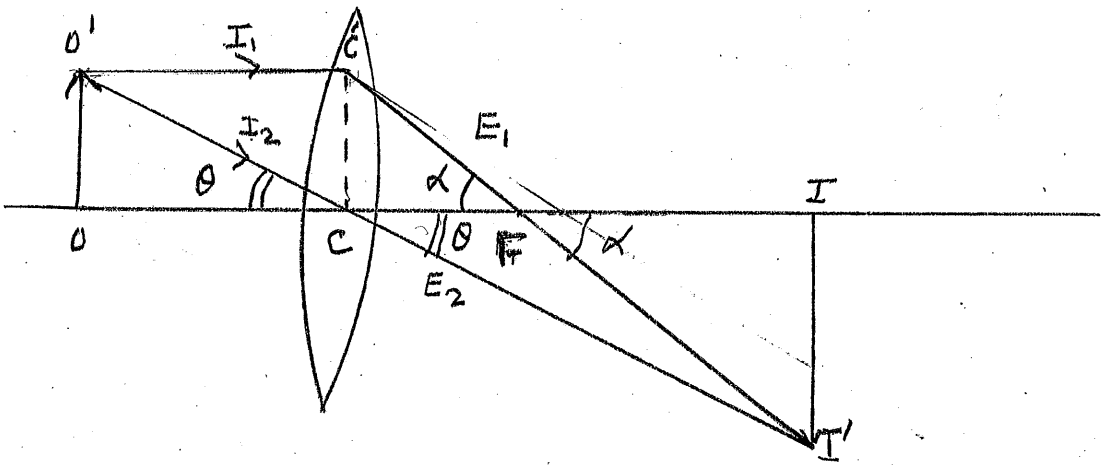
IMAGE FORMATION

Convergent Lens

$OC = p$ object distance

$IC = q$ image distance

$cf =$ focal length



Note: All angles are supposed to be small and thickness is small that is why lateral shift is negligible and the ray though C goes straight.

First look at magnification $m = \frac{-q}{p}$

$$\frac{II'}{OO'} = \left|\frac{q}{p}\right|$$

and indeed II' is inverted, m is negative!

Next from angles α

$$\frac{II'}{IF} = \frac{CC'}{CF}$$

$$\frac{II'}{CC'} = \frac{IF}{CF} = \frac{q-f}{f}$$

But $CC' = OO'$

So

$$\frac{II'}{OO'} = \frac{q-f}{f} = \frac{q}{p}$$

$$\frac{q}{f} - 1 = \frac{q}{p}$$

$$\frac{q}{f} = 1 + \frac{q}{p}$$

With

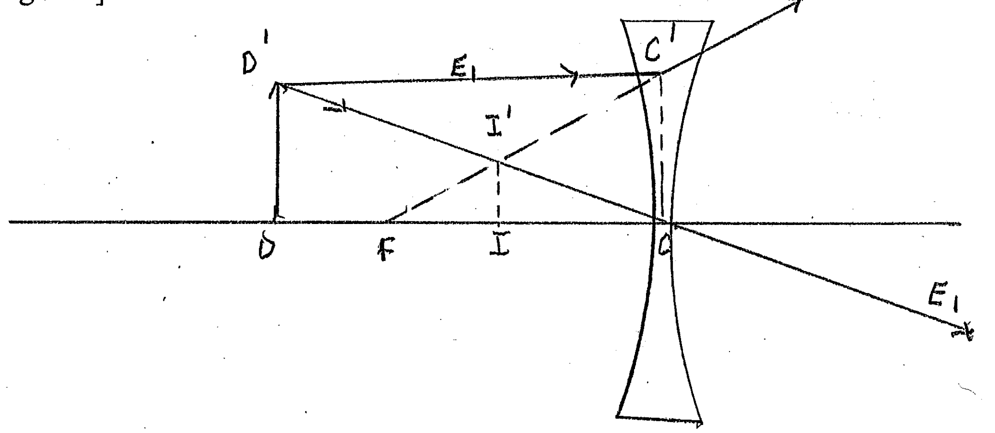
$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$$
$$m=\frac{-q}{p}$$

and these two equations describe all possible images formed by the lens. We can recall all the cases which arose in convergent mirror case –

i)	$p \to \infty$	$q \to f$	$m = 0$
ii)	$p > 2f$	$q < 2f$	m negative and less than 1
iii)	$p = 2f$	$p = 2f$	m negative and equal to 1
iv)	$p < 2f$	$p > 2f$	m negative and greater than 1
v)	$p < f$	q negative, virtual image	m positive

DIVERGENT LENS

$OC = p$
$IC = q$ [negative]
$FC = f$ [negative]
$CC' = OO'$



In this case ALL IMAGES ARE VIRTUAL UPRIGHT and REDUCED ($m < 1$). [like in convex mirror]

Now

$$\frac{CC'}{II'} = \frac{f}{f-q} = \frac{p}{q}$$

So

$$\frac{f-q}{f} = \frac{q}{p}$$

$$1 - \frac{q}{f} = \frac{q}{p}$$

$$\frac{q}{p} - 1 = \frac{-q}{p}$$

$$\frac{1}{p} - \frac{1}{q} = -\frac{1}{f}$$

But both f and q are negative, hence again

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f} \qquad \text{with} \qquad m = \frac{-q}{p}$$