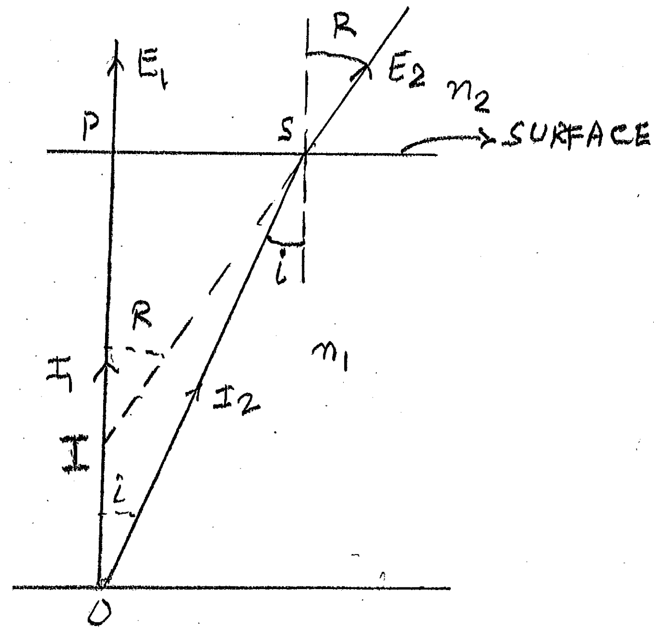
FORMATION OF IMAGES – REFRACTION AT A SINGLE SURFACE
[SIGN CONVENTION: ALONG LIGHT POSITIVE, AGAINST LIGHT NEGATIVE]

I Apparent Depth of Water in a Pool



Suppose you are standing at the edge of a swimming pool and look straight down. If the actual depth of water is *d* meters what value do you perceive? We can solve this problem by putting a point object *O* at the bottom and locate its image formed by the water as the light refracts [optical system so all distances are measured from *P*] through its surface.

Look at the picture. Take two rays starting from *O*:

I_1 makes angle of incidence zero and gives rise to E_1

I_2 makes angle of incidence i and causes E_2 satisfying

$$n_2 \sin R = n_1 \sin i$$

Since you are looking straight down all angles are small.

The virtual Image at

I [*q* is negative]

is located by intersection of E_1 and E_2 (extended backwards).

Next, from the picture we see

$$\tan R = \frac{SP}{IP} \quad (1)$$

$$\tan i = \frac{SP}{OP} \quad (2)$$

Divide Eq.(2) by Eq.(1)

$$\frac{IP}{OP} = \frac{\tan i}{\tan R}$$

$$\cong \frac{\sin i}{\sin R} \quad \begin{bmatrix} i << 1 \\ R << 1 \end{bmatrix}$$

$$= \frac{n_2}{n_1}$$

Clearly IP = apparent depth

OP = real depth

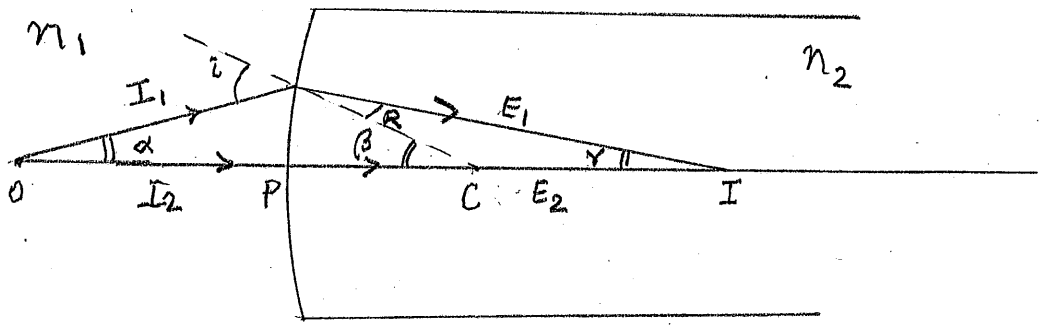
$$\frac{d_{app}}{d} = \frac{n_2}{n_1}$$

For water $n = 1.33$
For air $n = 1$

So $$\frac{d_{app}}{d} = \frac{3}{4}$$

So if water is 80cm deep, to a person at the edge it will appear to be only 60cm [small children should be warned before they jump in and suddenly find that they are too short].

II Single Convex Surface (positive r)



Here OP = object distance (p)
IP = image distance (q)
CP = radius of curved surface (r)

And all angles are small

Equations are $n_1 \sin i = n_2 \sin R$

Or $n_1 i = n_2 R$ -(1)

$\beta = \gamma + R$ -(2)

$i = \alpha + \beta$ -(3)

From Eq.(2)

$$\beta = \gamma + \frac{n_1}{n_2} i$$

$$= \gamma + \frac{n_1}{n_2}(\beta + \alpha)$$

$$\beta\left[1 - \frac{n_1}{n_2}\right] = \gamma + \frac{n_1}{n_2}\alpha$$

Or

$$(n_2 - n_1)\beta = n_2\gamma + n_1\alpha$$

$$(n_2 - n_1)\tan\beta = n_2 \tan\gamma + n_1 \tan\alpha$$

$$\boxed{\frac{(n_2 - n_1)}{r} = \frac{n_2}{q} + \frac{n_1}{P}} \qquad (4)$$

In order to access the magnification for a small object, imagine rotating *OPI* through a small angle you will get

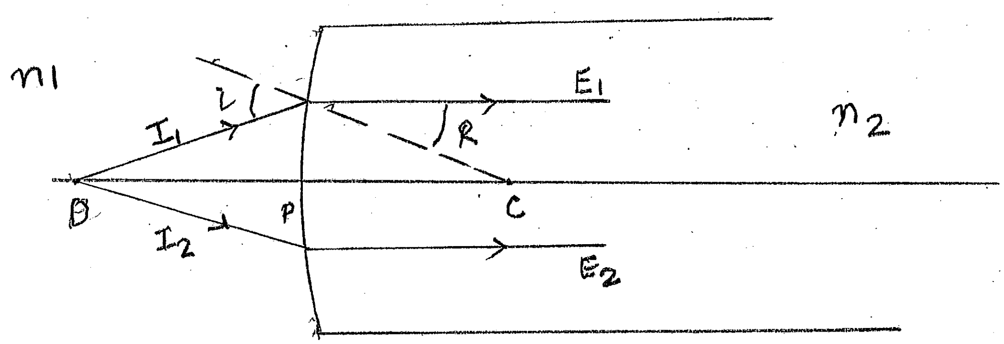
$$\tan i = \frac{OO'}{p}$$

$$\tan R = \frac{II'}{q} \qquad II' \text{ is negative!}$$

$$m = \frac{II'}{OO'} = \frac{-q \tan R}{p \tan i} = -\frac{n_2}{n_1}\frac{q}{p} \qquad (5)$$

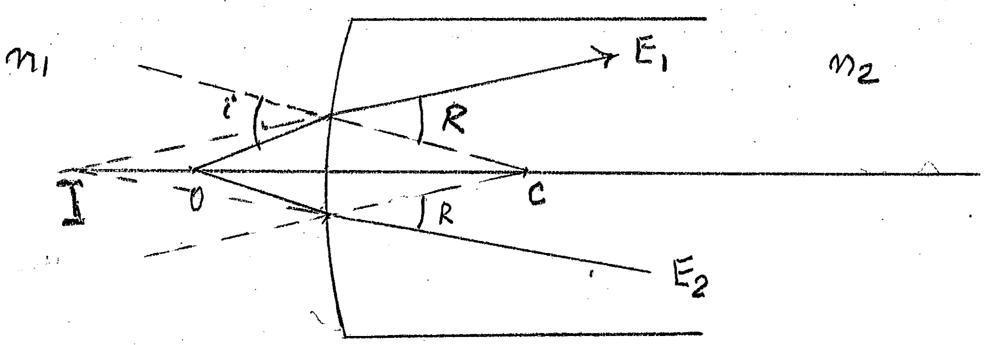
Special Cases

A p is such that $q \to \infty$, that is light becomes a parallel beam on entering the surface.



$$p = \frac{n_1 r}{n_2 - n_1} \qquad (6)$$

B If p becomes even smaller than that given by Eq.(6), E_1 and E_2 will diverge the image will switch to the left of the surface and become virtual.



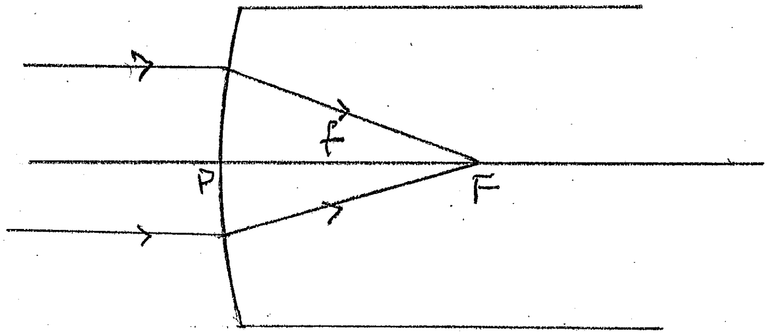
C Please note that if you set $r \to \infty$ in Eq.(4) you will recover the result of case I

$$\frac{q}{p} = -\frac{n_2}{n_1}$$

D If $p \to \infty$, INCIDENT BEAM is parallel

$$\frac{n_2}{f} = \frac{n_2 - n_1}{r}$$

where f is the focal length.



III Single Concave Surface (negative r)

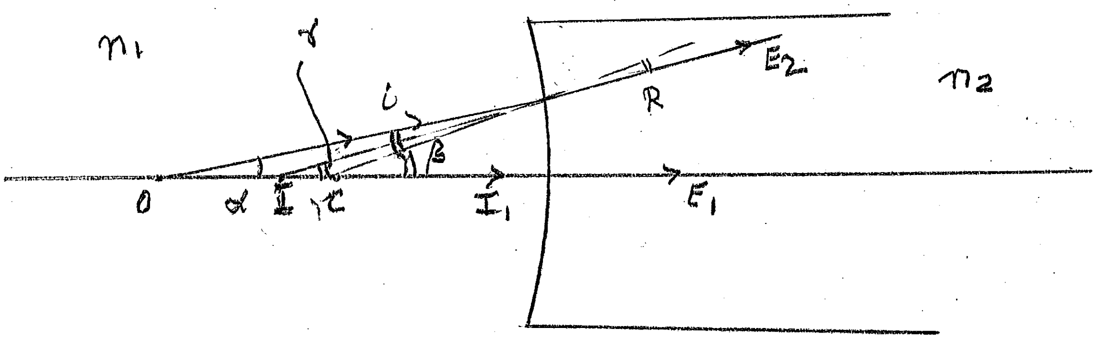


IMAGE IS ALWAYS VIRTUAL (q is negative)

Equations are $n_1 \sin i = n_2 \sin R$

Or $n_1 i = n_2 R$ -(1) Small Angles

$\beta = \alpha + i$ -(2)

$\beta = \gamma + R$ -(3)

$$\beta = \gamma + \frac{n_1}{n_2} i = \gamma + \frac{n_1}{n_2}(\beta - \alpha)$$

$$\tan\beta = \tan\gamma + \frac{n_1}{n_2}(\tan\beta - \tan\alpha)$$

$$\frac{1}{r}\left[1 - \frac{n_1}{n_2}\right] = \frac{1}{q} - \frac{n_1}{p}$$

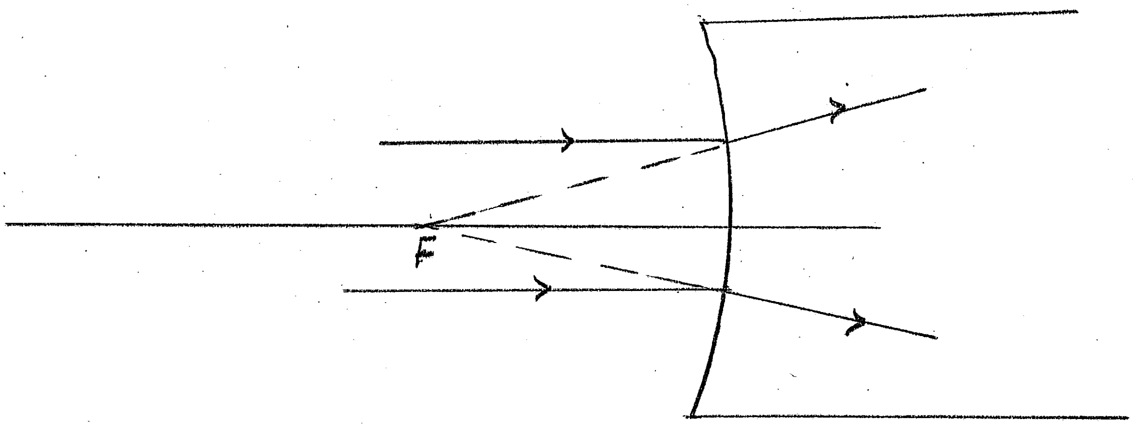
$$\frac{n_1}{p} - \frac{n_2}{q} = -\frac{1}{r}(n_2 - n_1)$$

But q and r are negative; Hence again

$$\boxed{\frac{n_2}{q} + \frac{n_1}{P} = \frac{n_2 - n_1}{r}}$$

q negative
r negative

Special Case $p \to \infty$ $q \to f$



f is negative

$$\frac{n_2}{f} = \frac{n_2 - n_1}{r}$$