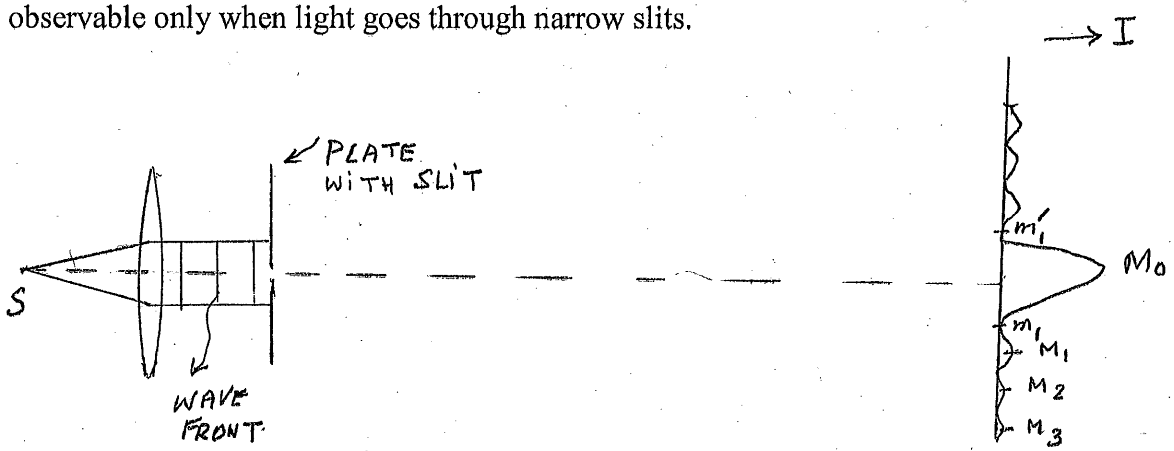
Diffraction – Single Slit

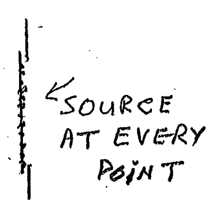
Diffraction arises because of superposition of a very large number of waves. Experimentally, it manifests itself by the spreading of a wave when it passes through an opening whose size is comparable to the wavelength that is why sound exhibits diffraction when it goes through doors and windows while diffraction of light is observable only when light goes through narrow slits.



EXPT: S is a point source of light of wavelength λ. It is placed at the focal point of a lens so after passing through the lens we get a parallel beam which is pictured as a plane wave front travelling to the right. We place a plate with a narrow slit of width w and let the light fall on a screen a distance D away. What you observe is a series of maxima M_0, M_1, M_2… where the central one (M_0) is brightest and the intensity reduces rapidly as you go from $M_0 \rightarrow M_1 \rightarrow M_2 \rightarrow M_3 \rightarrow$… Alternately m_1, m_2, m_3 locate the "dark" spots in between.

In the laboratory upstairs you use a laser as a light source as that produces a parallel beam and hence plane wave fronts.

Our challenge is to construct a simple model which will allow us to understand the observations. We begin by recalling Haygen's Construct that every point of a wave front is a source. Thus, it is quite reasonable to claim that the part of the wave front exposed by the slit gives rise to a large number (say N>>1) of waves all of which start in step (in phase) from the wave front. So now we must try to explain how N waves arriving at the screen conspire to produce the intensity pattern observed by us.



CENTRAL MAXIMUM (M_0): All of the waves arrive at the screen in phase because maximum path difference is $\frac{w^2}{4D}$ which is much smaller than λ for $w = 10^{-4}\,m$ and $D = 1m$. So if each one contributes the amplitude E_m, total amplitude at M_0 would be given by vector addition.

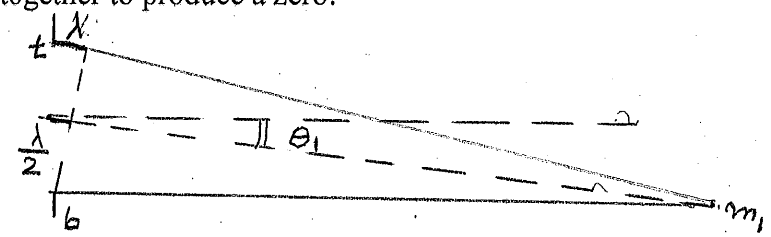
$$\rightarrow \rightarrow \rightarrow \rightarrow \ldots\ldots \rightarrow = NE_m$$

E_m

So intensity of M_0

$$I_0 \ \alpha \ N^2 E_m^{\ 2}$$ [Again constants ½ $\varepsilon_0 C$ omitted]

FIRST MINIMUM (m_1): Here total intensity is zero. So we want all N waves to add together to produce a zero:



(tm$_1$-bm$_1$)= λ

This will happen if the path difference between the wave coming from t and that coming from b is exactly λ. Then the slit can be split into two parts: for every wave coming from the lower half there will be one coming from the upper half which is $\frac{\lambda}{2}$ behind and

hence they cancel each other.

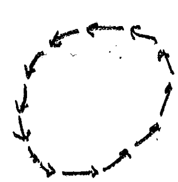
The angle ϑ_1 which locates m_1 is therefore given by

$$\sin \vartheta_1 = \lambda / w$$

Thus the central maximum, which is bounded by m_1 and m_1' will have a width of $2\vartheta_1$. The smaller the w the larger the spread due to diffraction.

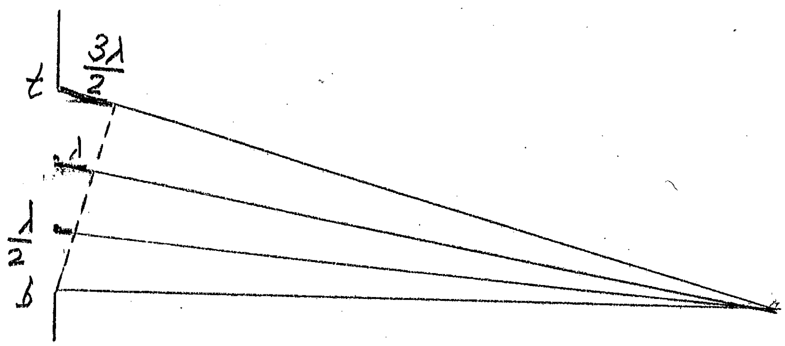
In terms of the E_m vectors m_1 happens because:

The string of length NE_m has been wound around so it closes on itself.



$\sum E_m = 0$

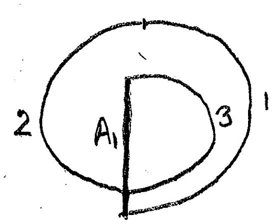
FIRST MAXIMUM M_1



$$(tM_1 - bM_1) = \frac{3\lambda}{2}$$

Now the waves arrange themselves so that the path difference between the wave from t and that from b is $\frac{3\lambda}{2}$. Effectively the slit splits into 3 equal parts, two of which cancel one another so that only $\frac{1}{3}rd$ of the sources contribute $\frac{1}{3}rd$ to the amplitude at M_1.

To calculate the amplitude at M_1 let us wind our string of length NE_m some more until it looks like.



The sum of all the vectors is A_1 and

$$\frac{3\pi}{2} A_1 = N E_m$$

Hence amplitude at M_1

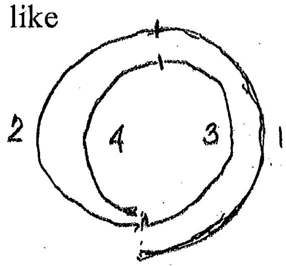
$$A_1 = \frac{2}{3\pi} NE_m$$

$$I_1 \; \alpha \; \frac{4}{9\pi^2} N^2 E_m^{\,2}$$

$$I_1 / I_0 = \frac{4}{9\pi^2}$$

M_1 is barely 1/20th as intense as M_0.

SECOND MINIMUM (m_2): This requires us to wind the string even more so it looks like



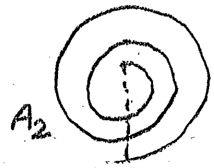
$$\Sigma E_m = 0$$

We need $t\, m_2 - b\, m_2 = 2\lambda$

The slit splits into 4 equal parts each quarter cancels its neighbor.

SECOND MAXIMUM M_2:

Continue winding further



$$\frac{5\pi}{2} A_2 = NE_m$$

$$A_2 = \frac{2}{5\pi} NE_m$$

$$\frac{I_2}{I_0} = \frac{4}{25\pi^2}$$

I_2 is nearly 62 times smaller than I_0. Subsequent minima/maxima follow from the above discussion.

TWO-SLIT EXPT: INTERFERENCE + DIFFRACTION

In discussing the two slit interference previously we assumed that $w \ll d$ so that the central maximum for diffraction became much broader than the width of the interference fringes and that allowed us to discuss the interference effect alone. In practice w and d can be quite comparable and what you observe is a diffraction-cum- interference pattern: diffraction maxima with interference fringes inside them.

Shown below are intensity patterns for the case $d = 3w$