DEVICES FOR NON - COULOMB $\underset{\rightarrow}{E}$

Inductor is a device which stores a $\underset{\rightarrow}{B}$-field. Consider the following: In such a device

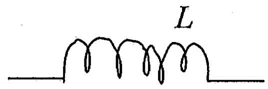
If i then B

$$\text{If } \frac{\Delta i}{\Delta t} \text{ then } \frac{\Delta B}{\Delta t} \text{ then } \frac{\Delta \Phi_B}{\Delta t} \text{ then } -\varepsilon$$

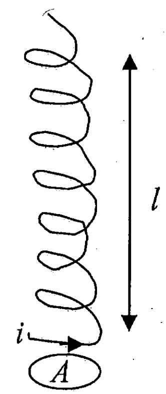
We define the

$$\text{Inductance } L = \frac{-\varepsilon}{\frac{\Delta i}{\Delta t}} \qquad \text{[effect/cause]}$$

Unit of L is $\frac{Volt - \sec}{Amp}$ and is called Henry. And the symbol is



Self Inductance $\frac{\Delta i}{\Delta t}$ and ε in the same coil. A tightly wound solenoid of area A and N turns spread over a length l. If the current is i



$$\underset{\rightarrow}{B} = \mu_0 n i \hat{y} \qquad \left[n = \frac{N}{l} \right]$$

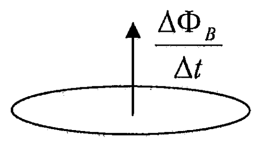
Parallel to the axis of the solenoid and essentially constant inside it with I varying

$$\frac{\Delta B}{\Delta t} = \mu_0 n \frac{\Delta i}{\Delta t}$$

and

$$\frac{\Delta \Phi_B}{\Delta t} = \mu_0 n A \frac{\Delta i}{\Delta t}$$

In one ring of the solenoid this will cause *emf*



$$\varepsilon_1 = -\mu_0 n A \frac{\Delta i}{\Delta t}$$

since there are N rings *emf*, for the entire solenoid

$$\varepsilon_N = -\mu_0 n N A \frac{\Delta i}{\Delta t}$$

So self inductance

$$L = \frac{-\varepsilon_N}{\frac{\Delta i}{\Delta t}} = \mu_0 n N A = \mu_0 n^2 A l = \mu_0 n^2 V$$

$V = Al$ being the volume of the solenoid

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Mutual Inductance $\frac{\Delta i}{\Delta t}$ in one coil, induces an *emf* in a second coil because the time varying flux of $\underset{\rightarrow}{B}$ also occurs inside the second coil. So now we have two coils each of length l one has n_1

turns per meter and the other has n_2 turns per meter and the second coil is wound on top of the first so both have the same cross-section A.

Once again due to current change in coil 1

$$\frac{\Delta\Phi_B}{\Delta t} = \mu_0 n_1 A \frac{\Delta i}{\Delta t}$$

Induced *emf* in one ring of coil 2

$$\varepsilon_{1n_1} = -\mu_0 n_1 A \frac{\Delta i}{\Delta t}$$

Induced *emf* in entire 2nd coil

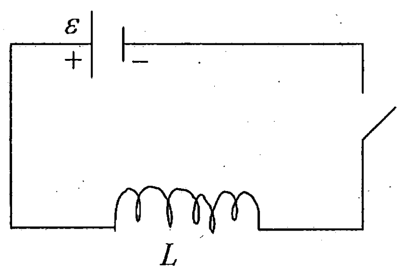
$$\varepsilon_{N_2 n_1} = -\mu_0 n_1 n_2 l A \frac{\Delta i}{\Delta t}$$

Mutual Inductance

$$M_{21} = \frac{-\varepsilon_{N_2 n_1}}{\frac{\Delta i}{\Delta t}} = \mu_0 n_1 n_2 V = \sqrt{L_1 L_2}$$

where L_1 and L_2 are the self-inductances of the two coils.

Next, we calculate the work done in establishing current in an inductor – Energy stored in a $\underset{\rightarrow}{B}$ field: When you close the switch the battery will try to establish a current in the inductor and of course the inductor will object by setting up an *emf*



$$\varepsilon_L = -L\frac{\Delta i}{\Delta t}$$

when the next Δq of charge makes its way through the inductor the work done will be

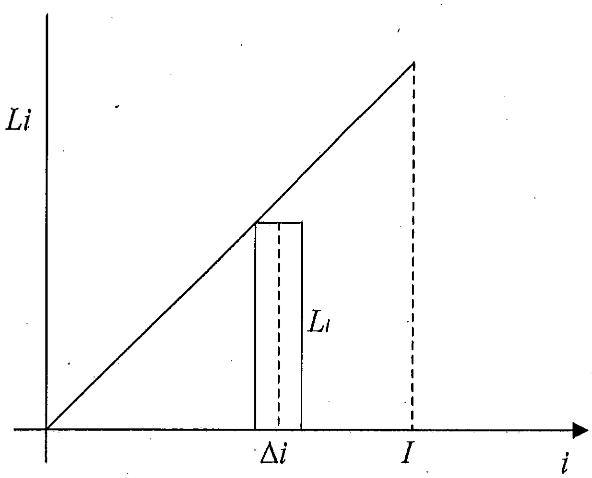
$$\Delta W = \Delta q L \frac{\Delta i}{\Delta t} = L i \Delta i$$

since $\frac{\Delta q}{\Delta t} = i$

To calculate work needed to increase current from 0 to I, plot Li as a function of i and we note that $Li\Delta i$ is the area of the rectangle. The area of the triangle tells us how much work is needed to establish I in L.

$$W_B = \frac{1}{2} L I^2$$

Where did all the energy go? Notice that once the current is established the inductor is not empty, there is a $\underset{\rightarrow}{B}$-field inside it and W_B is stored in this $\underset{\rightarrow}{B}$-field. If you apply it to a solenoid



$$L = \mu_0 n^2 V$$
$$B = \mu_0 n I$$
$$W_B = \frac{1}{2}\mu_0 n^2 I^2 V$$

So energy density of $\underset{\rightarrow}{B}$-field

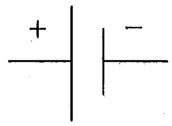
$$\eta_B = \frac{B^2}{2\mu_0}$$

Is the energy stored in 1 m^3 of $\underset{\rightarrow}{B}$-field.

Devices L-R Circuits

We have three devices:

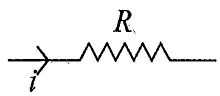
A Battery: Creates a Coulomb $\underset{\rightarrow}{E}$ using chemical energy



and output is the *emf* ε.

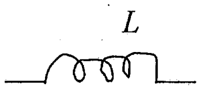
A Resistor (R): arises because it costs energy to transport charge through a conductor so that if you establish a current *i*, a voltage drop *V* appears across *R* such that

$$V = iR$$

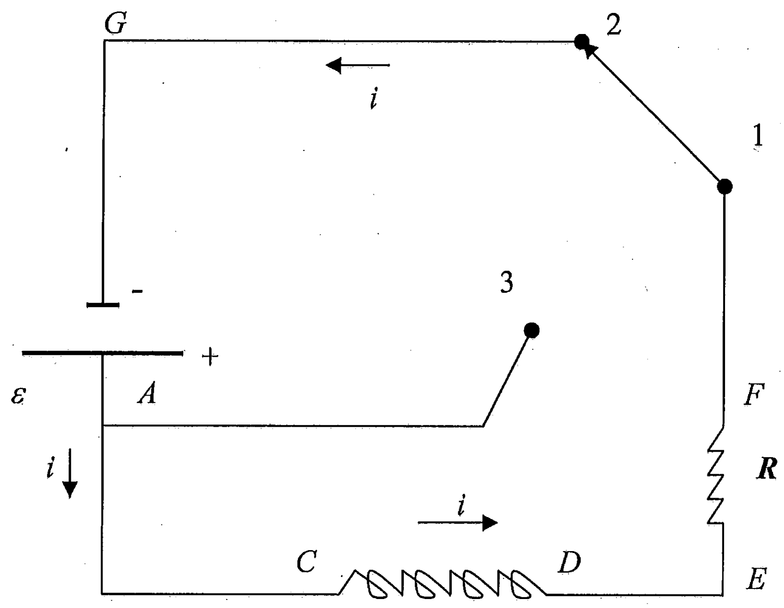


An Inductor (L): opposes any time variation of current through it by setting up an *emf*

$$\varepsilon = -L\frac{\Delta i}{\Delta t}$$



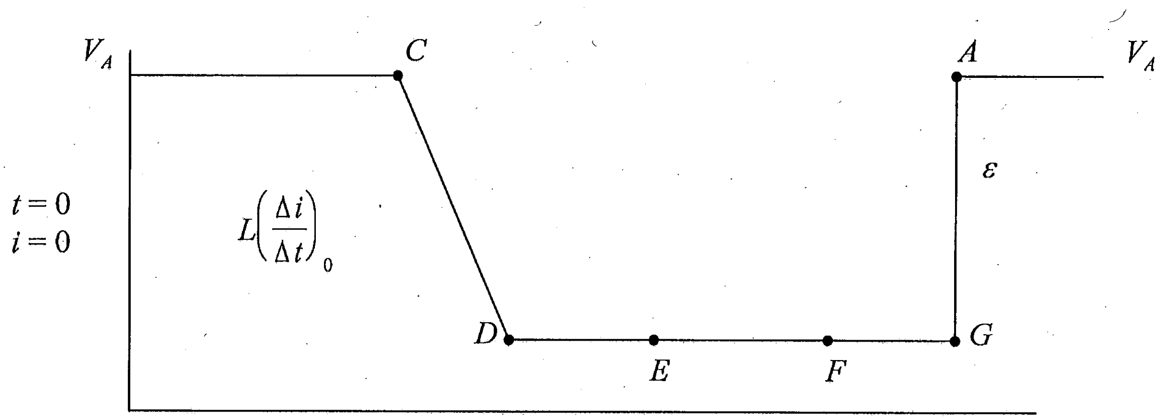
Now we want to study what happens when we connect our 3-devices, Battery, *R* and *L* in the circuit below.
At t=0, put the switch as shown (1→ 2). The battery immediately wants to set up a current.
However, *L* promptly objects* by setting up a "back" *emf*, which will oppose ε, $\varepsilon_B = -L\left(\frac{\Delta i}{\Delta t}\right)_0$



* The "minus" sign in the equation ensures that an inductor always opposes any change in current through it.

Thus it will take time for the current to grow.

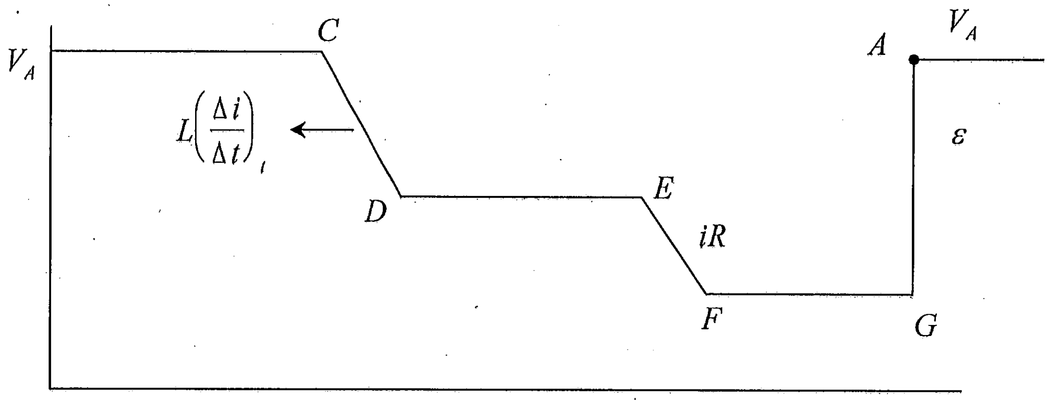
Step 1 at t=0, i=0, No potential drop across R. As a function of position on the circuit the potential values are



$$\varepsilon - L\left(\frac{\Delta i}{\Delta t}\right)_0 = 0$$

Step 2 A little time later i has grown some so now $v_R = iR$ and the potentials become

A little later

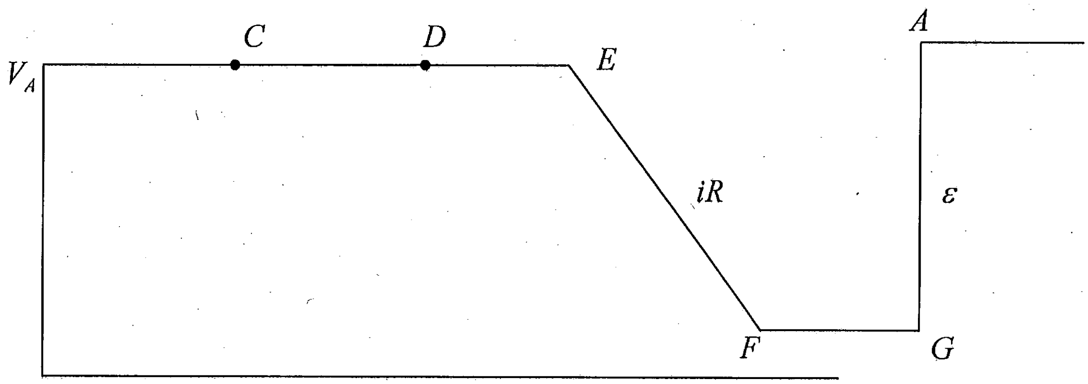


$$\varepsilon - L\left(\frac{\Delta i}{\Delta t}\right)_t = iR$$

Clearly, $\left(\frac{\Delta i}{\Delta t}\right)_t < \left(\frac{\Delta i}{\Delta t}\right)_0$ Rate of growth has slowed down.

Step 3 A long time later i has grown to its full value, i stops changing with time, $\frac{\Delta i}{\Delta t} \to 0$ so no more back *emf* from L. Potential variation looks like.

A long time later
i is constant



$$i(t \to \infty) = \frac{\varepsilon}{R}$$

Note:

$V_L = \varepsilon$ at $t=0$, $i=0$

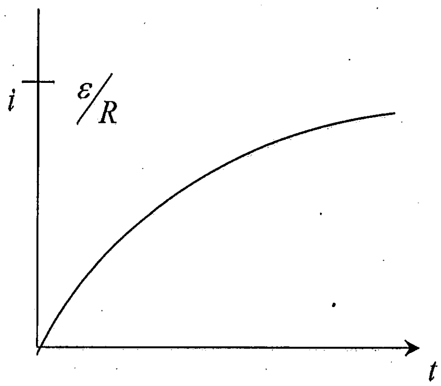
$V_L = 0$ at $t \to \infty$, $i = \frac{\varepsilon}{R}$

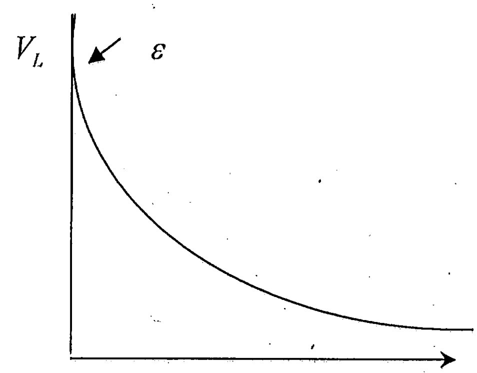
The time variations are controlled by the characteristic time

$$\tau = \frac{L}{R} \text{ (verify that this has dimensions of time)}$$

and look like

Growth of i





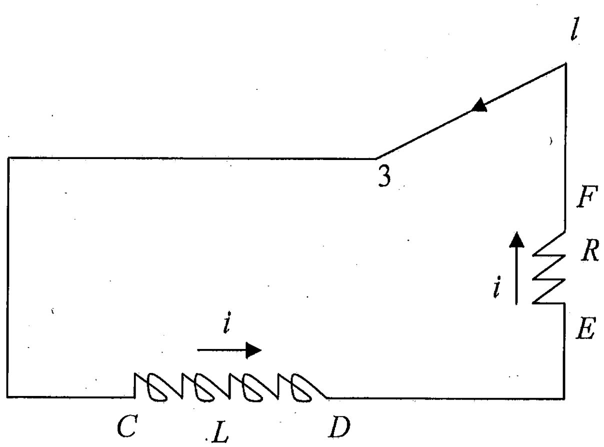
$$i = \frac{\varepsilon}{R}\left[1 - e^{-Rt/L}\right]$$

$$V_R = iR$$

$$= \varepsilon\left[1 - e^{-Rt/L}\right]$$

$$V_L = \varepsilon e^{-Rt/L}$$

After this long time $i = \frac{\varepsilon}{R}$ and a magnetic field has been established in L and stores $\frac{B^2}{2\mu_0}$ of energy per m^3. So now if we move the switch so 1→3 the circuit becomes



No Battery in circuit.
Magnetic Energy stored in L is source of current!

Let us start the clock again:

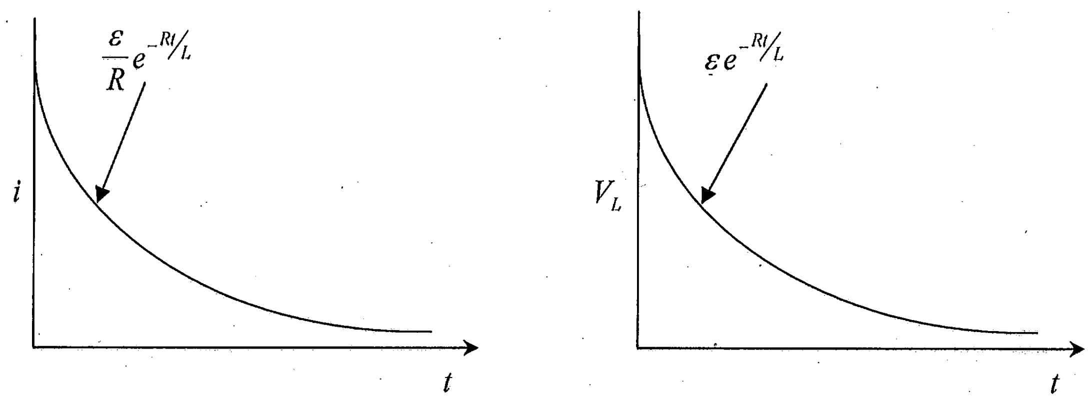
Step 1 t=0, $i = \frac{\varepsilon}{R}$ and the inductor wants to keep i going. Since $\frac{\Delta i}{\Delta t}$ is now –ive, V_L will be +ive and immediately jump to ε.

Step 2 As stored energy in L becomes smaller $\left|\frac{\Delta i}{\Delta t}\right|$ reduces with time.

Step 3 Long time later, all of the stored energy has been used up: $\frac{\Delta i}{\Delta t} \to 0$, $i \to 0$.

The time variations are

Decay of i

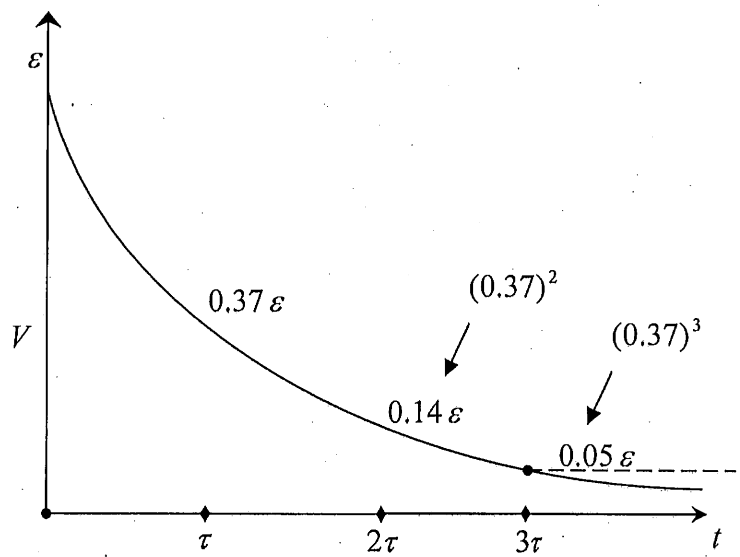


To Summarize:
- i) L keeps $i = 0$ when switch first closed
- ii) $V_L \to 0$ as $t \to \infty$
- iii) in an L-R circuit potential precedes current.

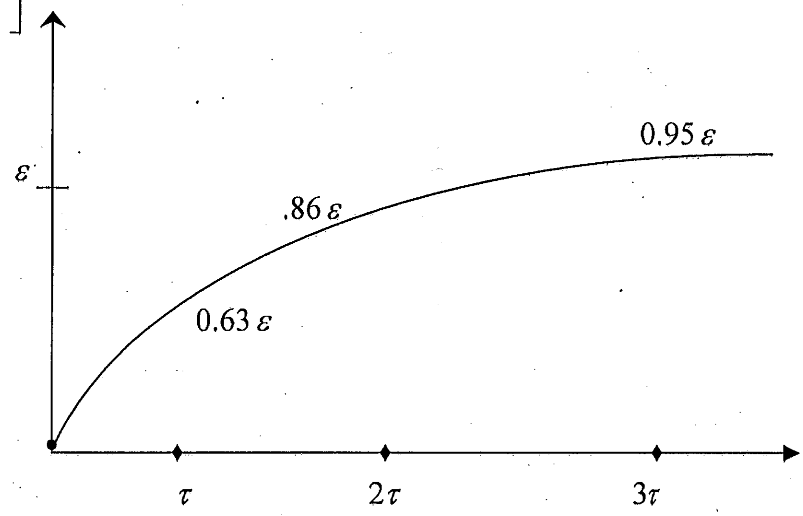
QUESTION: Why does τ depend on both L and R?

Exponential functions.

$V_L = \varepsilon e^{-t/\tau}$

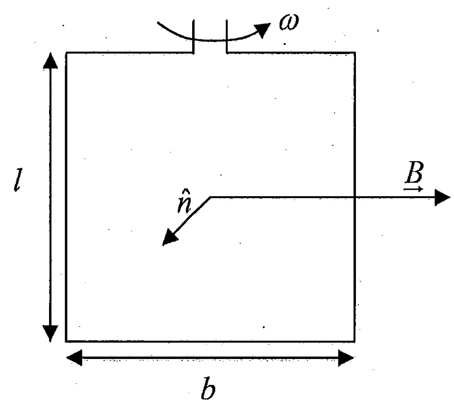


$$V_R = \varepsilon\left[1 - e^{-t/\tau}\right]$$



<u>Devices – AC Generator</u>

Take a coil of area $A(l \times b)$ mount it so that it can rotate freely about the y-axis in the presence of a constant $\underset{\rightarrow}{B} = B\hat{x}$. Rotate the coil about the y-axis at a constant angular velocity $\underset{\rightarrow}{\omega} = \omega\hat{y}$. The flux of the $\underset{\rightarrow}{B}$-field through the coil is



$$\Phi_B = A\hat{n} \bullet \underset{\rightarrow}{B}$$
$$= AB\cos(\hat{n}, \underset{\rightarrow}{B})$$

As the coil rotates the angle between $\hat{n}$ and $\underset{\rightarrow}{B}$ will change continuously according to the equation

$$\vartheta = \vartheta_0 + \omega t$$

and therefore

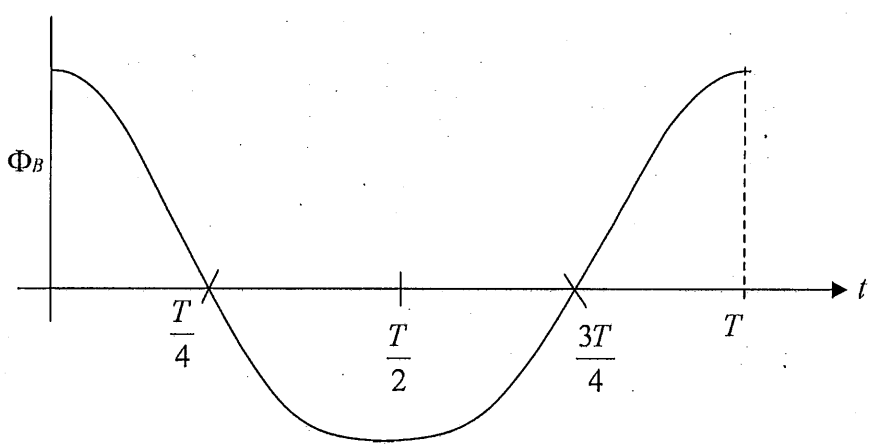
$$\Phi_B = AB\cos(\vartheta_0 + \omega t)$$

Let us make $\vartheta_0 = 0$, that is, make $\hat{n} \parallel \underset{\rightarrow}{B}$ at $t = 0$.

Plot Φ_B as a function of time

$$T = \frac{2\pi}{\omega}$$

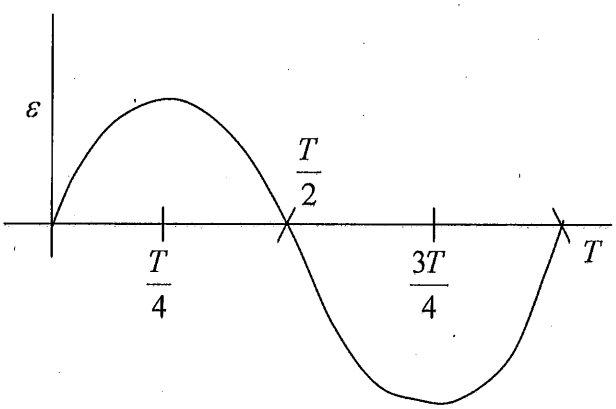
is the period



an εmf will appear in the coil according to

$$\varepsilon = -\frac{\Delta\Phi_B}{\Delta t}$$

which is the negative slope of the Φ_B vs. t curve



and we note

(i) ε is maximum when Φ_B is zero and zero when Φ_B is maximum

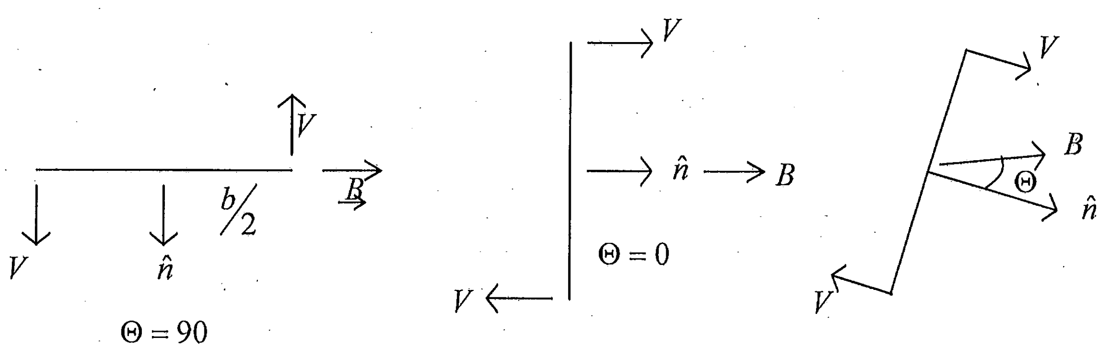
(ii) As T reduces (ω increases) the slope gets larger and therefore it is reasonable to write

$$\varepsilon = AB\omega \sin \omega t$$

Such a rotating coil generates an εmf which reverses every $\frac{1}{2}$ cycle so it is an alternating current generator. This is the principle of all power generation – when you switch on your light, the bulb goes on because you complete a loop in which someone is causing the flux of $\underset{\rightarrow}{B}$ to change by rotating a coil.

It is instructive to analyze the above device using the concept of motional εmf. As the coil rotates the vertical arms have velocities which vary from $v_Z = \frac{b\omega}{2}$ $[\hat{n} \perp \underset{\rightarrow}{B}]$ to zero when $\hat{n} \parallel \underset{\rightarrow}{B}$ so motional εmf goes from $\varepsilon_{\max} = \omega BA$ to zero. At other times the angle between $\underset{\rightarrow}{v}$ and $\underset{\rightarrow}{B}$ is also Θ $[\underset{\rightarrow}{v} \parallel \hat{n}]$ so

$$\varepsilon = \omega BA \sin \omega t$$



which is exactly the answer claimed above by using the Faraday-Lenz equation.