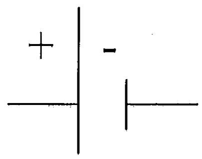
DEVICES BASED ON COULOMB $\underset{\to}{E}$ - FIELD

Battery: Produces an $\underset{\to}{E}$-field using chemical energy. Essentially consists of two metal plates dipped in a chemical slurry called an electrolyte which has mobile positive and negative ions. On account of chemical reactions, the positive ions accumulate on one plate while the negatives congregate on the other, thereby producing a Coulomb $\underset{\to}{E}$-field.



For historical reasons,

The output is called Electromotive force or *EMF* which is not a force at all. *EMF* is defined as the work done by an $\underset{\to}{E}$-field to move a unit charge and is measured in Volts, just like ΔV.

N.B. Inside a battery there are no mobile electrons, it is ions that move. Same is true for motion of charges through cell walls.

Capacitor: A device used to store an $\underset{\to}{E}$-field. Usually consists of two terminals separated by air/vacuum or an insulator. One establishes a potential difference V between the terminals; charges $\pm Q$ appear on them and "trap" an $\underset{\to}{E}$-field inside the device. No charge ever flows inside a capacitor. The "size" of the device is called capacitance and is defined by the equation

$$C = \frac{Q}{V} \qquad \text{[EFFECT / CAUSE]}$$

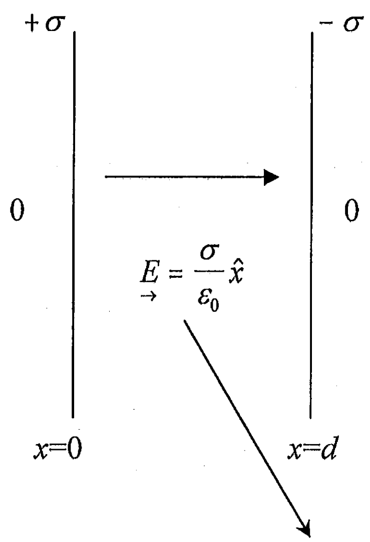
so, of course, the magnitude of C tells us how much charge the device will hold for a given V.

The unit of C is $\frac{Coulomb}{Volt}$ called a Farad (Fd) in honor of Faraday. 1 Fd is a very large unit. Most capacitors are in the range of $10^{-6}\,Fd (\equiv \mu Fd)$.

Examples

1. Parallel Plates of Area A separated by d, each plate has charge density $\sigma = \dfrac{Q}{A}$.

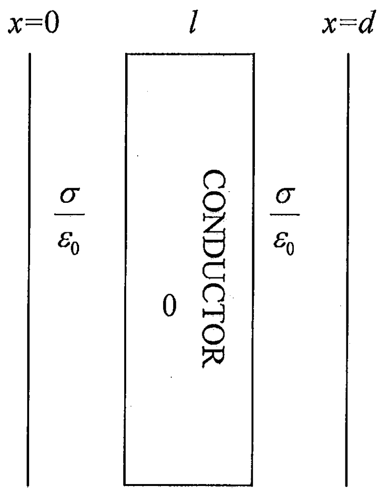
$\underset{\rightarrow}{E}$ -field is
$$\underset{\rightarrow}{E} = \frac{\sigma}{\varepsilon_0}\hat{x} \quad [0 < x < d]$$
$$\underset{\rightarrow}{E} = 0 \quad \textit{at all other } x.$$



air or vacuum between plates

$$\Delta V = -\underset{\rightarrow}{E}\bullet\underset{\rightarrow}{\Delta S} = -\frac{\sigma}{\varepsilon_0}d \quad so,\ V = \left|\frac{\sigma}{\varepsilon_0}d\right|$$

$$C = \frac{Q}{V} = \frac{\varepsilon_0 A}{d}$$



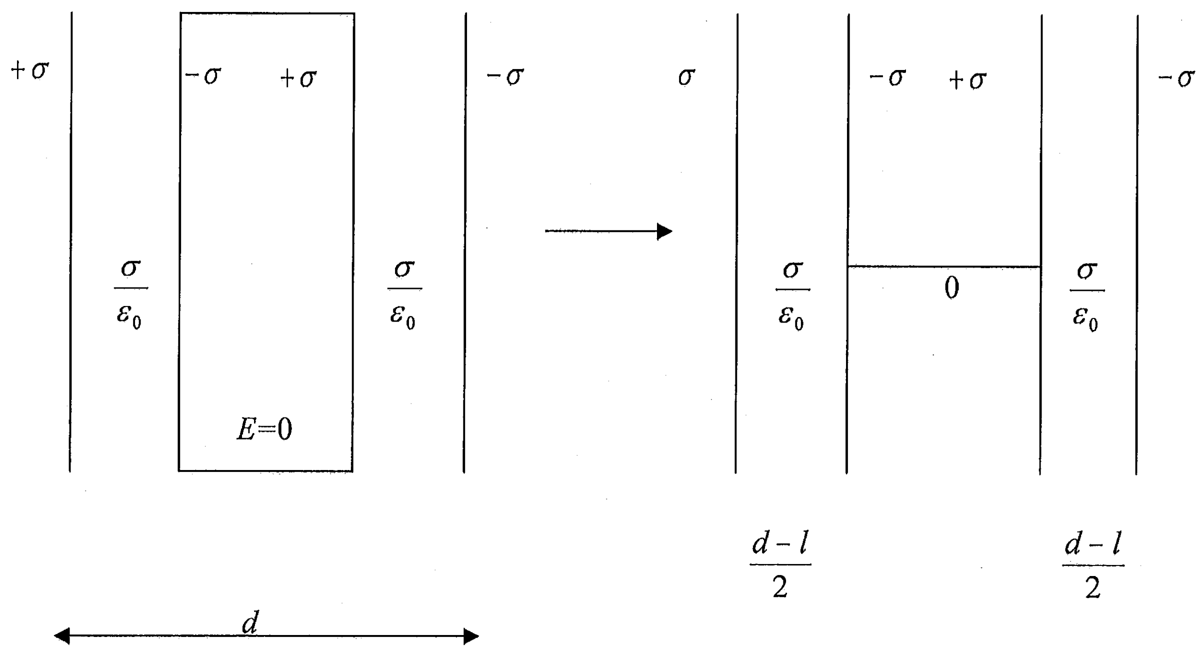
2. $$V = \frac{\sigma}{\varepsilon_0}(d - l)$$

$$C = \frac{\varepsilon_0 A}{d - l} \longrightarrow (1)$$

Put conductor of thickness l in middle. Now

$\underset{\rightarrow}{E}$ =0 inside conductor

Alternately, We have



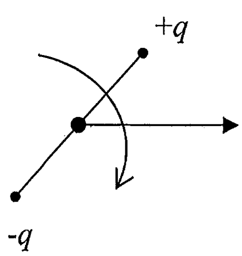
That is, as if two capacitors were in series, Each having $C_1 = C_2 = \dfrac{2\varepsilon_0 A}{d-l}$ (2)

And comparing Eqs. (2) & (1) you see $\dfrac{1}{C} = \dfrac{1}{C_1} + \dfrac{1}{C_2}$

So when capacitors are connected in series, total equivalent capacitance is given by

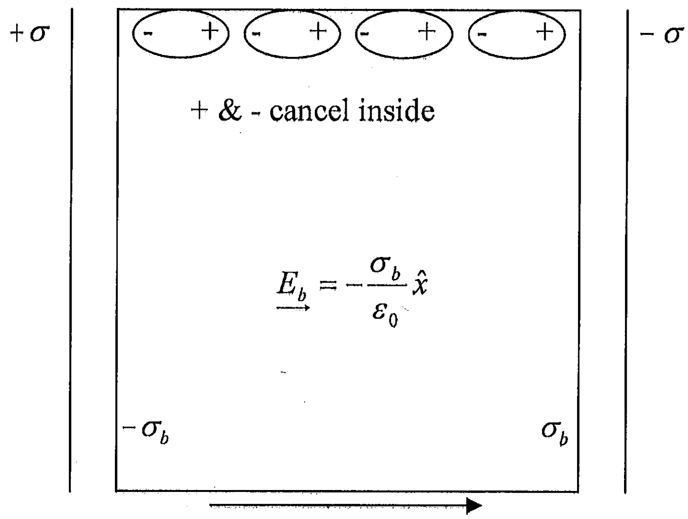
$\dfrac{1}{C_s} = \Sigma \dfrac{1}{C_i}$ SERIES CONNECTION (see below)

3. Dielectric between plates. Dielectric consists of Dipoles. Dielectric is an insulator.



$\vec{\tau} = \vec{p} \times \vec{E}$

Every dipole feels a torque which makes it parallel to the $\vec{E}$-field between the plates hence the system looks like what is shown on the next page:



$$\underset{\rightarrow}{E} = +\frac{\sigma}{\varepsilon_0}\hat{x}$$

Dipole in $\underset{\rightarrow}{E}$ -field experiences torque, which causes each Dipole to line up along $\underset{\rightarrow}{E}$.

On surfaces of dielectric charge sheets $+\sigma_b$ and $-\sigma_b$ appear.

$\underset{\rightarrow}{E}$ -field inside dielectric is the sum of field due to charge on plates $\left[+\frac{\sigma}{\varepsilon_0}\hat{x}\right]$ and that due to charge on surfaces of dielectric $\left[-\frac{\sigma_b}{\varepsilon_0}\hat{x}\right]$.

Hence,

$$\underset{\longrightarrow}{E_\kappa} = \frac{\sigma - \sigma_b}{\varepsilon_0}\hat{x}$$

$$\frac{E_\kappa}{E} = \frac{\sigma - \sigma_b}{\sigma} = \frac{1}{\kappa}$$

κ = Dielectric Const. [*N.B* κ *is always > 1*].

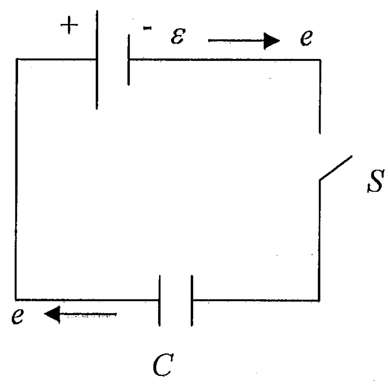
Now $V_\kappa = E_\kappa d = \frac{Ed}{\kappa} = \frac{\sigma d}{\varepsilon_0 \kappa}$

So $C_\kappa = \frac{Q}{V_\kappa} = \frac{\sigma A \varepsilon_0 \kappa}{\sigma d} = \frac{\kappa \varepsilon_0 A}{d}$

Capacitance is increased by factor κ.

Note: in both cases conductor of thickness l between plates or Dielectric of thickness d, potential difference is reduced but physics is totally different!

Next, put the two devices in a circuit:



Note: the lines connecting the devices are perfect conductors and so under stationary conditions they must become equipotentials. That is if we close the switch S and wait for a while the Potential difference across C will become ε.

$$V = \varepsilon$$
$$Q = \varepsilon C$$

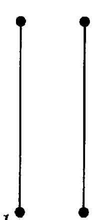
So the capacitor plates now have $+Q$ (left) and $-Q$ (right). How did this happen? Clearly, the +ive plate of the battery pulled electrons from the left sheet of capacitor while the –ive plate pushed electrons on the right sheet. Effectively, you start with q=0 on either sheet, transfer charge from one sheet to the other, so one becomes $-q$ & the other q [However, no flow of charge between capacitor plates]

Making $V = q/c$

Now take Δq from *rt.* To Left. Work done will be

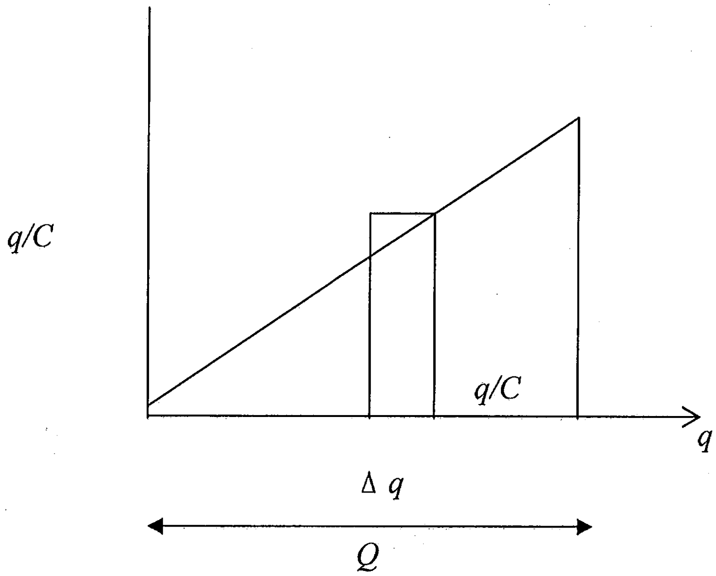
$DW = \Delta q \frac{q}{C}$ area of rectangle. (next page picture)

And we get



CONSERVATIVE FORCE: Work done is independent of the path.

To build up charge from 0 to Q you need area of Δ . This work is now $U_E = \frac{1}{2} \cdot Q(Q/C) = \frac{Q^2}{2C}$



Where does this work go? Notice, space between plates is not empty; there is an $\underset{\rightarrow}{E}$ -field in it.

This energy is stored as potential energy in that $\underset{\rightarrow}{\text{E}}$ - field .

Apply it to ||-plate [air bet plates] $Q = \sigma A, C = \frac{\varepsilon_0 A}{d}$

$$U_E = \frac{\sigma^2 A^2 d}{2\varepsilon_0 A}$$

$$= \frac{1}{2}\varepsilon_0 E^2 Ad \qquad\qquad E = \frac{\sigma}{\varepsilon_0}$$

Energy density $= \frac{U_E}{vol} = \frac{U_E}{Ad} = \frac{1}{2}\varepsilon_0 E^2$ this is like a "Pressure" $\eta_E = \frac{1}{2} \epsilon_0 \; E^2$

||-Plate [Dielectric]

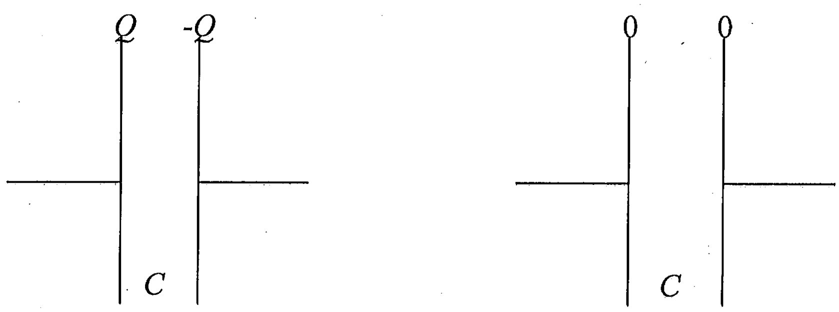
$$U_E = \frac{Q^2}{2C_k} = \frac{\sigma^2 A^2 d}{2k\varepsilon_0 A}$$

$$= \frac{1}{2} k\varepsilon_0 E_k{}^2 Ad$$

$$\eta_E(k) = \frac{1}{2} k\varepsilon_0 E_k{}^2$$

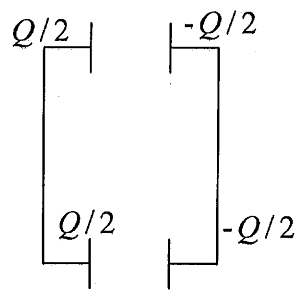
Next, consider the Expt: two identical capacitors.

First, put charges $\pm Q$ on one $U_E = \dfrac{Q^2}{2C}$



Next, connect left-to-left, right to right to make Equipotentials, now charge will be $\pm \dfrac{Q}{2}$ on each.

Total Energy $U_E = \dfrac{2 \cdot 1}{2}\left(\dfrac{Q}{2}\right)^2 \cdot \dfrac{1}{C} = \dfrac{Q^2}{4C}$



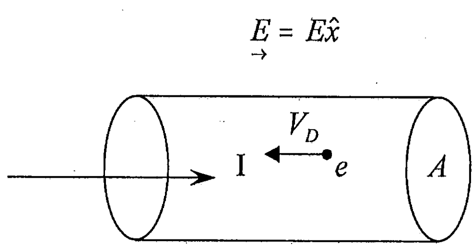
Important: What happened to half of the energy? In the second half of the experiment charge was transported from one set of plates to the other. This experiment tells us that it costs energy to transport charge through a conductor. This leads us to our third device.

-RESISTOR

First we define current

$$I = \lim_{\Delta t \to 0} \frac{\Delta Q}{\Delta t}$$

as the amount of charge flowing per second



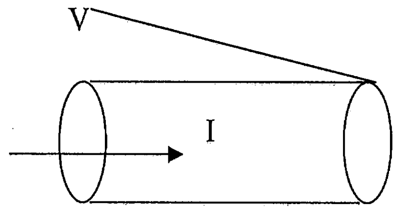
To calculate quantity of charge flowing per second, note that cross-sectional area is A and since only electrons are mobile one can write $I = n_e(-e)A(-V_D)$ in Coulomb/sec or Ampere (unit of current)

where n_e = # of mobile electrons/ m^3

$-e = 1.6x10^{-19} C$

$V_D = drift\ speed$

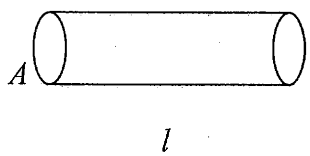
Notice: Direction of I is opposite to that of electron drift.
Recall: $V_D \cong 10^{-4} m/s$. While $V_{rms} \cong 10^5 m/s$ at 300K. This is because ions are stationary and act as scattering centers. Electron has a very tortuous path so although the speed between collisions is high the entire electron "cloud" drifts rather slowly.



Since transport costs energy, potential must drop. Hence, definition of resistance

$$R = \frac{V}{I}$$

For a particular piece of conductor $R = \frac{\rho l}{A}$



Unit of R is $\frac{Volt}{Amp}$
Called Ohm (Ω)

l=length, A= cross-section
ρ = resistivity (material property) $\rightarrow$ $(\Omega - m)$

[cf. conduction of heat $\frac{DQ}{\Delta t} = -kA\frac{\Delta T}{\Delta x}$]

$I = \frac{V}{R} = \frac{\Delta Q}{\Delta t} = \frac{VA}{\rho l} = \sigma A \frac{V}{l}$ $\qquad$ σ = electrical conductivity

It is instructive to write
$I = \vec{J} \bullet \vec{A}$ $\qquad$ $\vec{J}$ = current density vector

and we know that

$$E = \frac{V}{l}$$

so $\qquad \vec{J} = \sigma \vec{E}$

That is, if you apply an $\vec{E}$-field to a conductor it responds by setting up a current density proportional to E, the proportionality factor being the conductivity (electrical).

ELECTRICAL CONDUCTIVITY

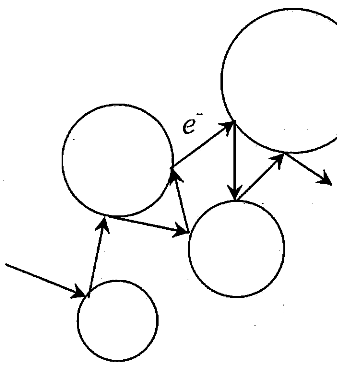
We have learnt that the Electrical Current is

$$I = n_e e A V_D$$

giving current density

$$\underset{\rightarrow}{J} = n_e e \underset{\rightarrow}{V_D}$$

Where V_D is the drift speed of the electrons and for a typical copper wire carrying a current of 1amp, V_D is about $10^{-4}\, m/s$ whereas at room temperature the thermal speed of the electron is $10^5\, m/s$. This huge reduction occurs because the ions whose mass is several thousand times larger are essentially at rest so that when an electron hits an ion it bounces right back. Between collisions.



$$\underset{\rightarrow}{\overleftarrow{E}} = -E\hat{x}$$

The applied $\underset{\rightarrow}{E}$ field causes the drift, so it acts effectively only during the time interval τ between two collisions.

Force on electron $\underset{\rightarrow}{F_E} = eE\hat{x}$

Since it acts for time τ the electron gets an impulse $\underset{\rightarrow}{P} = eE\tau\ \hat{x}$

And this gives rise to V_D so for an electron of mass m

$$mV_D = eE\tau$$

$$V_D = \frac{eE\tau}{m}$$

and current density

$$J = n_e \frac{e^2 E\tau}{m}$$

We have shown that

$$J = \sigma E$$

So the conductivity

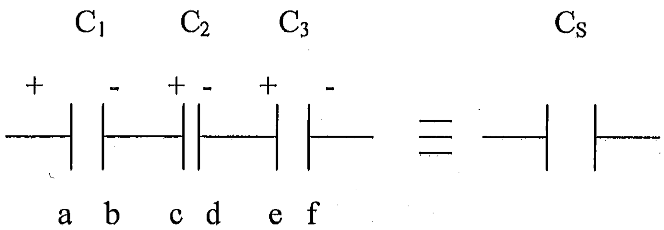
$$\sigma = \frac{n_e e^2 \tau}{m}$$

Which is the famous Drude forumula. When you solve problem 6-11 you will discover that for copper $\tau \sim 10^{-14}$ sec which tells you that the collisions are extremely frequent. Further, although each collision is close to being elastic the net loss of energy in transporting charge through a conductor is non-zero and that causes resistance.

Multiple Capacitors

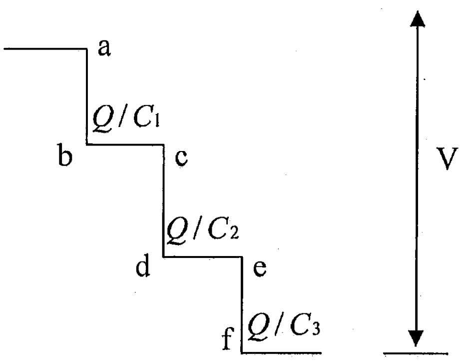
If there are more than one capacitors, then they can be connected either in

SERIES



In this case all plates carry the same charge $\pm Q$

However, the potentials add



Clearly,

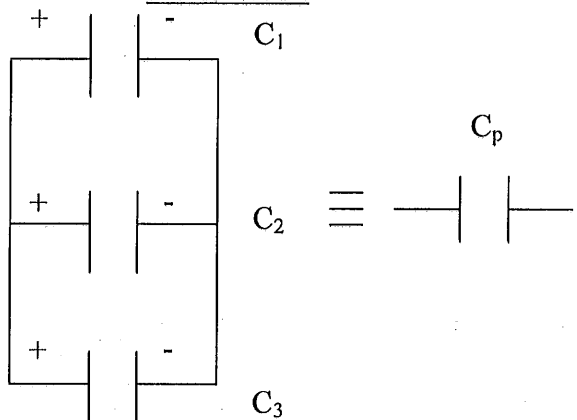
$$V = V_1 + V_2 + V_3$$
$$= \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}$$

So to replace C_1, C_2, C_3 by a single capacitor C_S

$$V = \frac{Q}{C_S} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}$$
$$\frac{1}{C_S} = \Sigma \frac{1}{C_i}$$

or

PARALLEL



The "lines are equipotentials so all positive plates are at the same potential and similarly all negative plates are at the same potential. Hence, charges are

$$Q_1 = C_1 V$$
$$Q_2 = C_2 V$$
$$Q_3 = C_3 V$$

And replacing C_1, C_2, C_3 by a single capacitor

$$C_P V = C_1 V + C_2 V + C_3 V$$
$$C_P = \Sigma C_i$$

<u>So Rule Is</u>

<u>For</u> Series Q is common

giving $\frac{1}{C_S} = \Sigma \frac{1}{C_i}$

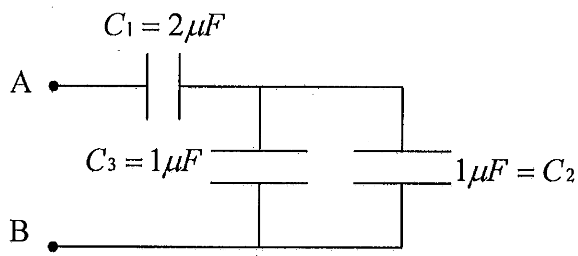
V's add

<u>For</u> Parallel V is Common

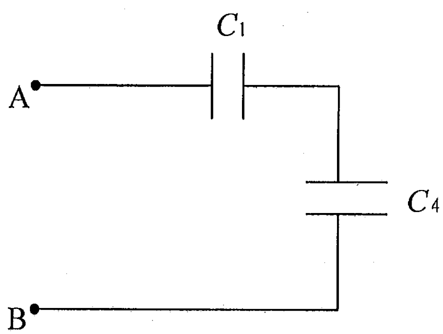
giving $C_P = \Sigma C_i$

Q's add

<u>Example</u>
What is C_{AB} ?



C_2, C_3 in parallel $\qquad C_4 = (1+1)\mu F = 2\mu F$

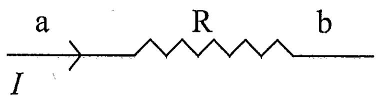


Now C_1, C_4 in series $\qquad \frac{1}{C_5} = \frac{1}{C_1} + \frac{1}{C_4} = \frac{1}{2} + \frac{1}{2}$

$C_{AB} = C_5 = 1\mu F$

Multiple Resistors

A resistor is drawn as

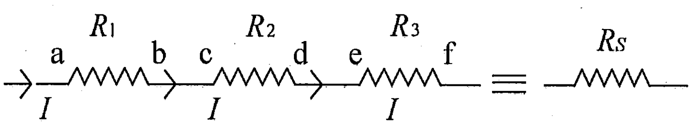


so if you establish a current I, the potential drop from a to b is $V_{ab} = IR$.

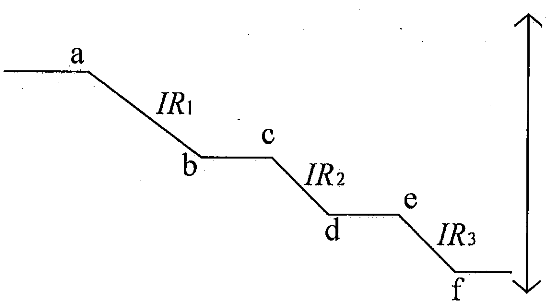
If you have more than one resistor, you can connect them in

Or

SERIES



Since current is flux of charge it must be the same for all otherwise charge cannot be conserved so potentials must look like



$V_{a-f} = IR_1 + IR_2 + IR_3$

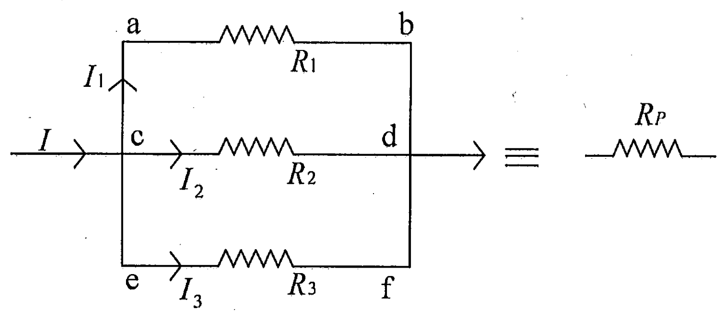
and if you replace them with a single resistor

R_S it must satisfy

$$V - IR_S = I(R_1 + R_2 + R_3)$$

so $\quad R_S = \Sigma R_i$

PARALLEL



First we note that change of potential is independent of path so

$$V_{ab} = V_{cd} = V_{ef} = V$$

Current is flux of charge and charge is conserved

so $\quad I_1 + I_2 + I_3 = I$

or $\quad \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3} = \frac{V}{R_P}$

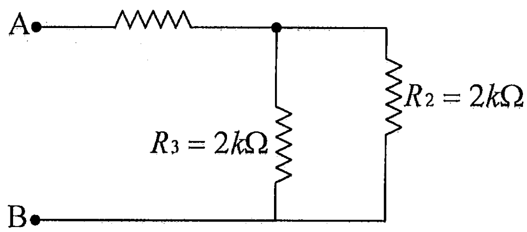
hence $\quad \frac{1}{R_P} = \Sigma \frac{1}{R_i}$

So the rule is:

For Series I is common V's add $\qquad R_S = \Sigma R_i$

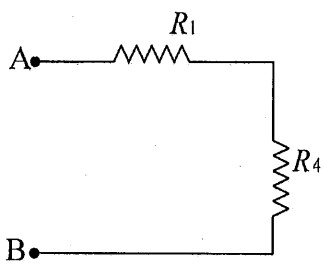
For parallel V is common I's add $\qquad R_P^{-1} = \Sigma R_i^{-1}$

<u>Example</u>



R_2 and R_3 are in parallel so
$$\frac{1}{R_4} = \frac{1}{R_2} + \frac{1}{R_3}$$
$$= \left(\frac{1}{2} + \frac{1}{2}\right)(k\Omega)^{-1}$$

$R_4 = 1k\Omega$



R_1 and R_4 are in series so

$$R_{AB} = R_1 + R_4 = (1+1)k\Omega = 2k\Omega$$