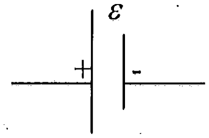
# DEVICES – AC CIRCUITS

Battery Source of Coulomb $\vec{E}$-field

Output is *emf*: $\varepsilon$

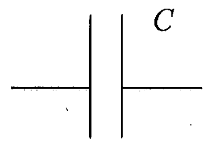



Capacitor: Container for $\vec{E}$-field $C = \frac{Q}{V}$

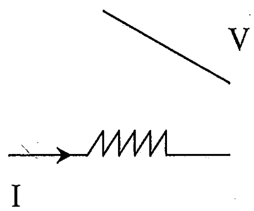
Potential Energy $U_E = Q^2/2C$

$\eta_E$ = Energy stored per $m^3 = \frac{1}{2}\epsilon_0 E^2$

$\epsilon_0 = 9\times 10^{-12}\ F/m$




Resistor: Represents that it costs energy to transport charge through a conductor



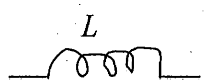

$R = \frac{V}{I}$ $\quad [\vec{J} = \sigma\vec{E}]$

Power loss $P = I^2 R = \frac{V^2}{R}$

Inductor: A time varying current causes a Non-Coulomb $\vec{E}$-field, or an induced *emf*, $L = \frac{-\varepsilon}{\left(\Delta i/\Delta t\right)}$

Container for $\vec{B}$-field, Potential Energy $U_B = \frac{1}{2}Li^2$

$\eta_B$ = Energy stored per $m^3 = \frac{B^2}{2\mu_0}$




A.C. Generator: Wire loops of area $A$ rotated at $\omega$ rad/s in a $\vec{B}$-field. Generates non-coulomb $\vec{E}$-field in the loops, produces an *emf*: $\varepsilon = \omega NBA\ Sin(\omega t)$

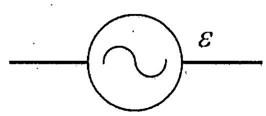
Where $N$= # of turns in the loop. Hence *ac-generator*

$\Phi_B = NBA\cos(\Theta)$

Where $\Theta$ is angle between $\hat{n}$ and $\vec{B}$

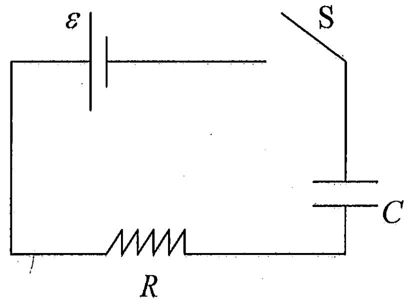
Rotation by $\omega$ *rads*/*sec* makes $\Theta = \omega t$, $\Phi_B = NBA\cos(\Theta)$

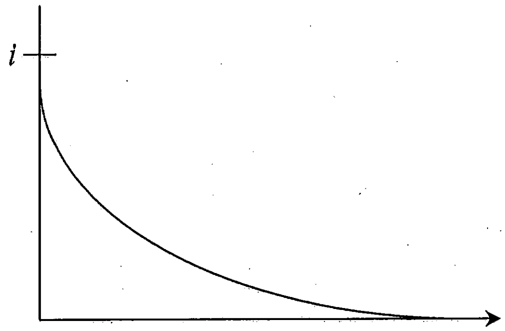
So $\frac{\Delta\Phi_B}{\Delta t} = -NBA\omega\sin\omega t$, $\varepsilon = \frac{-\Delta\Phi_B}{\Delta t} = \omega NBA\sin\omega t$



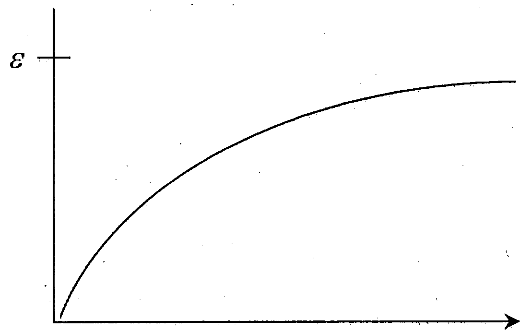
# CIRCUITS

I. RC with battery, close switch at $t$=0, current flows immediately, potential across $C$ appears later $\varepsilon = q/C + iR$




$$i = \frac{\varepsilon}{R} e^{-t/RC}$$



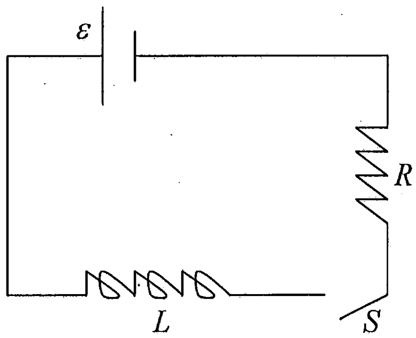

$$v_c = \varepsilon \left[1 - e^{-t/RC}\right]$$

$$\tau = RC$$

N.B. Current first, voltage later.

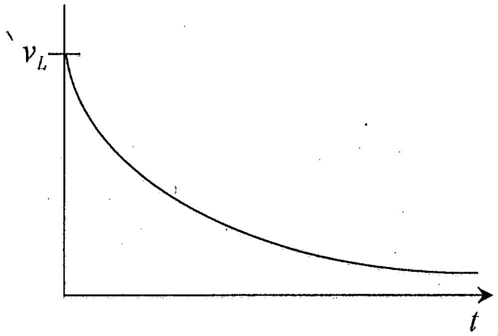
## II. RL With Battery

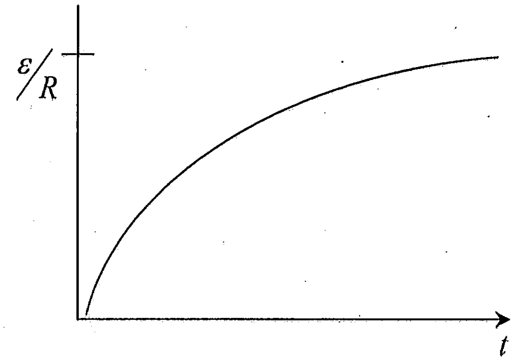
Close switch at $t$=0, $v_L$ immediately jumps to $|\varepsilon|$




Current builds slowly.

$$\varepsilon = L\frac{\Delta i}{\Delta t} + iR$$




$$v_L = \varepsilon e^{-Rt/L}$$

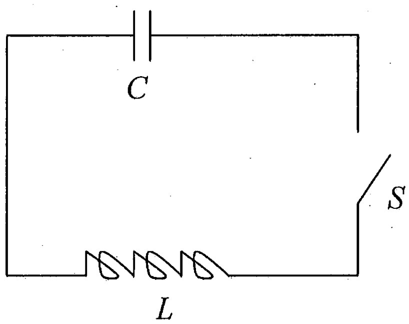
$$i = \frac{\varepsilon}{R}\left[1 - e^{-Rt/L}\right]$$

$$\tau = \frac{L}{R}$$

Note: Voltage First, Current Later.

III. LC-Circuit: Undamped Oscillator

First charge $C$ to $Q_0$. Close switch at $t$=0. Energy stored in capacitor $U_E = \dfrac{Q_0^2}{2C}$

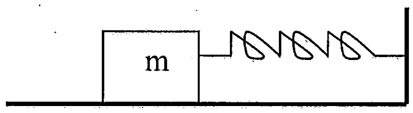



Charge begins to flow. Total Energy = (Energy in $\vec{E}$-field) + (Energy in $\vec{B}$-field)
= (Energy in $C$) + (Energy in $L$)

$$\frac{Q_0^2}{2C} = \frac{q^2}{2C} + \frac{1}{2}L\left(\frac{\Delta q}{\Delta t}\right)^2$$

Recognize, similarity to spring-mass oscillator

$$\frac{1}{2}kA^2 = \frac{1}{2}kx^2 + \frac{1}{2}m\left(\frac{\Delta x}{\Delta t}\right)^2$$




$x \rightarrow q$

$m \rightarrow L$

$k \rightarrow \frac{1}{C}$

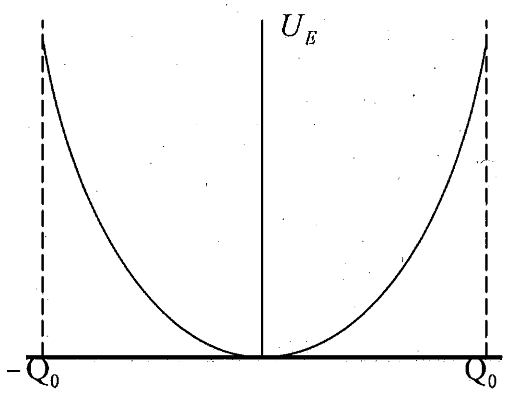
$x = A\cos\omega_0 t$

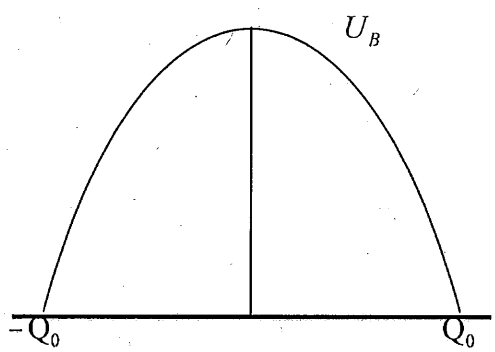
$$\omega_0 = \sqrt{\frac{k}{m}}$$

Now

$$q = Q_0\ Cos\,\omega_0 t$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

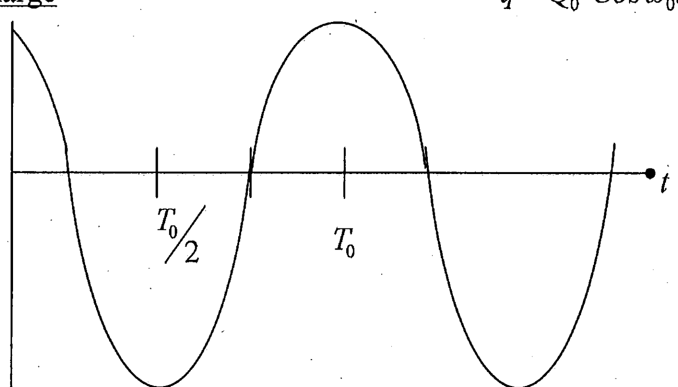


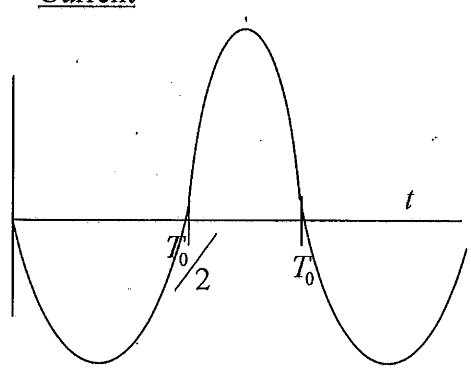

$\underset{\rightarrow}{E}$ -field collapses giving rise to $\underset{\rightarrow}{B}$ -field and vice versa.

Charge

$q = Q_0\ Cos\,\omega_0 t$




Current

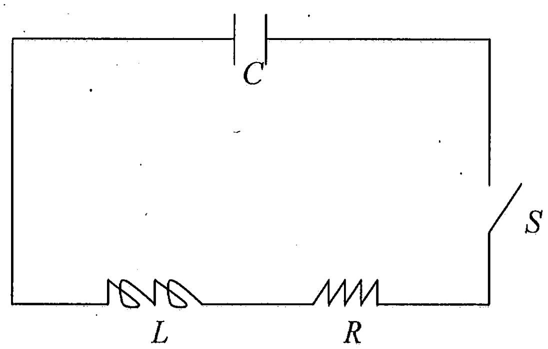



$i = -Q_0\omega_0\ Sin\,\omega_0 t$

Period $T = \frac{2\pi}{\omega_0}$

## IV. LCR-CIRCUIT: DAMPED OSCILLATOR.

At $t$=0, charge $C$ to $Q_0$ close switch. Now driving $i$ through $R$ costs $i^2R$ per second.




*So* $\left(\frac{q^2}{2C}+\frac{1}{2}Li^2\right)$ IS NOT CONSTANT

$$\frac{\Delta\left(\frac{q^2}{2C}+\frac{1}{2}Li^2\right)}{\Delta t} = -\,i^2R$$

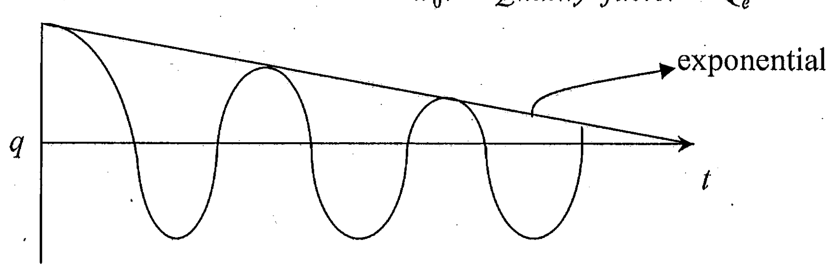
-ive sign on right because energy is being lost ($R$ is getting warmer).

Now $q = Q_0 e^{-Rt/2L}\, Cos\,\omega t$

$$\omega = \omega_0\left[1-\frac{1}{(2\omega_0\tau)^2}\right]^{1/2}$$

$$\tau = \frac{L}{R}$$

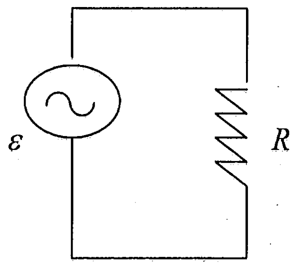
$$\omega_0\tau = \textit{Quality factor} = Q_e$$




Note 1: smaller $R$, larger the duration for which the oscillations persist.
Note 2: $R$ plays role of friction; as always energy lost goes to raise temperature. Electrical Equivalent of Heat.

# CIRCUITS: AC

## I. Resistor and Generator




$V_R = IR$

so

If $\varepsilon = \varepsilon_0 \, Sin\,\omega t$

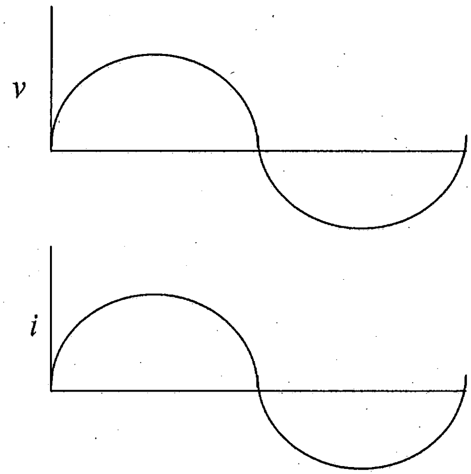
$i = \dfrac{\varepsilon_0}{R} Sin\,\omega t$

Current and voltage are in phase.

Power

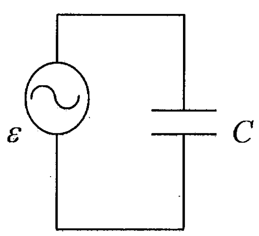
$P(t) = iv$

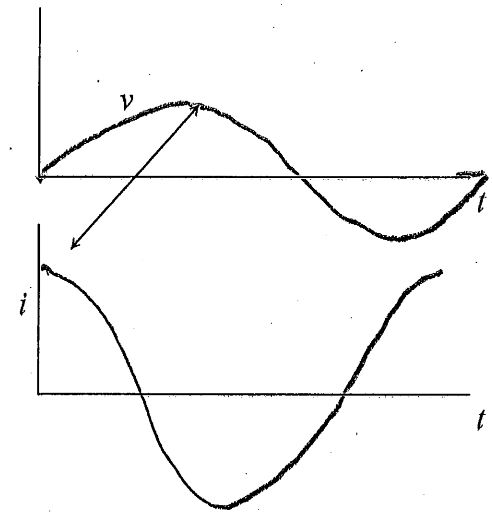
$= \dfrac{\varepsilon_0}{R} \, Sin^2 \; \omega t$




averaged over a cycle. $< P >= \dfrac{\varepsilon_0^{\;2}}{2R} = \dfrac{i_0 \varepsilon_0}{2} = \dfrac{i_0^{\;2} R}{2}$ and the power loss is as if $R$ was connected to a battery whose $\varepsilon = \dfrac{\varepsilon_0}{\sqrt{2}}$. In this sense one talks of $\dfrac{\varepsilon_0}{\sqrt{2}}$ *and* $\dfrac{i_0}{\sqrt{2}}$ as root-mean-square or r.m.s. voltage and current.

## II. Generator and Capacitor






now $q=Cv$

so charge and voltage in phase.

$v = \varepsilon_0 \, Sin\,\omega t$

$q = \varepsilon_0 \, Sin\,\omega t$

$$i = \frac{\Delta q}{\Delta t} = + \varepsilon_0 C\omega \; Cos\omega t$$

$i$ and $v$ are not in phase $i$ leads $v$ by $\pi/2$

$\frac{1}{\omega C}$ has dimensions of $R$ and we define capacitive reactance

$$X_c = \frac{1}{\omega C}$$

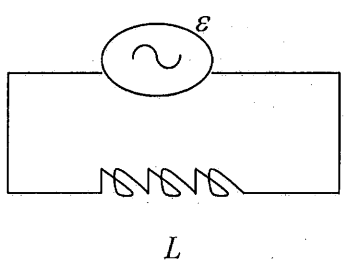
$$i = \frac{\varepsilon_0}{X_c} \cos\omega t$$

$$\text{Now} \quad P(t) = \frac{\varepsilon_0^{\,2}}{X_C} \, Sin\,\omega t \; Cos\,\omega t$$

$$\text{Average } \langle P \rangle = 0$$

No power loss on average.

III.



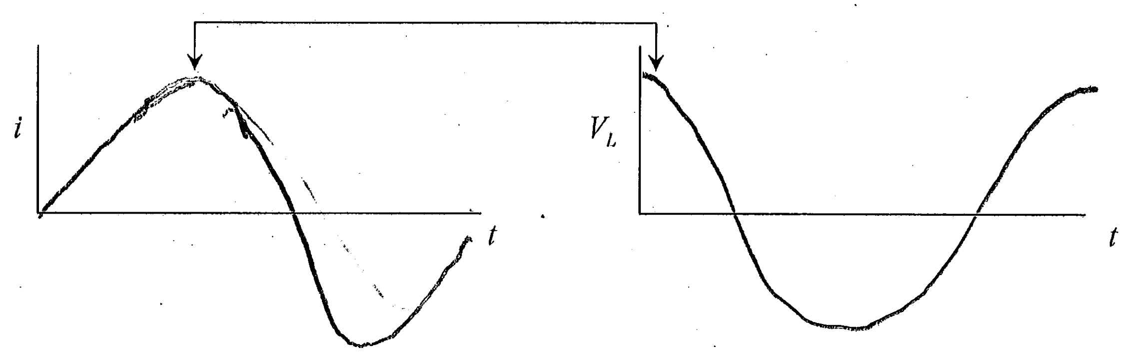

Now voltage in phase with slope of $i$ vs $t$ curve.

If $i = i_0 \, Sin\,\omega t$

$$\varepsilon - \frac{L\Delta i}{\Delta t} = 0$$

$$v_L = \frac{L\Delta i}{\Delta t}$$

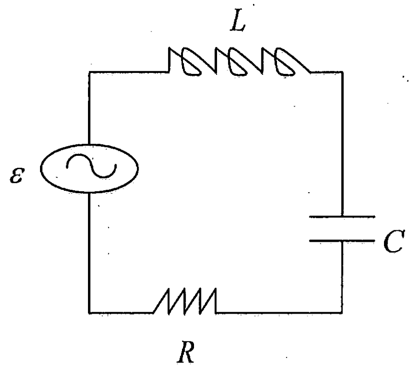
$v_L = i_0 \, \omega L \, Cos\,\omega t$ and now voltage leads current by $\frac{\pi}{2}$



$X_L = \omega L$

And again $<P> = 0$

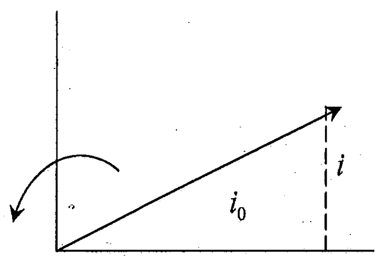
IV. Generator with all three.



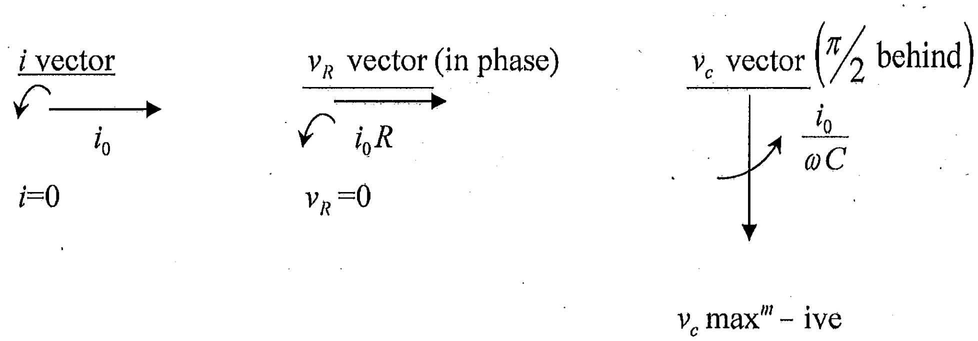

To work out behavior of L-C-R circuit we have to keep track of the relative phases in *R, C,* and *L.*

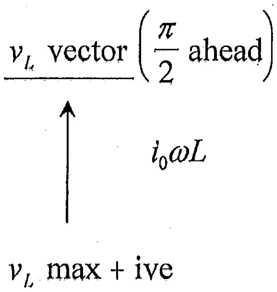
We introduce the concept of a Phasor. We have a series loop so current has to be same at all points. We begin with the current and build the voltage vector.

Take $i = i_0 \, Sin \, \omega t$ and represent it by current Phasor: vector of magnitude $i_0$ rotating at angular velocity $\omega$, i.e.

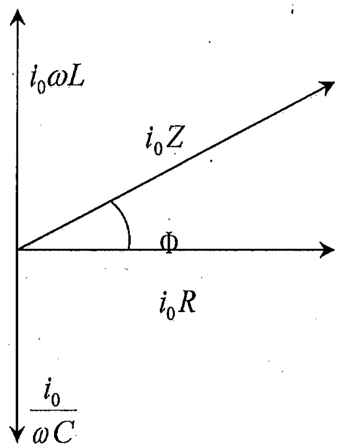



Concentrate on $t$=0






Total voltage vector




$$\varepsilon_0 = v_0 = i_0\left[R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right]^{1/2}$$

$$\tan\Phi = \frac{\omega L - \frac{1}{\omega C}}{R}$$

$$i = i_0 \, Sin\,\omega t$$
$$v = \varepsilon_0 \, Sin(\omega t + \varnothing)$$

<u>Power in Circuit</u>

$$P = iv = i_0\varepsilon_0 \, Sin\,\omega t \; Sin(\omega t + \varnothing)$$
$$= i_0\varepsilon_0\left[Sin^2\omega t \, Cos\varnothing + Sin\,\omega t \, Cos\,\omega t \, Sin\varnothing\right]$$
$$\langle P\rangle = \frac{i_0\varepsilon_0}{2} Cos\varnothing$$

This is the concept of Power factor $Cos\varnothing$

<u>Next,</u> Define Impedance

$$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{wC}\right)^2}$$

then $i_0 = \frac{\varepsilon_0}{Z}$ & $Cos\varnothing = \frac{R}{Z}$

so $< P >= \frac{\varepsilon_0{}^2}{2Z}\cos\Theta = \frac{\varepsilon_0{}^2}{2R}\cos^2\Theta$

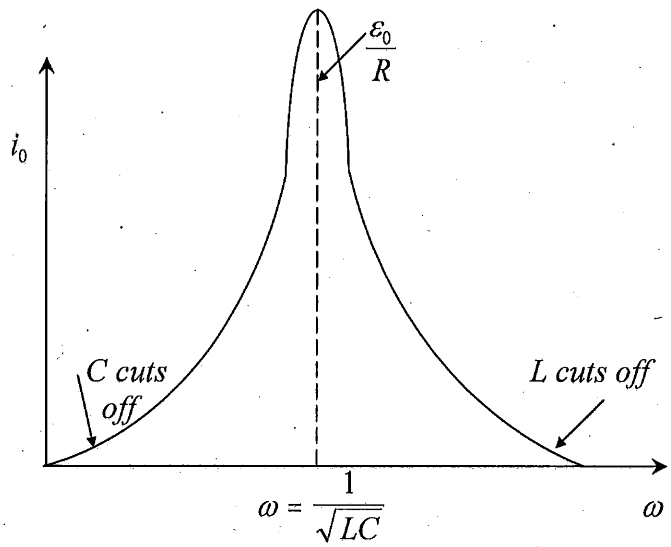
<u>RESONANCE</u> If generator frequency can be varied $i_0 = \dfrac{\varepsilon_0}{\sqrt{R^2 + \left(\omega L - \dfrac{1}{\omega C}\right)^2}}$

Gives $\quad i_0 \to 0$ when $\omega \to 0$ b/c $\dfrac{1}{\omega C} \to \infty$ [capacitor cuts off $i_0$]

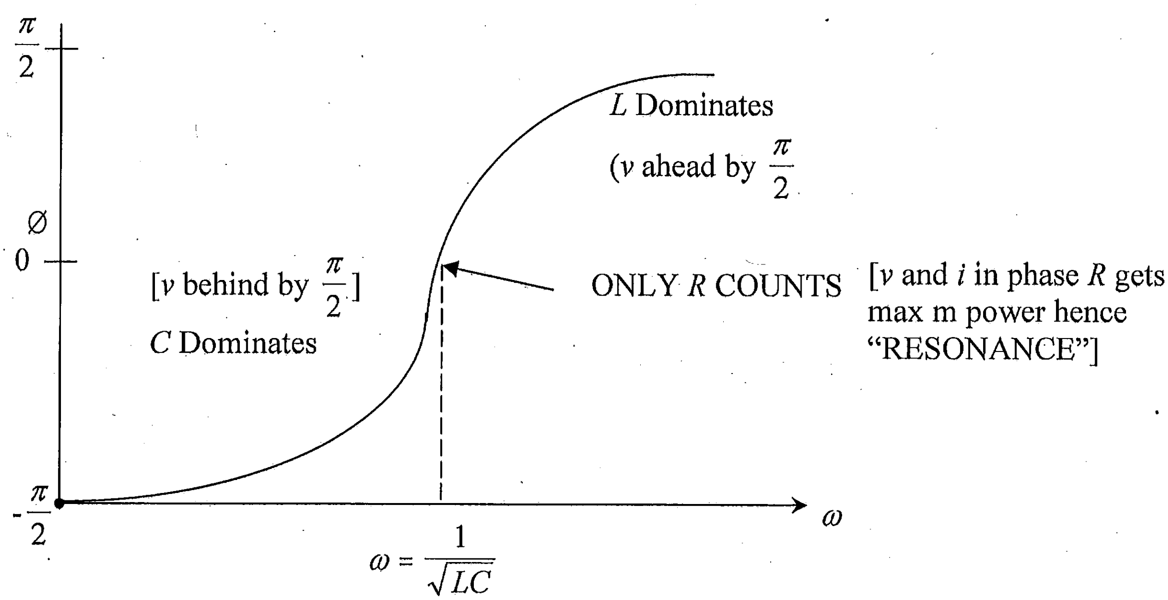
$\quad i_0 \to 0$ when $\omega \to \infty$ b/c $\omega L \to \infty$ [Inductor cuts off $i_0$]

$\quad i_0$ is max$^m$ when $\omega L = \dfrac{1}{\omega C}$

This is the phenomenon of Resonance




The phase difference ∅ is also a function of frequency



Note: Resonance occurs when Generator frequency $\omega$ is equal to natural frequency of $LC$ Circuit $\omega_0 = \frac{1}{\sqrt{LC}}$