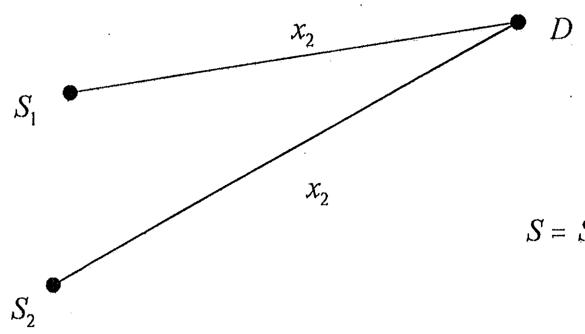
SOUND WAVES-INTERFERENCE

What happens if two sound waves start in phase but have traveled different distances before they arrive at the detector? S_1, S_2 are two sources of sound, which emit waves in phase.



$$S = S_m Sin(kx - \omega t)$$

Let us put both phases equal to zero at the starting points. Note: when a wave travels a distance λ its phase must change by 2π. Hence, the wave from S_1 is when it arrives at D will have the phase $\Phi_1 = \frac{2\pi}{\lambda} x_1$.

When this wave arrives at D there is nothing there. However, when the wave from S_2 arrives at D its phase will be $\Phi_2 = \frac{2\pi}{\lambda} x_2$ and since wave from S_1 is already there, the two waves superpose. If $(\Phi_1 - \Phi_2) = 0, 2\pi, 4\pi, 6\pi$ the two waves will be in phase at D and will combine to produce a maximum at D.

We call this constructive interference. 

CONDITION FOR MAXIMUM AT D:

$$(\Phi_1 - \Phi_2) = 2M\pi \quad M = 0,1,2...$$

$$\frac{2\pi}{\lambda}(x_1 - x_2) = 2M\pi \qquad \text{(I)}$$

$$or \; (x_1 - x_2) = M\lambda \quad M = 0,1,2...$$

In other words, if the path DIFFERENCE is a whole # of λ's, the waves which started in phase will again be in phase at D and produce a maximum there.

However, if

$$\left(\Phi_1 - \Phi_2\right) = \pi, 3\pi, 5\pi \ldots$$

or equivalently (II)

$$\left(x_1 - x_2\right) = (2m+1)\frac{\lambda}{2}$$

with m=0, 1, 2,...

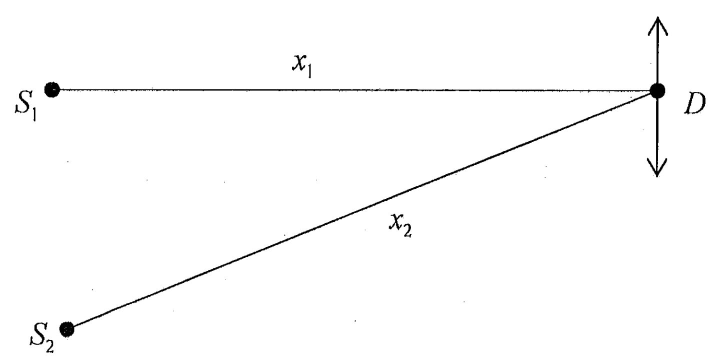
When the waves meet at D, they will be exactly out of phase
And cancel each other.

causing DESTRUCTIVE INERFERENCE

CONDITION FOR MINIMUM

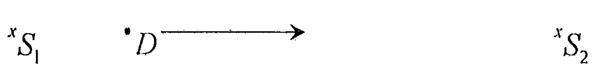
$$\left(x_1 - x_2\right) = (2m+1)\frac{\lambda}{2}, m = 0,1,2,\ldots \qquad \text{(II)}$$

If you keep S_1 and S_2 fixed and move D up and down as shown, $(x_1 - x_2)$ will vary and you will encounter a series of maxima (loud sound) and minima (no sound) as you go alternately from Eq I to Eq II and vice versa.



INTERESTING CASE

S_1 and S_2 are separated by $M\lambda$ and D (your ear) moves along line joining S_1 and S_2.



a) When D is at S_1:

$x_1 = 0$

$x_2 = M\lambda$

$(x_1 - x_2) = -M\lambda$ $\qquad$ *MAX*

b) When D is at Mid-point:

$$x_1 = x_2 = \frac{M\lambda}{2}$$

$(x_1 - x_2) = 0$ $\qquad$ *MAX*

c) When D is at S_2:

$x_1 = M\lambda$

$x_2 = 0$

$(x_1 - x_2) = M\lambda$ $\qquad$ *MAX*

In all you hear ($2M+1$) Maxima
In between where will be $2M$ minima

N.B. CRUCIAL QUANTITY IS PATH DIFFERENCE!!!

TRIG IDENTITY

$\sin A + \sin B$

$$= \sin\left(\frac{A + A + B - B}{2}\right) + \sin\left(\frac{B + B + A - A}{2}\right)$$

$$= \sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right) + \cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right) + \sin\left(\frac{B+A}{2}\right)\cos\left(\frac{B-A}{2}\right) + \cos\left(\frac{B+A}{2}\right)\sin\left(\frac{B-A}{2}\right)$$

Now

$$\cos\left(\frac{B-A}{2}\right) = \cos\left(\frac{A-B}{2}\right)$$

But

$$\sin\left(\frac{B-A}{2}\right) = -\sin\left(\frac{A-B}{2}\right)$$

Hence

$$\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

Similarly

$$\cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

So when you superpose

$y_1 = A\sin(kx_1 - \omega t)$ and $y_2 = A\sin(kx_2 - \omega t)$

You get

$$y = 2A\cos\frac{k(x_1 - x_2)}{2}\sin\left[\frac{k(x_1 + x_2)}{2} - \omega t\right]$$

That is, a wave whose amplitude is controlled by $(x_1 - x_2)$ leading to INTERFERENCE effects.