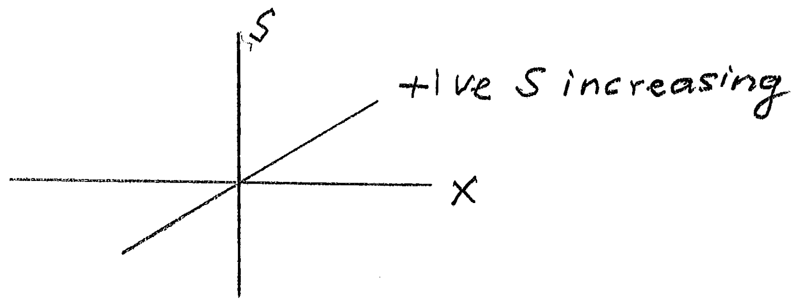
SOUND

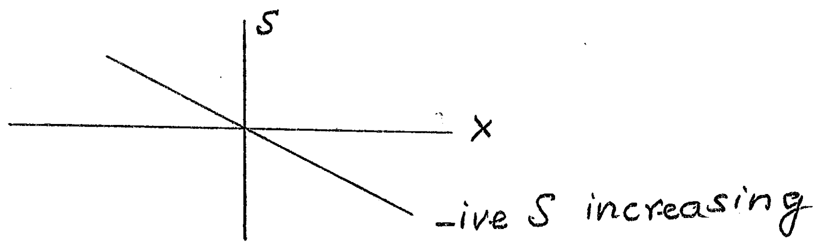
a) There is NO sound in vacuum; you need matter to propagate a sound wave.
b) SOUND: Any mechanical wave whose frequency lies between $20Hz$ and $20,000Hz$, that is, $20Hz \le f \le 20kHz$ (It is called sound because you can hear it!).
c) We will work with sound in Gases only-then sound is a purely Longitudinal wave.
d) Sound is a longitudinal displacement wave or a longitudinal pressure wave.
e) Periodic Sound wave properties

DISPLACEMENT	PRESSURE
Sine wave, $\varnothing = 0$ $S = S_m Sin(\kappa x - \omega t)$ amplitude $\underset{\rightarrow}{S_m} \parallel \hat{x}$ $\omega = vk$	To write corresponding pressure wave we have to realize that the variation occurs so rapidly that there is no possibility for exchange of heat (DQ) to ensure equilibrium with surroundings, so DQ=0, sound is an adiabatic process: Pressure and Volume satisfy: $P_0 V_0^{\gamma} =$ constant. P_0 = ambient pressure $\gamma = \frac{C_p}{C_v}$, C_p = sp ht at const P C_v = sp ht at const V $\gamma_{monoatomic} = \frac{5}{3}$ $\gamma_{diatomic} = \frac{7}{5}$ $\rightarrow \varnothing = \frac{-\pi}{2}$ $P = P_0 - \gamma P_0 S_m \kappa \; Cos(\kappa x - \omega t)$
Displacement oscillates about zero.	Pressure oscillates about P_0 Amp of pressure wave $P_m = \gamma P_0 S_m \kappa$ $\underset{\rightarrow}{P_m} \parallel \hat{x}$ Pressure is $\frac{\pi}{2}$ out of phase with displacement where S is max. $(P-P_0)=0$!

Displacement wave → Pressure wave
If the displacement of the gas atoms is a function of x then near a point where $S = 0$ either the gas is <u>expanding</u>



Or <u>contracting</u>



In either case, volume is changing so pressure must change and therefore we must know the relation between these changes to proceed further. The frequencies are higher than 20Hz so all the variations are extremely rapid, heat exchange with surrounding is effectively zero. Sound is an adiabatic process.

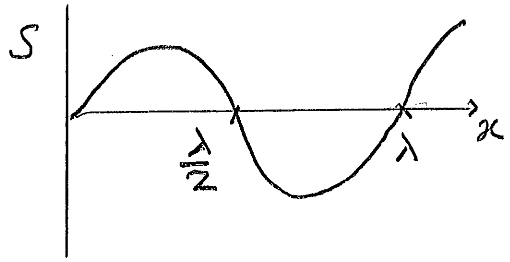
<u>SPECIAL NOTE</u>
Detailed interpretation of displacement and pressure curves in a sound wave

Or

Why is pressure variation $\frac{\pi}{2}$ out of phase with displacement as a function of position?

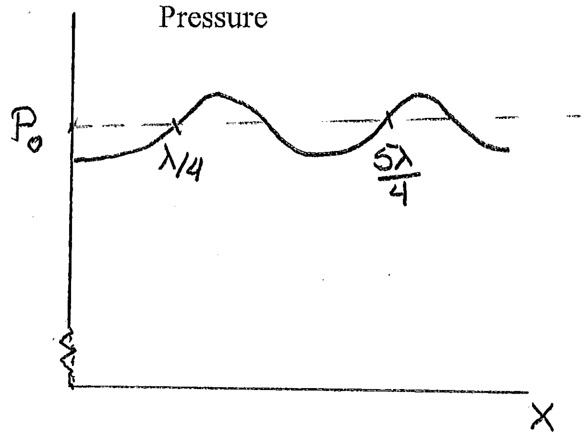
The curves are (at t = 0)

Displacement



$S = S_m \sin kx$

Pressure



$P = P_0 - P_m \cos kx$

Near x = 0, Displacements look like

$$\overset{0}{\leftarrow\leftarrow\leftarrow \bullet \rightarrow\rightarrow\rightarrow}$$ All displacement away from 0

That is displacements of particles increase rapidly as you go away from x = 0. Consequently, gas is in <u>expansion</u> that is why pressure is at <u>MINIMUM.</u>

Near $x = \frac{\lambda}{4}$, Displacements look like

$$\overset{\frac{\lambda}{4}}{\rightarrow\rightarrow\rightarrow \bullet \rightarrow\rightarrow\rightarrow}$$

That is, <u>all</u> the displacements are nearly equal so there is little change in volume and hence P is at its equilibrium value. Deviation from $\equiv m$ is <u>zero.</u>

Near $x = \frac{\lambda}{2}$, Displacements look like

$$\overset{\frac{\lambda}{2}}{\longrightarrow \rightarrow \rightarrow \bullet \leftarrow \leftarrow \longleftarrow}$$

Displacements are toward $\frac{\lambda}{2}$ and increase as you go away from $\frac{\lambda}{2}$ so here gas is in contraction and that is why pressure is at a <u>MAXIMUM.</u>

CRUCIAL Pt. is that change of volume and hence change of pressure happens only if displacement everywhere is not the same.

f) Speed of sound in a gas

$v = \sqrt{\frac{\gamma P_0}{\rho_0}}$ $\qquad \rho_0 =$ ambient density

since $P_0 V_0 = N k_B T$ $\qquad [k_B = 1.383x10^{-23} J/K]$

If gas particles have mass *m*, we can write

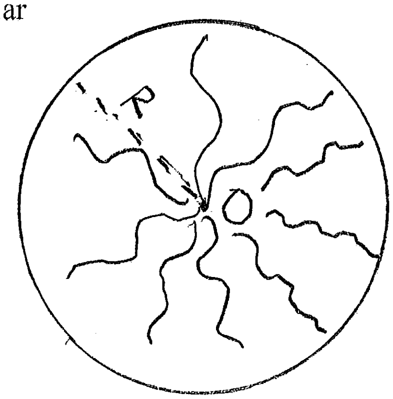
$$P_0 = \frac{Nm}{V_0}\frac{k_B}{m}T, or \frac{P_0}{\rho_0} = \frac{k_B T}{m}$$

$$v = \sqrt{\frac{\gamma k_B T}{m}} = \sqrt{\gamma}\,\frac{v_{rms}}{\sqrt{3}}$$ [T in Kelvin Scale]

<u>INTENSITY OF SOUND</u> – Experiment

Sound wave, of course, transports energy and we define the intensity as energy transport per unit area per unit time.

Intensity (I) MT^{-3} $Watt/m^2$ Scalar



To calculate I we begin by putting a point source of sound at the origin. Let it emit P_w watts (Joules/sec) of power. The wave spreads equally in all directions and after travelling a distance R from O is spread evenly on the surface of a sphere of radius R. So

$$I = \frac{P_w}{4\pi R^2}$$

As we move away from the source the intensity will reduce as the square of the distance from the source.

Example: Let $P_w = 126 Watts$ $(4\pi \cong 12.6)$

We can construct the table

R (m)	I (Watts/m^2)
1	10
10	10^{-1}
10^2	10^{-3}
10^3	10^{-5}
10^4	10^{-7}
10^5	10^{-9}
10^6	10^{-11}
3.2×10^6	10^{-12}

The numbers in the table look awfully small but it is useful to know that the human ear can detect $10^{-12}\,Watt/m^2$.

Animals are even more sensitive. Incidentally, your ear "drum" is essentially a disk of radius 1cm so the quietest sound delivers only 3×10^{-18} Joules/sec. In the sequel, we will calculate the amplitude of this wave.

LEVEL OF SOUND – DECIBEL SCALE

It was Alexander Graham Bell (the inventor of the telephone) who discovered that the human ear is not a linear detector. In fact, if I goes from $10^{-9}\,Watt/m^2$ to $10^{-6}\,Watt/m^2$ we will claim that intensity has just increased by a factor of 2. This has led to the convention that instead of speaking of intensity of sound we speak of the sound level β defined by the equation

$$\beta = (10dB)\log_{10}\frac{I}{I_0}$$

Where dB is the abbreviation for Decibel (in honor of A.G.) and $I_0 = 10^{-12}\,Watt/m^2$ so for the quietest sound $\frac{I}{I_0} = 1$, $\beta = 0$.

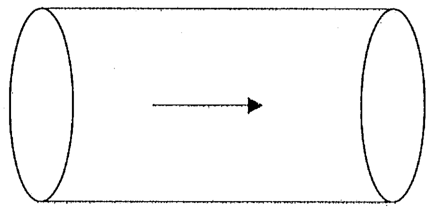
Again, let us make a table and you see as I goes from $10^{-9}\,Watt/m^2$ to $10^{-6}\,Watt/m^2$, β increases only by a factor of 2.

Further, the logarithmic variation implies that $63dB = 2\times10^{-6}\,Watt/m^2$, $66dB = 4\times10^{-6}\,Watt/m^2$, and $69dB = 8\times10^{-6}\,Watt/m^2$. Incidentally 120dB represents the threshold of pain!

β (dB)	I/I_0	I (W/m^2)
0	1	10^{-12}
10	10^{1}	10^{-11}
20	10^{2}	10^{-10}
30	10^{3}	10^{-9}
40	10^{4}	10^{-8}
50	10^{5}	10^{-7}
60	10^{6}	10^{-6}

INTENSITY OF SOUND – PERIODIC WAVE

Imagine that the wave is traveling with velocity v through a tube of cross-section A.



Since $S = S_m Sin\,(\kappa x - \omega t)$ the particle velocity is
$V_p = S_m \omega \cos(\kappa x - \omega t)$ and kinetic energy per unit volume is $K \cdot E = \frac{1}{2}\rho_0 S_m^{\ 2}\omega^2$.
Volume of wave traveling past every cross-section will be Av in one second.
(Energy transport per second through area,
$A) = \frac{1}{2} A\,\rho_0 S_m^{\ 2}\omega^2 v$
Intensity I=energy transport per second per m^2

$$I = \frac{1}{2}\rho_0 S_m^{\ 2}\omega^2 v \qquad \left[\rho_0 = \frac{\gamma P_0}{v^2}\right]$$

$$= \frac{1}{2}\gamma P_0 S_m^{\ 2}\frac{\omega^2}{v}$$

Please compare this with energy transport per second on wire $\langle P\rangle = \frac{1}{2}A^2\frac{\omega^2}{v}, F$

Amplitude of displacement wave for $I_0\ (\omega = 10^3\, rad/s, \gamma = 1.4, P_0 = 10^5\, N/m^2)$,

$$10^{-12} = \frac{1}{2} x 1.4 x 10^5 x \frac{S_m^{\ 2} x 10^6}{340}$$

$$S_m \cong 10^{-10} m$$

Roughly equal to diameter of hydrogen atom. REMARKABLE!!!*

*Your ear can discern motion of air molecules whose displacement is equal to the diameter of a hydrogen atom.