Physics – Introduction

Where have we been and where are we going? When we began our study of Physics in 121 we referred to two fields of intellectual endeavor. The Physicist attempts to (i) provide the simplest and most economical description of nature as we know it and, at a more basic level, (ii) use intuition and imagination to propose a new fundamental idea and go back to nature to ascertain its applicability and develop a deeper understanding of its origin. Most of our study in 121 focused on (i) where we discussed the motion of particles and rigid bodies followed by a brief discussion of Thermodynamics.

The only example of (ii) was Newton's brilliant introduction (in the mid 1600's) of the Universal Law of Gravitation

$$\underrightarrow{F_G} = -\frac{GM_1M_2}{r^2}\hat{r} \qquad - (1)$$

whose origins Newton did not understand and in fact were not elucidated until the early 1900's: First Cavendish had to measure *G*, then Faraday introduced (1800's) the concept of a field and nearly 100 years later, Einstein developed the bases for this extremely elegant equation. A single powerful idea led to 350 years of hard work for some of the foremost thinkers. In 122 we will build on what we learned in 121 to understand the nature of sound and light. Again, most of it will be focused on explaining observations but there will be one example of an absolutely astounding power of the human mind.

We begin by placing on record the physical relationships developed in 121 in terms of the four fundamental physical dimensions:

Length (L)

Time (T)

Mass (M)

Temperature (Θ)

The master table lists all these physical quantities and their dimensions, units and whether they are scalars (magnitude only) or vectors (magnitude and direction).

Dimensions – Units – Scalar or Vector

Time	T	*sec.*	S
Mass	M	*kg*	S
Length	L	*m*	S
Area	L^2	m^2	V
Volume	L^3	m^3	S
Angle	L^0	*radian*	V
Speed	LT^{-1}	ms^{-1}	S
Velocity	LT^{-1}	ms^{-1}	V
Displacement	L	m	V
Acceleration	LT^{-2}	m/s^2	V
Force	MLT^{-2}	$kg-m/s^2$ *(newton)*	V
Work	ML^2T^{-2}	$N-m$ *(Joule)*	S
Energy	ML^2T^{-2}	*Joule*	S
Momentum	MLT^{-1}	$kg-m/s$	V
Angular Velocity	L^0T^{-1}	*rad/sec*	V
Angular Acceleration	L^0T^{-2}	$rad/\sec^2$	V
Torque	ML^2T^{-2}	N-m	V
Moment of Inertia	ML^2	$kg-m^2$	S
Temperature	θ	°C, °F, °K	S
Heat	ML^2T^{-2}	*Joule*	S
Specific Heat	$L^2T^{-2}\theta^{-1}$	*Joule/kg/K*	S
Thermal Conductivity	$MLT^{-3}\theta^{-1}$	*Joule/m-s-C*	S
Pressure	$ML^{-1}T^{-2}$	N/m^2	S
Density	ML^{-3}	kg/m^3	S
GR Constant	$M^{-1}L^3T^{-2}$	$N-m^2/(kg)^2$	
Boltzman Constant	$ML^2T^{-2}\theta^{-1}$	*Joule/K*	
Stefan Constant	$MT^{-3}\theta^{-4}$	$J-\sec^{-1}-m^{-2}-K^{-4}$	
Power	ML^2T^{-3}	*Joule/sec (watt)*	
Coefficient of Friction:	**Dimensionless Ratio**		
Expansion Coefficient	θ^{-1}	$(°C)^{-1}$	S
Angular Momentum	ML^2T^{-1}	$kg-m^2/s$	V
Entropy	$ML^2T^{-2}\theta^{-1}$	*J/K*	S
Frequency	T^{-1}	*hertz*	S

Formulae

Angle $\Theta = \frac{S}{R}$

Trig. Functions $\sin\Theta = \frac{o}{h}$, $\cos\Theta = \frac{a}{h}$, $\tan\Theta = \frac{o}{a}$

Pythagoras Theorem $a^2 + o^2 = h^2$; $\sin^2\Theta + \cos^2\Theta = 1$

Quadratic $ax^2 + bx + c = 0$, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Circle $Area = \pi R^2$

Sphere $Surface\ Area = 4\pi R^2$ $volume = \frac{4\pi}{3}R^3$

Speed $s = \frac{\text{Distance Travelled}}{\text{Time of Travel}}$

Unit Vectors $\hat{x}, \hat{y}, \hat{z}$ Magnitude is 1 (one), directions along, *x, y, z* axis respectively

Displacement (on x-axis) $\Delta\underset{\to}{x} = (x_f - x_i)\hat{x}$

Average Velocity $<\underset{\to}{v}> = \frac{(x_f - x_i)}{t_f - t_i}\hat{x}$

Instantaneous Velocity $\underset{\to}{v} = \lim_{\Delta t \to 0}\left(\frac{\Delta \underset{\to}{x}}{\Delta t}\right)$

Average Acceleration $<\underset{\to}{a}> = \frac{\underset{\to}{v_f} - \underset{\to}{v_i}}{t_f - t_i}$

Instantaneous Acceleration $<\underset{\to}{a}> = \lim_{\Delta t \to 0}\left(\frac{\Delta \underset{\to}{v}}{\Delta t}\right)$

Kinematics

Constant $\underset{\to}{v} = v\hat{x}$; $\underset{\to}{x} = (x_i + vt)\hat{x}$

Constant $a = a\hat{x}$; $\underset{\to}{v} = (v_i + at)\hat{x}$

$\underset{\to}{x} = \left(x_i + v_i t + \frac{1}{2}at^2 \right)\hat{x}$; $v^2 = v_i^{\,2} + 2a(x - x_i)$

Free Fall

$\underset{\to}{a} = -9.8\frac{m}{s^2}\hat{y}$

$\underset{\to}{v} = (v_i - 9.8t)\hat{y}$

$\underset{\to}{y} = (y_i + v_i t - 4.9t^2)\hat{y}$

$v^2 = v_i^{\,2} - 19.6(y - y_i)$

Vector Algebra

$\underset{\to}{R} = \underset{\to}{A} + \underset{\to}{B}$

$R = \sqrt{A^2 + B^2 + 2AB\cos\Theta}$ [Θ is angle between $\underset{\to}{A}$ and $\underset{\to}{B}$]

$\tan\Theta_R = \frac{B\sin\Theta}{A + B\cos\Theta}$

Component of a Vector

$v_d = v\cos(\underset{\to}{v}, \hat{d})$

In xy-plane $\underset{\to}{v} = v_x\hat{x} + v_y\hat{y}$

$v_x = v\cos\Theta$, $v_y = v\sin\Theta$

$R = \underset{\to}{v_1} + \underset{\to}{v_2} + \ldots\ldots + \underset{\to}{v_N} = \Sigma \underset{\to}{v_i} = \Sigma v_{ix}\hat{x} + \Sigma v_{iy}\hat{y}$

$R_x = \Sigma v_{ix}$, $R_y = \Sigma v_{iy}$

$R = \sqrt{R_x^{\,2} + R_y^{\,2}}$ $\tan\Theta_R = \frac{R_y}{R_x}$

<u>Trig. Identities</u>

$\sin(\Theta_1 + \Theta_2) = (\sin\Theta_1\cos\Theta_2 + \cos\Theta_1\sin\Theta_2)$

$\cos(\Theta_1 + \Theta_2) = (\cos\Theta_1\cos\Theta_2 - \sin\Theta_1\sin\Theta_2)$

Projectile Motion

$\underset{\to}{a} = 0\hat{x} - 9.8\frac{m}{s^2}\hat{y}$

Initial Velocity $\underset{\rightarrow}{v_i} = (v_i \cos\Theta_i)\hat{x} + (v_i \sin\Theta_i)\hat{y}$

Velocity $v_x = v_i \cos\Theta_i$ $v_y = (v_i \sin\Theta_i - 9.8t)$

Position $x = (v_i \cos\Theta_i)t$ $y = (v_i \sin\Theta_i)t - 4.9t^2$

$y_{top} = \frac{v_i^2 \sin^2\Theta_i}{19.6}$ $t_{top} = \frac{v_i \sin\Theta_i}{9.8}$

Range $R = \frac{v_i^2 \sin 2\Theta_i}{9.8}$

Equation of Parabolic Path $y = y_i + x\tan\Theta i - 4.9\left(\frac{x}{v_i \cos\Theta_i}\right)^2$

Dynamics

Equilibrium $\Sigma_i \underset{\rightarrow}{F_i} \equiv 0$

Non-Zero $\underset{\rightarrow}{a}$ $M\underset{\rightarrow}{a} = \Sigma \underset{\rightarrow}{F_i}$ AT THAT POINT

AT THAT TIME
(Free Body Diagram!)

Forces
Weight $\underset{\rightarrow}{W} = -Mg\hat{y}$
Spring Force $\underset{\rightarrow}{F_{sp}} = -k\Delta x\hat{x}$

Friction
$f_s \le \mu_s n$ (Static)
$f_k = \mu_k n$ (Kinetic)

Circular Motion (Uniform in xy-plane)

Period $=T$ seconds

Angular Velocity $\underset{\rightarrow}{\omega} = \pm\frac{2\pi}{T}\hat{z}$ (Right hand rule)

Position $\underset{\rightarrow}{r} = R\hat{r}$ Velocity $\underset{\rightarrow}{v} = R\omega\hat{\tau} = \frac{2\pi R}{T}\hat{\tau}$

Centripetal Acceleration $\underset{\rightarrow}{a_c} = -R\omega^2\hat{r} = \frac{-v^2}{R}\hat{r}$

Centripetal Force (Required) $\underrightarrow{F_c} = -MR\omega^2\hat{r} = \frac{-Mv^2}{R}\hat{r}$

Gravitational Force

Two Point Masses $\underrightarrow{F_G} = \frac{-GM_1M_2}{r^2}$

Point Mass and Shell $r < R_{shell}$ $F_G = 0$

$r > R_{shell}$ $\underrightarrow{F_G} = \frac{-GM_{shell}m}{r^2}\hat{r}$

Point Mass and Uniform Sphere (Density d) of Mass M

$r < R$ $\underrightarrow{F_G} = \frac{-4\pi}{3}Gdmr\hat{r}$

$r > R$ $\underrightarrow{F_G} = \frac{-GMm}{r^2}\hat{r}$

Keplerian Orbits (Circular)

Planets $T_p^{\ 2} = \frac{4\pi^2}{GM_{Sun}}R_p^{\ 3}$

Earth Satellites $T_S^{\ 2} = \frac{4\pi^2}{GM_{Earth}}R_S^{\ 3}$

Conservation Laws

Mechanical Energy: Work $\Delta W = \underrightarrow{F} \bullet \underrightarrow{\Delta S} = F\Delta S\cos(\underrightarrow{F}, \underrightarrow{\Delta S})$

$= F_{\|}\Delta S$

[Vector Algebra: Scalar Product $\underrightarrow{A} \bullet \underrightarrow{B} = AB\cos(\underrightarrow{A}, \underrightarrow{B})$]

Kinetic Energy: $K = \frac{1}{2}MV^2$

Change of Potential Energy: $\Delta P = -\underrightarrow{F_{co}} \bullet \underrightarrow{\Delta S}$

$\underrightarrow{F_{co}}$: Conservative Force (Work done independent of path, only end-points matter)

Earth-Mass $P_g = Mgh$

Spring $P_{sp} = \frac{1}{2}kx^2$

Conservation of Mechanical Energy $K_f + P_f = K_i + P_i + W_{NCF}$

W_{NCF} = Work done by Non-Conservative Force

Potential Energy for $\underset{\rightarrow}{F_G}$

Two Point Masses $P_G = \frac{-GM_1M_2}{r}$

Point Mass and Shell $r > R_{Shell}$ $P_G = \frac{-GmM}{r}$

$r < R_{Shell}$ $P_G = \frac{-GmM}{R_{Shell}}$

Point Mass and Uniform Sphere $r > R$ $P_G = \frac{-GmM}{r}$

$r < R$ $P_G = \frac{-GmM}{R} - \frac{GMm}{2R}\left[1 - \frac{r^2}{R^2}\right]$

Linear Momentum

$\underset{\rightarrow}{p} = m\underset{\rightarrow}{v}$; Kinetic Energy $K = \frac{p^2}{2M}$

$\frac{\Delta \underset{\rightarrow}{p}}{\Delta t} = \Sigma \underset{\rightarrow}{F_i}$ $\underset{\rightarrow}{J} = < \underset{\rightarrow}{F_i} > \Delta t$

Impulse

Conservation Law (Many Finite Objects) If $\underset{\rightarrow}{F_{ext}} = 0$

$\Sigma \underset{\rightarrow}{p_i} = \text{constant}$

Two Body Collisions $\underset{\rightarrow}{p_1}' + \underset{\rightarrow}{p_2}' = \underset{\rightarrow}{p_1} + \underset{\rightarrow}{p_2}$ ALWAYS

$\underset{\rightarrow}{r_{cm}} = \frac{M_1\underset{\rightarrow}{r_1} + M_2\underset{\rightarrow}{r_2}}{M_1 + M_2}, \underset{\rightarrow}{v_{cm}} = \text{constant}$

Totally Elastic Collisions $\frac{1}{2}M_1v_1'^2 + \frac{1}{2}M_2v_2'^2 = \frac{1}{2}M_1v_1^2 + \frac{1}{2}M_2v_2^2$

(Kinetic Energy also conserved)

Totally Inelastic Collisions $\underset{\rightarrow}{v_1}' = \underset{\rightarrow}{v_2}'$

(Objects stick together)

Totally Elastic Head-On Collision

$$\underset{\rightarrow}{v_1}' = \left(\frac{M_1 - M_2}{M_1 + M_2}\right)\underset{\rightarrow}{v_1} + \left(\frac{2M_2}{M_1 + M_2}\right)\underset{\rightarrow}{v_2}$$

$$\underset{\rightarrow}{v_2}' = \left(\frac{M_2 - M_1}{M_1 + M_2}\right)\underset{\rightarrow}{v_2} + \left(\frac{2M_1}{M_1 + M_2}\right)\underset{\rightarrow}{v_1}$$

Non-Uniform Circular Motion (xy-plane)

Angular Acceleration $\underset{\rightarrow}{\alpha} = \alpha\hat{z}$

Tangential Acceleration $\underset{\rightarrow}{a_t} = R\alpha\hat{\tau}$

Angular Velocity $\underset{\rightarrow}{\omega} = (\omega_i + \alpha t)\hat{z}$

Tangential Velocity $\underset{\rightarrow}{v_t} = R\omega\hat{\tau}$

Angular "Position" $\underset{\rightarrow}{\Theta} = \left(\Theta_i + \omega_i t + \frac{1}{2}\alpha t^2\right)\hat{z}$

Displacement on Circle $S = R\Theta$

$\omega^2 = \omega_i^2 + 2\alpha(\Theta - \Theta_i)$

Rigid Body Motions

$\underset{\rightarrow}{r_{CM}} = \underset{\rightarrow}{r_{C \bullet G}} = \frac{\Sigma m_i r_i}{\Sigma m_i}$

$\Sigma m_i = M$

Translation

$\underset{\rightarrow}{v}$ is common $M\underset{\rightarrow}{a} = \Sigma \underset{\rightarrow}{F_i}$

$\underset{\rightarrow}{a}$ is common

Rotation

$\underset{\rightarrow}{\alpha}$ is common

$\underset{\rightarrow}{\omega}$ is common

To cause $\underset{\rightarrow}{\alpha}$ need Torque $\underset{\rightarrow}{\tau} = [\underset{\rightarrow}{r} \times \underset{\rightarrow}{F}]$

Vector Algebra: Cross Product $\underset{\rightarrow}{C} = [\underset{\rightarrow}{A} \times \underset{\rightarrow}{B}]$, $C = AB\sin(\underset{\rightarrow}{A}, \underset{\rightarrow}{B})$, $\underset{\rightarrow}{C} \perp \underset{\rightarrow}{A}$ and $\underset{\rightarrow}{B}$

$\tau = rF\sin(\underset{\rightarrow}{r}, \underset{\rightarrow}{F}) = rF_\perp = r_\perp F$

Dynamics $I\underset{\rightarrow}{\alpha} = \Sigma\underset{\rightarrow}{\tau_i}$

I: Moment of Inertia $I = \Sigma m_i r_i^2$

Kinetic Energy

Translation $K_{Tr} = \frac{1}{2}Mv^2$

Rotation $K_{Rot} = \frac{1}{2}I\omega^2$

Angular Momentum

Single Mass $\underset{\rightarrow}{l} = [\underset{\rightarrow}{r} \times \underset{\rightarrow}{p}]$ $\underset{\rightarrow}{l} = mr^2\omega\hat{z}$

Rigid Body $\underset{\rightarrow}{L} = I\underset{\rightarrow}{\omega}$

Conservation Law: If $\underset{\rightarrow}{\tau_{Ext}} = 0$, $\underset{\rightarrow}{L} = \text{Constant}$

Thermodynamics

<u>Pressure</u> $P = \frac{F}{A}$

Near Earth $\Delta P = -dg\Delta y$

In Liquid at Depth $P = P_A + dgh$

$P_A = 10^5 \frac{N}{m^2}$ (Atmospheric)

<u>Temperature</u> (Θ) NEEDED TO DEFINE EQUILIBRIUM

<u>Scales</u> $\frac{C}{5} = \frac{F-32}{9}$ $K = C + 273$

<u>Ideal Gas</u> $PV = Nk_BT = \mu RT$

$k_B = 1.38 \times 10^{-23}\frac{J}{K}$, $R = 8.36 J/mol/K$

$P = \frac{1}{3}m\frac{N}{V} < C^2 >$; $<\underset{\rightarrow}{C}> = 0$

$\frac{1}{2}m < C^2 > = \frac{3}{2}k_BT$

$C_{rms} = \sqrt{< C^2 >} = \sqrt{\frac{3k_BT}{m}}$

<u>Expansion:</u>
<u>Solids</u>

Linear $l = l_0[1 + \alpha(\Theta - \Theta_i)]$

Volume $V = V_0[1 + 3\alpha(\Theta - \Theta_i)]$

<u>Liquids</u> $V = V_0[1 + \beta(\Theta - \Theta_i)]$

Heat

Solids/Liquids $DQ = mC\Delta\Theta$ or mL

Calorimetry $\Sigma m_i C_i \Delta\Theta_i + \Sigma m_j L_j = 0$

Modes of Transfer
<u>Conduction</u> Solids/Immobile Liquids

Steady State $\frac{DQ}{\Delta t} = -KA\frac{\Delta T}{\Delta x}$

Radiation $\frac{DQ}{\Delta t} = Ae\sigma T^4$

$\sigma = 6\times 10^{-8}\ J/\sec/m^2/K^4$

Laws of Thermodynamics

First Law: Conservation of Energy

$\pm DQ \pm DW \pm dU = 0$

U = Internal Energy

Monatomic Gas (per Mol) $U_{MA} = \frac{3}{2}RT$ Diatomic Gas $U_{DA} = \frac{5}{2}RT$

Specific Heats: Gas

Constant Volume $(C_V)_{MA} = \frac{3}{2}R$, $(C_V)_{DA} = \frac{5}{2}R$ (per Mol)

Constant Pressure $C_P = (C_V + R)$

$\gamma = \frac{C_P}{C_V}$ (>1)

Processes

Isochoric $(\Delta V = 0)$ Constant Volume

$DQ = dU$ $C_V = \left(\frac{dU}{\Delta T}\right)_V$

Isobaric $(\Delta P = 0)$ Constant Pressure

$DQ = dU + P\Delta V$ $C_P = C_V + R$

Isothermic $(\Delta T = 0)$ Constant Temperature

$dU = 0$ $DQ = DW = RT\ln\left(\frac{V_f}{V_i}\right)$

Adiabatic $DQ = 0$ $PV^{\gamma} =$ Constant

Or

$TV^{\gamma-1} =$ Constant

Cyclic $dU_{Cycle} = 0$

$DW_{Cycle} =$ (Area of Loop in P vs. V Diagram)

Second Law: Direction of Thermodynamic Processes (Entropy)

Carnot Cycle: (4 Reversible Processes)

$$\frac{DQ_H}{T_H} + \frac{DQ_C}{T_C} = 0$$

Change of Entropy in Reversible Process

$$dS \overset{R}{=} \frac{DQ}{T}$$

Efficiency Engine $$\eta = \frac{DQ_H + DQ_C}{DQ_H} = 1 - \frac{T_C}{T_H}$$

Heat Pump, Coefficient of Performance

$$COP = \frac{T_H}{T_H - T_C}$$

Change of Entropy in any adiabatic process $dS \geq 0$!