MECHANICAL WAVES (TRAVELLING)

We begin our discussion of the wave phenomenon by considering waves in matter. The simplest definition of a wave is to call it a traveling disturbance (or equivalently, deviation from equilibrium). For instance, if you drop a stone on the surface of an undisturbed body of water you can watch the "disturbance" traveling radially out of the "point" of contact.

Formally, we can "construct" a wave in several steps. For simplicity, we take a wave traveling along x-axis.

Step 1. We need a disturbance D.
Step 2. D must be a function of x.
Step 3. D must also be a function of t.
Step 4. If x and t appear in the function in the combinations $(x \mp vt)$ the disturbance D cannot be stationary. It must travel along x with speed v.

Further,

$(x\text{-}vt)$ implies $\underset{\rightarrow}{v} = v\hat{x}[travel\ in + ive\, x - direction]$

$(x\text{+}vt)$ implies $\underset{\rightarrow}{v} = -v\hat{x}[travel\ in - ive\, x - direction]$

EXERCIZE: Put $D = A(x - t)^2$ and show that "parabola" travels.

Periodic Waves

The simplest wave is when $(x\text{-}vt)$ appears in a *sin* or *cos* function. $D = sin\,(x\text{-}vt)$ But this equation is not justified. First, since D is a disturbance it must have dimensions so we need

$$D = A\ Sin(x - vt)$$

Where A has the dimensions of D. Next, argument of *Sin* cannot have dimensions, so we need

$$D = A\ Sin\frac{(x - vt)}{\lambda}$$

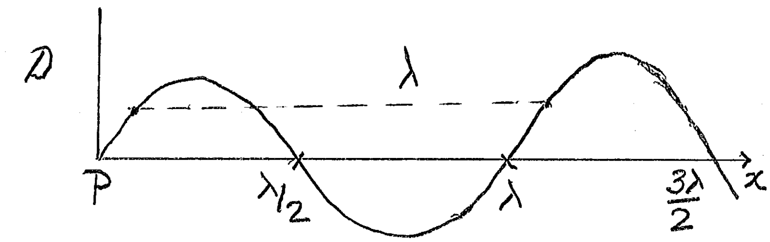
Where λ is a length. Since $\frac{v}{\lambda}$ has dimension of (1/Time), put $\frac{v}{\lambda} = \frac{1}{T}$

NOTE:

Periodic Wave

$$D = A\sin\left(\frac{2\Pi x}{\lambda} - \frac{2\Pi t}{T}\right)$$

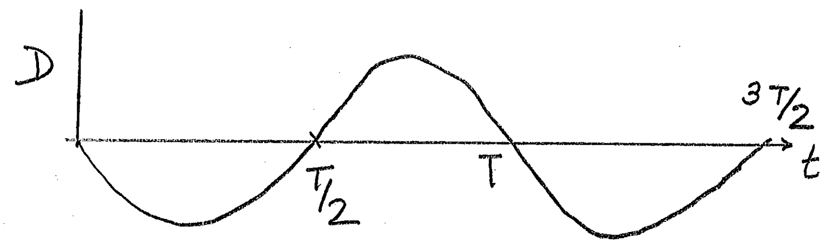
What is λ? Plot D as a function of x at t = 0.



The "wave" function repeats every λ meters so

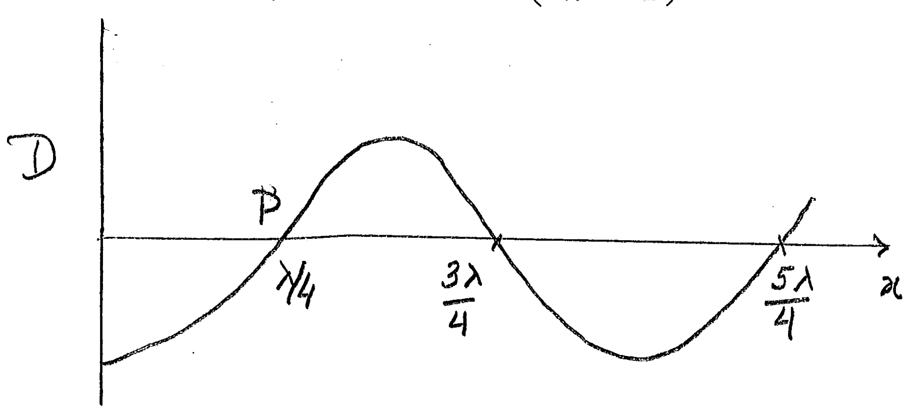
$\lambda =$ wavelength

As before, T is the period, Plot D at x = 0, D repeats every T seconds.



How far will travel in time $\frac{T}{4}$

$$t = \frac{T}{4} \qquad D = A\sin\left(\frac{2\Pi x}{\lambda} - \frac{\Pi}{2}\right)$$



Note: P has travelled to the right by $\frac{\lambda}{4}$ meters so

$$\text{Speed} = \frac{\lambda/4}{T/4} = \frac{\lambda}{T}$$

Velocity $$\underset{\rightarrow}{V} = \frac{\lambda}{T}\hat{x} = \lambda f\hat{x}$$

This makes sense because λ is distance moved in one period and the frequency f is the number of periods in $1\,\text{sec}$, so distance travelled in $1\,\text{sec}$ is λf.

EXERCISE: Take $D = A\sin\left(\frac{2\Pi x}{\lambda} + \frac{2\Pi t}{T}\right)$ and convince yourself that in time $\frac{T}{4}$ D moves TO THE LEFT by $\frac{\lambda}{4}$ so velocity

$$\underset{\rightarrow}{V} = -\lambda f\hat{x}$$

Note

Two kinds of period waves:

If $\underset{\rightarrow}{A} \parallel \hat{x}$ Longitudinal

If $\underset{\rightarrow}{A} \perp \hat{x}$ Transverse

Longitudinal: Variation of D along direction of propagation

Transverse: Variation of D perpendicular to direction of propagation.

Next, introduce a phase angle $\varnothing$ and we get $D = A\ Sin\left(\frac{2\pi x}{\lambda} - \frac{2\pi t}{T} + \varnothing\right)$ as the most general periodic wave. Note that 2π has been put in, as we know repeat angle for *Sin*. If you put $\varnothing = \pi$ you recover the Equation in some books.

$$D = A\ Sin\left(\frac{2\pi t}{T} - \frac{2\pi x}{\lambda}\right)$$

As shown

λ = Repeat Distance= wavelength

T = period, $\frac{1}{T} = f$ (frequency)

And $v = \lambda f$

Next, define $k = \frac{2\pi}{\lambda}$ (wave vector)

$\omega = 2\pi f$ (angular frequency)

$\omega = vk$

And we can write $D = A\ Sin(kx - wt + \varnothing)$ for any periodic wave traveling a long +*ive* x-axis with velocity $\underset{\rightarrow}{v} = \frac{\omega}{k}\hat{x}$

Similarly, $D = A\ Sin(kx + wt + \varnothing)$ is any periodic wave along –ive x-axis with

$$\underset{\rightarrow}{v} = -\frac{w}{k}\hat{x}$$