ENERGY CONSERVATION PRINCIPLE REVISITED: ELECTRIC POTENTIAL

Now that we have a new force

$$\vec{F}_E = \frac{k_e Q_1 Q_2}{r^2}\hat{r} \qquad \longrightarrow (1)$$

And a new field

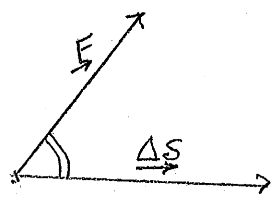
$$\vec{E} = \frac{\vec{F}_E}{q} \qquad \longrightarrow (2)$$

We need to take another look at the Principle of Conservation of Energy.

First, let us recall some of our discussion from 121 where we talked only of mechanical energy:

MECHANICAL WORK

$$\Delta W = \vec{F} \bullet \Delta\vec{S} = F\Delta S\cos(\vec{F}, \Delta\vec{S})$$



Where

$\vec{F}$ = Force Vector

$\Delta\vec{S}$ = Displacement Vector

Note $\Delta W = 0$ if $\vec{F} \perp \Delta\vec{S}$, that is only component of $\vec{F} \parallel \Delta\vec{S}$ does work

KINETIC ENERGY

Work stored in motion: if an object of mass M is sitting at rest, the work required to give it a speed V is stored as Kinetic Energy

$$K = \frac{1}{2}MV^2$$

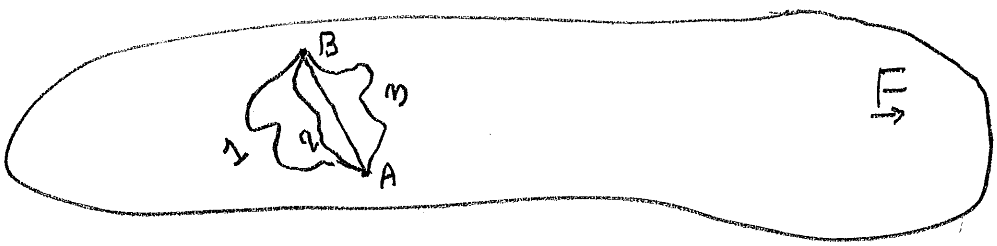
or since linear momentum $\vec{p} = M\vec{V}$ we can write $K = \frac{p^2}{2M}$

Potential Energy (U) Presents a greater conceptual challenge.

U is the mechanical work stored in a system when it is prepared (or put together) in the presence of a prevailing conservative force.

Supposed we have a region of space in which there is a prevailing force (weight near Earth's surface comes to mind). That is, at every point in this region an object will experience a force.

Let the object be at point B [First, notice that you can't let the object go as $\underset{\rightarrow}{F}$ will immediately cause $\underset{\rightarrow}{a}$ and object will move].



To define U at B we have to calculate how much work was needed to put the object at B in the presence of $\underset{\rightarrow}{F}$. Let us pick some point A, where we can claim that U is known, and calculate the work needed to go from A to B. As soon as we try to do that we realize that the only way we can get a meaningful answer is if the work required to go from A to B is independent of the path taken. So our prevailing force has to be special. Such a force is called a CONSERVATIVE FORCE – WORK DONE DEPENDS ONLY ON END-POINTS AND NOT ON THE PATH TAKEN.

If that is true we have a unique answer

$$\Delta W_1 = \Delta W_2 = \Delta W_3 = \Delta W_{AB}$$

and we can use this fact to calculate change in U in going from A to B

$$\Delta U_{AB} = -\underset{\rightarrow}{F} \bullet \underset{\longrightarrow}{\Delta S_{AB}}$$

NOTE THE NEGATIVE SIGN: it comes about because as stated above we cannot let the object go. In fact, the displacement from A to B must be carried out in such a way that the object cannot change its speed (if any). That is, we need to apply a force $-\underset{\rightarrow}{F}$ to balance the ambient $\underset{\rightarrow}{F}$ at every point. The net force will become zero at all points. ΔU_{AB} is work being done by $-\underset{\rightarrow}{F}$.

So when $\underset{\rightarrow}{F}$ is conservative ΔU_{AB} is unique. In the final step we can choose A such that $U_A = 0$. Then $U_B = -\underset{\rightarrow}{F} \bullet \underset{\longrightarrow}{\Delta S_{AB}}$

Using the above definition we discussed two cases

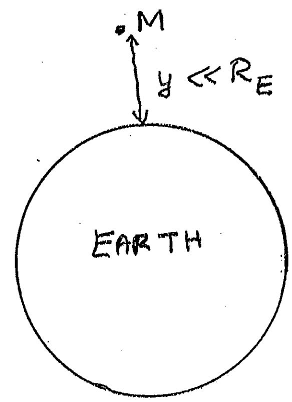
(i) U for Earth-Mass
System: Near earth
The Conservative Force is

$$\underset{\rightarrow}{F_g} = -Mg\hat{y}$$

Hence

$$U_g(y) = +Mgy \qquad (3)$$

Taking $U_g = 0$ for $y = 0$



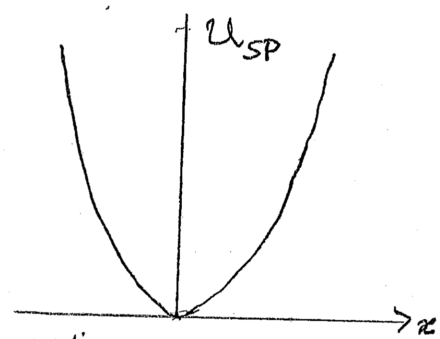
(ii) U for Mass attached to a spring.
Here the conservative force is

$$\underset{\rightarrow}{F} = -kx\hat{x}$$

Hence

$$U_{SP}(x) = \frac{1}{2}kx^2 \qquad (4)$$

Taking U = 0, when x = 0 [spring relaxed]



We were then able to write the principle of Mechanical Energy Conservation

$$K_f + U_{gf} + U_{spf} = K_i + U_{gi} + U_{spi} + W_{NCF} \qquad (5)$$

Where i and f refer to the initial and final state and W_{NCF} takes account of work done by non-conservative forces (friction for instance) in going from i to f.

When we got to thermodynamic systems we learnt that the system can change its energy in 3 ways.

Exchange heat <u>DQ</u> with its surroundings because of a temperature difference across a conducting wall

Have mechanical work <u>DW</u> done on or by it

Change the internal energy stored within it (dU)

The Conservation Law (First Law of Thermodynamics) Became

$$\pm DQ \pm DW \pm dU = 0$$

That is in any thermodynamic process the total change in energy must be <u>ZERO</u>

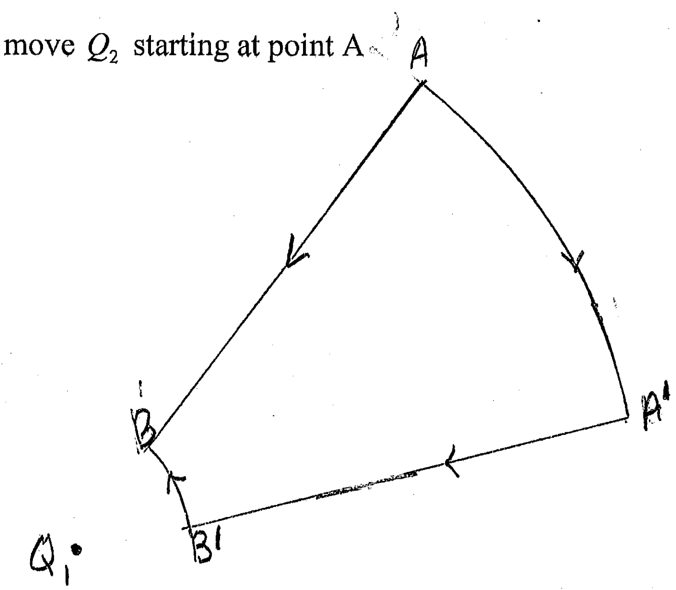
Now, let us look at $\underrightarrow{F_E}$ from equation (1). The first step is to examine if it is a conservative force. If so, we can define a potential energy for $\underrightarrow{F_E}$.

Note: in the following proof it is CRUCIAL to recognize that $\underrightarrow{F_E}$ acts only along the line joining the two charges and so work done by $\underrightarrow{F_E}$ being

$$\Delta W = \underrightarrow{F_E} \bullet \underrightarrow{\Delta S}$$

Will be ZERO for any displacement along the circumference (i.e. $\perp \hat{r}$)

Let us fix Q_1 and move Q_2 starting at point A.



First Path

$\underrightarrow{\Delta S}$ is AB along $\hat{r}$ work done will be

$$\Delta W_{AB} = \underrightarrow{F_E} \bullet \underrightarrow{\Delta S_{AB}}$$

Second Path

First, go from $A \to A'$ along circumference

$$\Delta W_{AA'} = 0$$

Next go along $\hat{r}$ from A' to B' when $A'B' = AB$

$$\Delta W_{A'B'} = \underrightarrow{F_E} \bullet \underrightarrow{\Delta S_{A'B'}} = \Delta W_{AB}$$

Next go from B' to B along circumference

$$\Delta W_{B'B} = 0$$

Hence

$$\Delta W_{AA'B'B} = \Delta W_{AB}$$

Work is independent of path $\underrightarrow{F_E}$ is indeed conservative, potential energy is definable.

$$\Delta U_E = -\underrightarrow{F_E} \bullet \underrightarrow{\Delta S}$$

Note the minus sign. As always, the force which does the work must be equal and opposite to the conservative force. ΔU_E is the change in electrostatic potential energy consequent upon a displacement $\underrightarrow{\Delta S}$.

The Energy Conservation Equation will now read

$$K_f + U_{gf} + U_{spf} + U_{Ef} = K_i + U_{gi} + U_{spi} + U_{Ei} + W_{NCF} \qquad (7)$$

Where again f and i, respectively refer to the final and initial states of the system, and W_{NCF} is work done by non-conservative forces while system goes from i to f.

In the present case it is useful to define a new quantity called Electrostatic Potential which is related to ΔU_E by the equation

$$\Delta V = \frac{\Delta U_E}{q} = \frac{-\underrightarrow{F_E} \bullet \underrightarrow{\Delta S}}{q}$$

Potential $\qquad = -\underrightarrow{E} \bullet \underrightarrow{\Delta S}$

ΔV = change of potential energy per unit charge

Like ΔU_E, ΔV is also a scalar, the dimensions are $ML^2T^{-2}Q^{-1}$ and the unit is Joule/coulomb which is called a Volt.