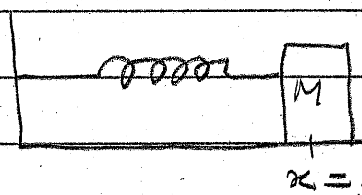
TEST QUESTIONS – Exam 1

1. An object of mass M is acted on by a force

$$\vec{F} = Cx\,\hat{x}$$

where C is a positive number. What are the dimensions of C? Will this object have linear harmonic oscillations? Justify your answer.

2. A spring mass oscillator consists of a mass M attached to a spring of spring constant k. M is placed on a frictionless horizontal table, as shown:



i) At x = 0, the spring is unstretched. If you pull the mass by an amount of x meters what is the force it will experience? why?

ii) If the mass is pulled by an amount A = 0.05m, what is its potential energy? why?

iii) What is the angular frequency of the oscillator in terms of k and M?

iv) By what factor would you change k if you wish to double the a) period, b) frequency.

v) For what values of x is the potential energy i) zero ii) maximum

vi) For what values of x is the potential energy equal to the kinetic energy? why?

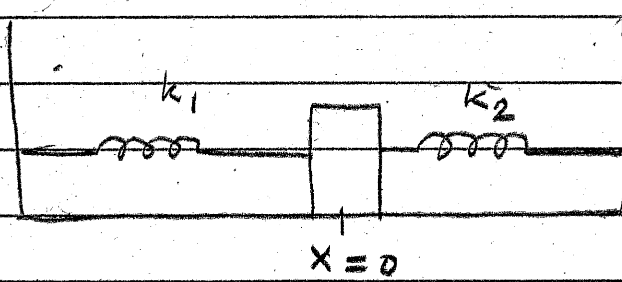
3a For a given value of k and M, if you hang the oscillator vertically will its frequency be different from $\frac{1}{2\pi}\sqrt{\frac{k}{M}}$? Justify your answer.

3b The potential energy of an object when it is displaced from equilibrium (at $x=0$) is given by

$$P = \frac{1}{2} C x^3$$

will this object exhibit linear harmonic oscillations? Justify your answer.

3c As shown, an object of mass 0.1 kg is held in place by two springs $k_1 = 490$ N/m, $k_2 = 510$ N/m. At $x=0$ both springs are unstretched. If the object is displaced by 0.1 m and let go, what is the frequency of the oscillation? Why?

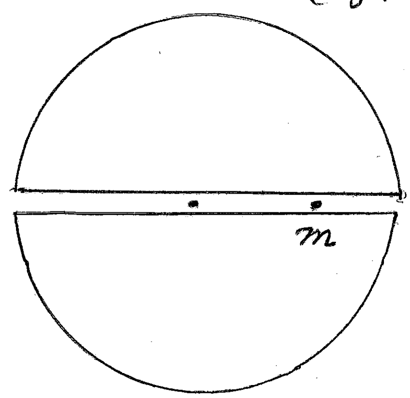


4. For a vertical spring-mass oscillator the amplitude can be controlled either by the experimenter (you!) or by the mass itself. How come? Explain clearly.

5. If a point mass m is located at a distance r from the center of a uniform solid sphere of density d, it will experience a force

$$\vec{F}_G = -\frac{4\pi}{3} G d m r \hat{r}$$

where G is the universal constant $6.7 \times 10^{-11} \frac{N\text{-}m^2}{(kg)^2}$.
If you make a small hole along a diameter and release m what will its motion be? Why?

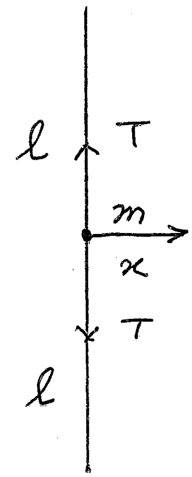


6. A linear harmonic oscillator is described by the equation

$$x = 0.05 \sin(6.28t + 0.1)$$

where distances are in meters and times in seconds. Write down its (a) amplitude, (b) frequency (c) phase angle (d) position at $t = 0$. (e) maximum velocity, (f) maximum acceleration. If the mass is 0.01 kg what is the spring constant?

7 A small mass m is held in place on a horizontal frictionless table by two strings each of length l and having tension T. If you displace it by a small amount x ($\ll l$) so that T doesnot change what is the ensuing motion when the mass is released. Why?



8 In the experiment with a simple pendulum why do you need small amplitudes to obtain linear harmonic oscillations?

9 Show that in a simple pendulum when the angle (θ) of swing is small ($\theta \ll 1$) the potential energy is quadratic in θ [Hint: assume that P.E. is zero when $\theta = 0$].

10 A grand father clock (which is essentially a pendulum) loses 5 minutes every hour. What should you do to its length to make it run right? Why?

11 A pendulum has a period of 1sec on Earth. What is its length? If you take it to the moon where $g_{moon} = \frac{g_{Earth}}{6}$ will its period increase or decrease? By what factor? If you want the period to stay unaltered, what should you change? Why?

12 A linear harmonic oscillator is represented by the equation

$$x = 0.01 \cos(2\pi t + \pi/3)\ m$$

Draw its position, velocity and acceleration as functions of time.

13 A wave is written as

$$D = 10\,N/m^2 + 11\,N/m^2 \sin(6.38x + 12.56t)$$

where x is in meters and t in secs

(i) What kind of wave is this?

(ii) Do you think that such a wave can exist?

14 FILL IN THE BLANKS IN THE EQUATION

$$D = \quad \sin \quad (x - vt)$$

For a travelling wave and explain the physical meaning of the symbols that you write.

15 What is the difference between a transverse and a longitudinal wave?

16 What is a travelling wave?

17 Two strings have the same length but one has a mass which is nine times that of the other. If both have the same tension in them which will have the higher wave velocity? Why? If you want both of them to have same wave velocity what must you do to the tension in the lighter string?

18 Show that on a stretched string a small amplitude periodic wave transports

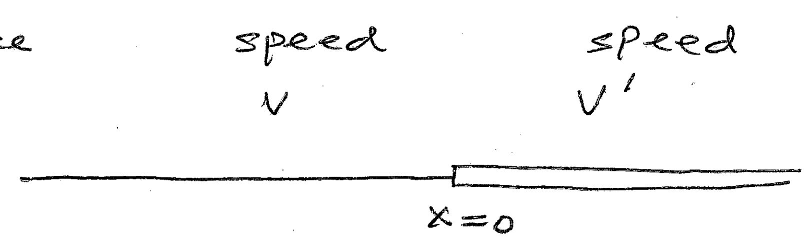
$$\eta = \frac{1}{2} \frac{A^2 \omega^2 F}{V}$$

Joules of energy per second.

where $A =$ amplitude
$\omega =$ angular frequency
$F =$ tension in string
$V =$ wave velocity.

19 How would η in Prob 18 change if you a) Double ω or b) Double V or c) double F or d) reduce A by a factor of 4.

20 Two strings are tied together at $x=0$. An incident wave arrives at $x=0$ as



$$y_i = A_i \sin(kx - \omega t)$$

and gives rise to a reflected wave

$$y_r = A_r \sin(kx + \omega t)$$

and a transmitted wave

$$y_t = A_t \sin(k'x - \omega' t)$$

i) What is the relation between ω and ω'?
ii) What determines k'?
(iii) Given that $\frac{A_r}{A_i} = \frac{V - V'}{V + V'}$, $\frac{A_t}{A_i} = \frac{2V'}{V + V'}$

show that if $V' = 0$, there is a phase change of π during reflection. That is a crest (trough) incident at $x=0$ is reflected as a trough (crest).

16 A wave $y_i = A_i \sin(kx - \omega t)$ travelling on a string arrives at $x=0$ where velocity changes from v to v' and it gives rise to a reflected wave $y_r = A_r \sin(kx + \omega t)$ and a transmitted wave $y_t = A_t \sin(k'x + \omega' t)$.

i) are these waves longitudinal or transverse?

ii) What is the relationship between ω' and ω?

(iii) We are told that the amplitudes are related by the equations

$$\frac{A_r}{A_i} = \frac{v - v'}{v + v'}, \quad \frac{A_t}{A_i} = \frac{2v'}{v + v'}$$

(iv) The energy transported per second by a wave is

$$P = \frac{1}{2} A^2 \omega^2 \frac{T}{v}$$

where T is the Tension in the string. Show that if $v' \gg v$ very little energy goes into the transmitted wave.

(v) Show that on reflection there is no change of phase if $v' \gg v$.

17. A mass M is subjected to a force $\vec{F} = Cx\hat{x}$ where C is a positive quantity. Will it exhibit linear harmonic oscillations? Justify your answer.

Test QUESTIONS - Contd.

1 What is Sound?

2 What is the speed of sound on the moon [The moon has no atmosphere]?

3 Sound can be thought of as a displacement wave or a pressure wave. Why is there a phase difference of $\frac{\pi}{2}$ between the displacement and the pressure? That is, where displacement is maximum pressure variation is zero and vice versa?

4 When reflection occurs at a fixed end (x = 0), there is a phase change of π and incident and reflected waves are written as: $y_i = A_i \sin(kx - \omega t)$; $y_r = A_i \sin(kx + \omega t)$. Show that the superposition of these two waves produces Nodes ~~separated~~ every ~~by~~ $\frac{\lambda}{2}$ starting from x = 0?

5 In a standing wave what are the separations between i) Two neighboring nodes, (ii) Two neighboring Antinodes (iii) A node and its antinode neighbor?

6 What is the speed of the wave:

$$y = 0.5 \sin(6.28x) \cos(12.56t) \text{ m}$$

where t is in secs? Write down its i) amplitude, (ii) frequency

7. The speed of sound in a gas is written as

$$V_s = \sqrt{\frac{\gamma k_B T}{m}}$$

Why is there a $\gamma \left(= \frac{C_p}{C_v}\right)$ in this equation?

8. The speed of sound in air is 340m/s. Can mechanical waves of wavelengths i) 100m, (ii) 10m, (iii) 0.1m and (iv) 0.001m be called "SOUND"?

9. Draw a periodic (Sine) sound wave as
i) a displacement wave
(ii) a pressure wave
at $t=0$.

10. Draw the first three modes of vibration of a wire fixed at both ends. If the length of the wire is 1 meter and the wave speed in it is 100m/s what are the frequencies of these modes?

11. The intensity of a sound wave in air is

$$I = \frac{1}{2} S_m^2 \omega^2 \rho_0 V_s = \frac{1}{2} \gamma P_0 \frac{S_m^2 \omega^2}{V_s}$$

Calculate the amplitude S_m of this wave if $\omega = 500$ rad/s, $\rho_0 = 1.2$ kg/m^3 and $V_s = 340$ m/s while $I = I_0 = 10^{-12}$ Watt/m^2

12. How would the answer to Prob 11 change if the Intensity was 60 db?

13. The amplitude of the pressure wave of Prob 12 is

$$P_m = \gamma k S_m P_0$$

where

$\gamma = 1.4$, $P_0 = 10^5$ N/m^2. How large is P_m for 60 db Sound?

14. When a tube (length L) is open at both ends the wavelengths of the modes in it are given by

$$\lambda_n = \frac{2L}{n} \quad n = 1, 2, 3, \ldots$$

If it is open at one end and closed at the other

$$\lambda_n = \frac{4L}{(2n-1)}, \quad n = 1, 2, 3, \ldots$$

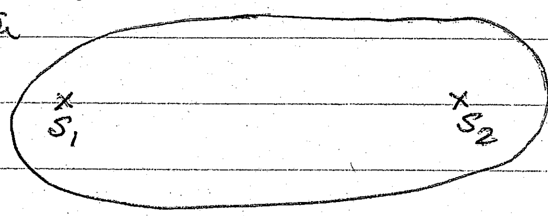
Why this difference?

15. A piano tuner finds that after some initial tuning of the "A" string she hears 4 beats with respect to a 440 Hz tuning fork. What are the possible frequencies of the sound

emitted by the string? (ii) If after the initial tuning she loosens the string very slightly and finds that now she hears 6 beats which of the two answers to part (i) is correct?

16. You are travelling toward a hill when you blow your horn ($f = 500Hz$). If your speed is 30 mph and the speed of sound is 340 m/s, how many beats will you expect to discern between your horn and the sound reflected by the hill?

17. Two sources of sound having same frequency and wavelength are 10 meters apart. If the wavelength of sound is <u>One meter</u> and the waves leave S_1 and S_2 in phase, how many <u>maxima</u> will you encounter as you walk around the path shown.



18. If in Prob 17 you were to stand exactly in the middle of the line joining S_1 and S_2 and heard NOTHING. What would it tell you about the phase difference of the waves starting at the same time from S_1 and S_2.