Y

SOLUTIONS - WEEK 3

(3-1) Since $\beta = 60$ dB, then $\log_{10}\left(\frac{I}{I_0}\right) = 6$. It means that $I = 10^{-6}$ W/m^2.
The power of the sound wave is $P_W = I \cdot 4\pi R^2$, where $R = 0.5$ cm is the radius of the ear. It yields

$$P_W = 3.14 \times 10^{-9} \text{ Watts}$$

The pressure amplitude P_m is given by $P_m = \gamma P_0 S_m K$, where $\gamma = 1.4$, $P_0 = 10^5$ N/m^2, $K = \frac{\omega}{v}$ is the wavenumber, and S_m is given by

$$I = \frac{1}{2}\gamma P_0 S_m^2 \frac{\omega^2}{v}$$

or,
$$S_m^2 = \frac{2Iv}{\gamma P_0 \omega^2}$$

It yields

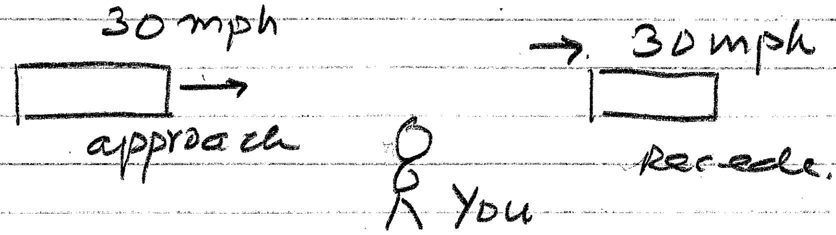
$$P_m = \gamma P_0 S_m K$$

$$= \gamma P_0 \sqrt{\frac{2Iv}{\gamma P_0 \omega^2}}\ \frac{\omega}{v}$$

$$= \sqrt{\frac{2I\gamma P_0}{v}} \qquad ; v = 340 \text{ m/s}$$

$$= 0.029 \text{ N/m}^2$$

3-2 $f = 300\,\text{Hz}$



speed of train $= 30\,\text{mph}$
$= 13.4\ \text{m/s}$.

Train's Source $V_{sou} = 13.4\ \text{m/s}$

speed of sound $V_s = 330\ \text{m/s}$

approach
$$f_A' = \frac{f}{1 - \frac{V_s}{V}} = \frac{300}{1 - \frac{13.4}{330}} = 313\ \text{Hz}$$

recede
$$f_R' = \frac{f}{1 + \frac{V_s}{V}} = \frac{300}{1 + \frac{13.4}{330}} = 288\ \text{Hz}$$

So frequency changes by 25 Hz

(3-3) We divide the process into two parts. The first part is when the wave reaches the hill from our car, and the second part is when the wave bounces back from the hill to our car.

For the first part, the frequency received by the hill is

$$f_{hill} = \frac{1}{1-\frac{V_{so}}{V_s}} f_0 \qquad ; \; V_{so} = 60 \text{ mph}, \quad V_s = 340 \text{ m/s}$$

$$= 542.8 \text{ Hz}$$

For the second part, the perceived frequency after the wave is reflected by the hill is (NO CHANGE DURING REFLECTION).

$$f = \left(1+\frac{V_o}{V_s}\right) f_{hill} \qquad ; \; V_o = 60 \text{ mph}$$

$$= 585.6 \text{ Hz}$$

(3-4) The frequency of the fork is 440 Hz. Since she receives 4 beats, then the frequencies of the sound from the string are either 436 Hz or 444 Hz. If, then, she tightens the string, which makes the string frequency larger, she will get 6 beats. It is possible only if the frequency after initial tuning is 444 Hz, for the number of beats will decrease (instead of increase) if the frequency is 436 Hz.

(3-5) Number of electrons is $\frac{10^{-9}}{1.6 \times 10^{-19}} = 6.25 \times 10^{9}$.

(3-6) In a volume of 1 m^3, there will be 8.95×10^6 grams of Cu. It is obtained by multiplying the density 8.95 grams/cm^3 with the volume. Since the molar mass of Cu is 64 grams, then in a volume of 1 m^3, there are

$$\frac{8.95 \times 10^6}{64} \text{ moles of Cu}.$$

The number of Cu atoms then can be obtained by multiplying this number with Avogadro's number, 6.02×10^{23}. So, the number of Cu atoms is

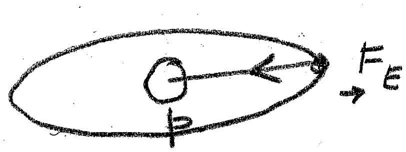
$$\frac{8.95 \times 10^6}{64} \times 6.02 \times 10^{23} = 8.4 \times 10^{28} / m^3$$

which is also the number of mobile electrons contained in the same volume because each atom contributes one free electron

(3-7) ~~A~~ The required centripetal force is $\vec{F}_c = -\frac{m_e v_e^2}{r}\hat{r}$.

It is provided by the Coulomb force $\vec{F}_E = -\frac{ke^2}{r^2}\hat{r}$

Hence $$\frac{ke^2}{r^2} = \frac{m_e v_e^2}{r}$$



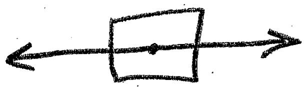
So, $$v_e^2 = \frac{ke^2}{m_e r}$$

where $k = 9 \times 10^9$ $N \cdot m^2/C^2$, $|e| = 1.6 \times 10^{-19}$ C, $m_e = 9 \times 10^{-31}$ kg, $r = 0.5 \times 10^{-10}$ m. It gives

$$v_e = 2.26 \times 10^6 \text{ m/s}.$$

(3-8) (i) The ~~s~~ spring must be compressed because the Coulomb force between these two charges is a repelling force.

(ii) Let's draw the force diagram for the right charge.

$-k_{sp}\Delta x\,\hat{x} = \vec{F}_{spring}$  $\vec{F}_{Coulomb} = \frac{k_e q^2}{d^2}\hat{x}$

Since the charge is in equilibrium, then

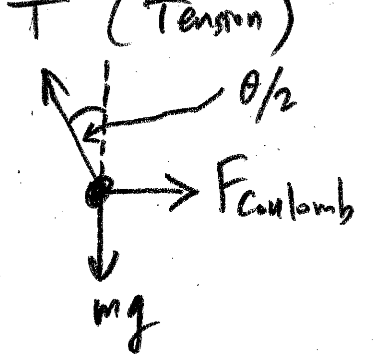
$$\vec{F}_{Coulomb} + \vec{F}_{spring} = 0$$

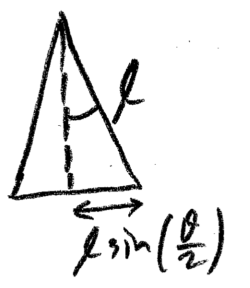
$$\frac{kq^2}{d^2} = k_{spring}\cdot \Delta X$$

where $d = 2$ mm is the separation between two charges at equilibrium, $k_{spring} = 10^3$ N/m is the spring constant, $k = 9\times 10^9\ N\cdot m^2/C^2$ is Coulomb constant, and $\Delta X = 1$ mm is the compression of the spring. It yields

$$q = 2.1\times 10^{-8}\ C.$$

(3-9) Let's draw the force diagram for any one of the two charges.





The Coulomb force is given by

$$\vec{F}_{Coulomb} = +\frac{kq^2}{d^2}\hat{x}$$

where $d = 2\ell\sin\left(\frac{\theta}{2}\right)$ is the separation between the two charges at equilibrium.

The charge is in equilibrium so the total force on it must be zero.

along x $\quad T\sin\left(\frac{\theta}{2}\right) - F_{coulomb} = 0$

along y $\quad T\cos\left(\frac{\theta}{2}\right) - mg = 0$

such that we get

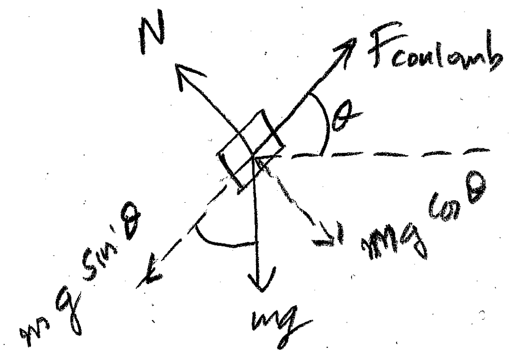
$$\tan\left(\frac{\theta}{2}\right) = \frac{F_{coulomb}}{mg}$$

$$mg\tan\left(\frac{\theta}{2}\right) = \frac{kq^2}{d^2}$$

$$q^2 = \frac{4mg\ell^2\tan\left(\frac{\theta}{2}\right)\sin^2\left(\frac{\theta}{2}\right)}{k}$$

$$q = 1.9\times10^{-8}\ C.$$

(3-10) Draw the force diagram on the upper charge.



Charge is in eqm.
(total force is zero.
Along Incline

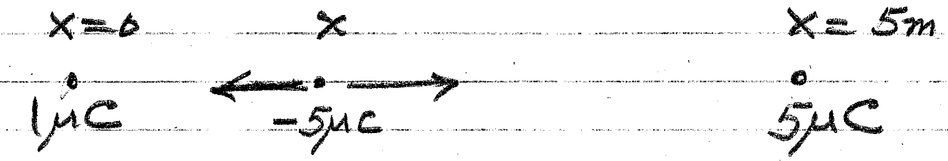
$$F_{coulomb} - mg\sin\theta = 0$$

$$\frac{kq^2}{\ell^2} = mg\sin\theta$$

$$\ell^2 = \frac{kq^2}{mg\sin\theta}$$

$$\ell = 0.96\ m.$$

3-11 The charge $-5\mu C$ must be placed between the $1\mu C$ and $6\mu C$ charges so that the total force on it is zero. Let it be at x meters



The two forces on it are

$$\underset{\rightarrow}{F}_{1,5} = -k \frac{1 \times 5 \times 10^{-12}}{x^2} \hat{x}$$

$$\underset{\rightarrow}{F}_{5,16} = +k \frac{5 \times 16 \times 10^{-12}}{(5-x)^2} \hat{x}$$

we need $\underset{\rightarrow}{F}_{1,5} + \underset{\rightarrow}{F}_{5,16} = 0$

$$+\frac{k \times 1 \times 5 \times 10^{-12}}{x^2} = \frac{k \times 5 \times 16 \times 10^{-12}}{(5-x)^2}$$

$$\frac{1}{x^2} = \frac{16}{(5-x)^2}$$

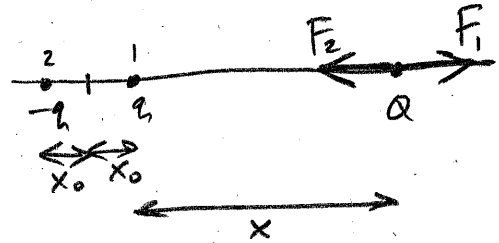
$$\frac{1}{x} = \frac{4}{5-x}$$

$$5 - x = 4x$$

$$x = 1m$$

(3-12) First, let's put a charge Q at $x = 1\,m$. We want to find the force on Q due to the dipole, where the two charges of dipole are located at $x_0 = \pm 0.01\,m$. Using the usual Coulomb interaction,

$$\vec{F} = \left(\frac{kQq}{(x-x_0)^2} - \frac{kQq}{(x+x_0)^2} \right) \hat{x}$$



$$\underset{\to}{F} = \left[\frac{kQq}{x^2}\left(1-\frac{x_0}{x}\right)^{-2} - \frac{kQq}{x^2}\left(1+\frac{x_0}{x}\right)^{-2}\right]\hat{x}$$

Using the approximation $(1+a)^n \simeq 1+na$ if $a \ll 1$, then

$$\left.\begin{aligned}\left(1-\frac{x_0}{x}\right)^{-2} &\approx 1+2\frac{x_0}{x}\\ \left(1+\frac{x_0}{x}\right)^{-2} &\approx 1-2\frac{x_0}{x}\end{aligned}\right\} \text{ since } \frac{x_0}{x} \ll 1.$$

So,

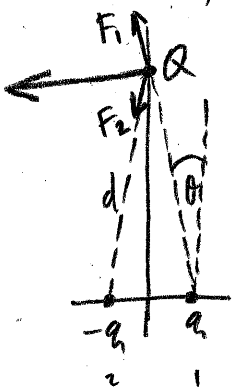
$$\begin{aligned}\underset{\to}{F} &= \left[\frac{kQq}{x^2}\left(1+\frac{2x_0}{x}\right) - \frac{kQq}{x^2}\left(1-\frac{2x_0}{x}\right)\right]\\ &= \left[\frac{4kQqx_0}{x^3}\right]\hat{x}\\ &= \left[\frac{2kQp}{x^3}\right]\hat{x}\end{aligned}$$

where the force points to positive x-direction. In vector notation,

$$\vec{F} = \frac{2kQ\vec{p}}{x^3} \quad \hat{x}$$

$= 360\,Q\ \hat{x}$ Newton

Second, if we put the charge Q at $y = 1m$, then



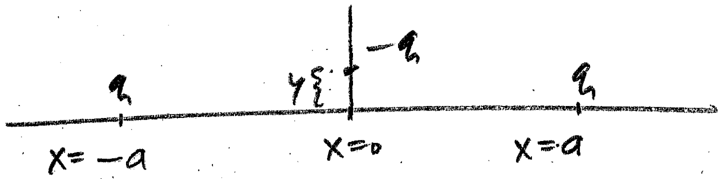
$$\begin{aligned}\sum F_y &= F_1\cos\theta - F_2\cos\theta\\ &= \frac{kQq}{d^2}\cos\theta - \frac{kQq}{d^2}\cos\theta \quad ; \ d=\sqrt{x_0^2+y^2}\\ &= 0\end{aligned}$$

$$\begin{aligned}\sum F_x &= -F_1\sin\theta - F_2\sin\theta\\ &= -2\frac{kQq}{d^2}\sin\theta\\ &= -2\frac{kQq}{x_0^2+y^2}\frac{x_0}{d}\\ \underset{\to}{F} &= -\frac{2kQqx_0}{(x_0^2+y^2)^{3/2}}\hat{x}\end{aligned}$$

Make $Y \gg x_0$

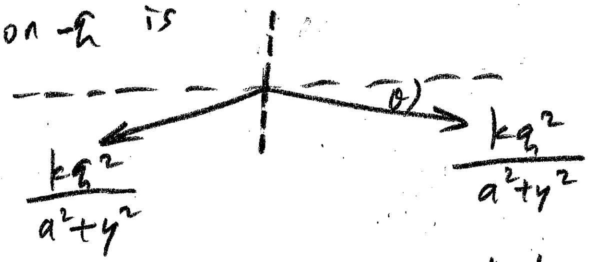
$$\underset{\to}{F} = -\frac{2kQqx_0}{y^3}\hat{x} = -\frac{kQ\underset{\to}{p}}{y^3}$$

(3-13)



$y \ll a$.

The force on $-q$ is



Since the system is symmetric relative to the y-axis, then the total force on $-q$ in x-direction is zero. On the y-direction, we have

$$\Sigma F_y = -2\frac{kq^2}{a^2+y^2}\sin\theta$$

$$= -2\frac{kq^2}{a^2+y^2}\frac{y}{\sqrt{a^2+y^2}}$$

$$= -\frac{2kq^2 y}{(a^2+y^2)^{3/2}}$$

Since $y \ll a$, $(a^2+y^2)^{3/2} \approx a^3$. So,

$$\vec{F} = -\frac{2kq^2}{a^3}\, y\, \hat{y}$$

(i) This force is proportional to the displacement y, so "the effective spring constant" for this oscillatory motion is

$$\frac{2kq^2}{a^3}.$$

The frequency of the motion then is given by

$$f = \frac{1}{2\pi}\sqrt{\frac{2kq^2}{ma^3}} = 2.14 \text{ Hz}.$$

(ii) The period of the motion is given by

$$T = \frac{1}{f} = 0.47 \text{ s.}$$

If it is the period of a simple pendulum, its length should be given by the relation

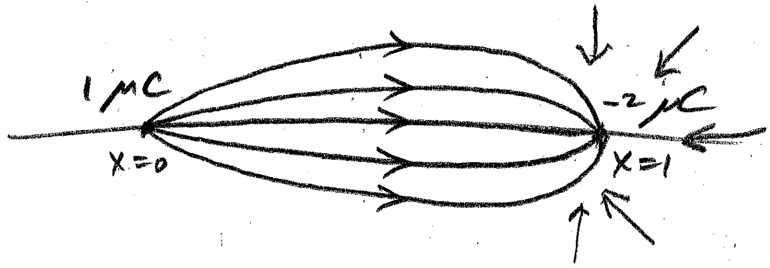
$$T = 2\pi\sqrt{\frac{L}{g}}$$

$$L = \frac{gT^2}{4\pi^2}$$

$$= 5.4 \text{ cm}$$

(3-14) The superposition of the forces due to the three charges on the charge at the center of circle is zero. If it is not zero, then it should point somewhere. But the system is symmetric, so there is no reason for any direction to be special. It implies that the total force on the charge at the center is zero.

(3-15)



No of lines must be proportional to the charge.