FORMULAE FOR WK-11

Maxwell's EQUATIONS

$$\sum_c \vec{E} \cdot \Delta\vec{A} = \frac{1}{\epsilon_0} \sum Q_i$$

$$\sum_c \vec{B} \cdot \Delta\vec{A} = 0$$

$$\sum_c \vec{E}_{NC} \cdot \Delta\vec{l} = -\frac{\Delta\phi_B}{\Delta t}$$

$$\sum_c \vec{B} \cdot \Delta\vec{l} = \mu_0 \sum I_c + \mu_0 I_D$$

$$I_D = \epsilon_0 \frac{\Delta\phi_E}{\Delta t} \qquad I_c = \frac{\Delta Q}{\Delta t}$$

EMwave speed $c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$ in Vacuum

$$\vec{E} = E_m \sin(kx - \omega t)\,\hat{y} \qquad E = cB$$

$$\vec{B} = B_m \sin(kx - \omega t)\,\hat{z}$$

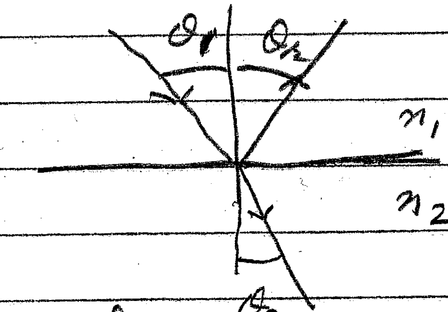
$$I = \frac{P_w}{4\pi r^2} \qquad \langle I \rangle = \frac{1}{2}\epsilon_0 c E_m^2 = \frac{c B_m^2}{2\mu_0} = \frac{E_m B_m}{2\mu_0}$$

$$k = \frac{2\pi}{\lambda}, \quad \omega = 2\pi f$$

Geom Optics - Fermat's Principle

Reflection Angle of Refl = Angle of Incidence

$$\theta_2 = \theta_1$$



Refraction

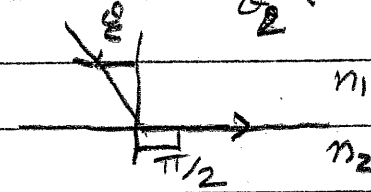
Snell's law $n_2 \sin\theta_2 = n_1 \sin\theta_1$

$$n = \frac{c}{v}$$

If $n_2 < n_1$, $\theta_2 > \theta_1$

Critical angle $\theta_2 = \pi/2$

$$\sin\theta_c = \frac{n_2}{n_1}$$



11-1 Units of E: N/C or V/m.

Units of B: $\frac{N\cdot s}{C\cdot m} = T$

Units of μ_0: $N/A^2 = N\cdot s^2/C^2$ $\left[\frac{T\text{-}m}{A}\right]$

$\Rightarrow$ Units of $\frac{EB}{\mu_0} = \frac{N}{C}\,\frac{N\cdot s}{C\cdot m} \times \frac{C^2}{Ns^2} = \frac{N}{m\cdot s} = \frac{N\cdot m}{m^2\cdot s} = \frac{J}{m^2\cdot s}$,

i.e, dimensions of $\frac{EB}{\mu_0}$ is $\frac{\text{Energy}}{\text{Area}\cdot\text{Time}}$

Alternate $B \rightarrow$ Tesla $\frac{B}{\mu_0} \rightarrow \frac{\text{Amp}}{m}$.

$\mu_0 \rightarrow$ Tesla-m/Amp.

$E \rightarrow \frac{\text{Volt}}{m}$ (Volt × Amp) = Pwr = J/s

$\frac{EB}{\mu_0} = \frac{\text{Volt} \times \text{Amp}}{m^2} = \frac{J}{s\text{-}m^2}$

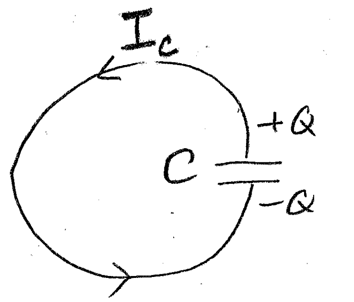
11-2 A light year is the distance that light travels in one year. 1 light yr = $(365 \times 24 \times 3600\ s) \times 3\times10^5\ km/s$

2×10^6 lt yrs $= 2\times10^6 \times (365\times24\times3600\ s)\times 3\times10^5\ km/s$

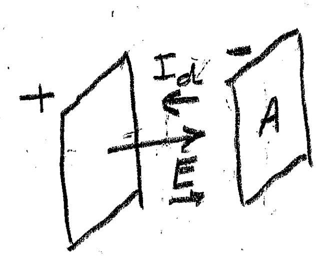
$= 1.9\times10^{19}$ km

11-3 conduction

The current in the wire, defined in the direction indicated by the arrows, is $I_c = \frac{\Delta Q}{\Delta t}$, where Q is the amount of charge on the capacitor.



The displacement current is defined as $\varepsilon_0 \frac{\Delta \Phi_E}{\Delta t}$, where Φ_E is the flux of $\vec{E}$ between through the capacitor plates.



For the capacitor,

$$\Phi_E = |\vec{E}| A = \frac{Q}{A\varepsilon_0} A = \frac{Q}{\varepsilon_0}$$

Hence

$$I_{displacement} = \varepsilon_0 \frac{\Delta \Phi_E}{\Delta t} = \varepsilon_0 \cdot \frac{1}{\varepsilon_0} \frac{\Delta Q}{\Delta t} = \frac{\Delta Q}{\Delta t} = I_C$$

∴ Conduction current in wire = Displacement current between capacitor plates

NO CONDUCTION IN A CAPACITOR!

11-4

Light, or more precisely visible light, is a transverse EM wave with a wavelength between 400 nm and 700 nm in vaccum.

Note that since light is a kind of EM wave, it inherits all the properties of EM waves, i.e, speed in vacuum is 3×10^8 m/s, is a transverse wave, etc.

(5) Sound & light are both waves. However there are notable differences between them. For example:

SOUND	LIGHT
There is no sound in vacuum.	Light propagates in vacuum.
It is a mechanical wave	It is an electromagnetic wave.
Speed IN AIR Is about 340 m/s	Speed is HUGE (3×10^8 m/s) in Vac.
It is a longitudinal wave in Gases	It is a transverse wave
Its frequency ranges between $20\ Hz < f < 20{,}000\ Hz$	Its frequency is about $4 \rightarrow 7 \times 10^{14}\ Hz$

11-6

Radiation is an EM wave.

Different types of radiation are distinguished by their frequencies and wavelengths.

Heat radiation (In Fra-Red).

wavelengths ~ a few microns.

freq ~ 10^{13} Hz

FM radio freq ~ 100 MHz, $\lambda \sim 3$ m

x-rays, wavelength $\lambda \sim 1$ nm

$f \sim 10^{17}$ Hz

γ-rays, $\lambda \sim 10^{-12}$ nm

$f \sim 10^{20}$ Hz

11-7 If $E = cB = \frac{B}{\sqrt{\mu_0 \varepsilon_0}}$,

then $\eta_E = \frac{1}{2}\varepsilon_0 E^2 = \frac{1}{2}\varepsilon_0 \frac{B^2}{\mu_0 \varepsilon_0} = \frac{B^2}{2\mu_0} = \eta_B$.

$\therefore$ If $E = cB$, then $\eta_E = \eta_B$

11-8. Let the distance that the flash occurs be x. Then 5 sec is the difference between the time it takes for light and sound to traverse x, i.e

$$\left| \frac{x}{c} - \frac{x}{330\,m/s} \right| = 5\,s$$

Since $c >> 330\,m/s$, this means $\frac{x}{330\,m/s} \simeq 5\,s$

$\therefore x = 330\,m/s \times 5\,s = 1650\,m$

11-9 ε_0 has units of $F/m = \frac{C^2}{N\cdot m}/m = \frac{C^2}{N\cdot m^2}$ $[M^{-1}Q^2L^{-3}T^2]$

μ_0 has units of $H/m = \frac{m^2 kg}{C^2}/m = \frac{m\,kg}{C^2}$ $[M Q^{-2} L]$

$\therefore$ Units of $\mu_0\varepsilon_0 = \frac{C^2}{N\cdot m^2}\cdot\frac{m\,kg}{C^2} = \frac{kg}{N\cdot m} = \frac{kg}{kg\cdot m/s^2\cdot m} = \frac{s^2}{m^2}$

Units of $\frac{1}{\sqrt{\mu_0\varepsilon_0}} = \frac{m}{s}$

$\Rightarrow$ Dimensions of $\frac{1}{\sqrt{\mu_0\varepsilon_0}} = LT^{-1}$ $\leftarrow$ $\mu_0\varepsilon_0 = L^{-2}T^2$

11-10 The intensity of EM waves is given by

$I = \frac{1}{2}\varepsilon_0 E_m^2 c$ where E_m is the amplitude of the $\vec{E}$-field.

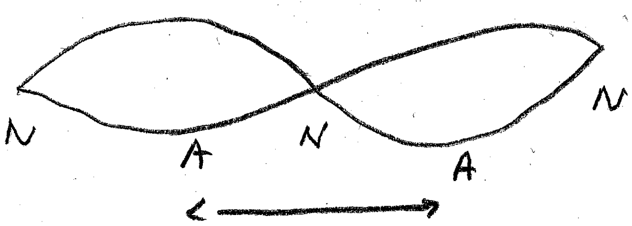
$$\Rightarrow E_m = \sqrt{\frac{2I}{\varepsilon_0 c}} = \sqrt{\frac{2 \times 1.4\times10^3\ W/m^2}{9\times10^{-12}\ F/m \times 3\times10^8\ m/s}} = 1.0\times10^3\ \frac{N}{C}$$

11-11 The antinodes of a standing EM wave, just as for standing waves on a string, are half-wavelengths apart. So the wavelength of the EM waves in the microwave must be $2\times6\,cm = 0.12\ m$.

For a traveling wave, we have the relation $v = \lambda f$.

So $v = 0.12\,m \times 2.45\times10^9\,Hz = 2.94\times10^8\ m/s$.

$\Rightarrow$ You can measure the speed of light using a microwave! (Provided you have the frequency).



11-12.

$$f = \frac{c}{\lambda} = \frac{3\times10^{8}\ m/s}{632\ nm} = \frac{3\times10^{8}}{632\times10^{-9}}\ Hz = 4.75\times10^{14}\ Hz$$

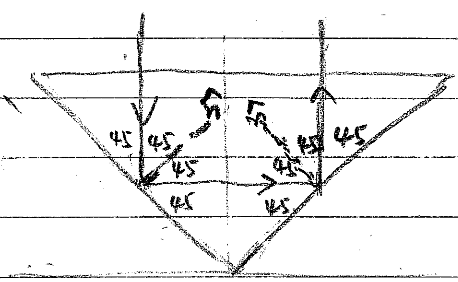
If laser travels in glass. $v = \frac{c}{n}$.

but frequency DOES NOT CHANGE!

So $f' = f = 4.75\times10^{14}\ Hz$.

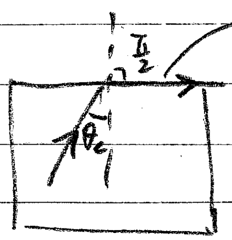
$$\lambda' = \frac{v}{f'} = \frac{c}{nf} = \frac{\lambda}{n} = \frac{632\ nm}{1.457} = 434\ nm.$$

11-13.



angle of reflection = angle of incidence both must be measured from the normal to the mirror.

11-14



$n_1 \sin\theta_c = n_2 \sin\frac{\pi}{2}$.

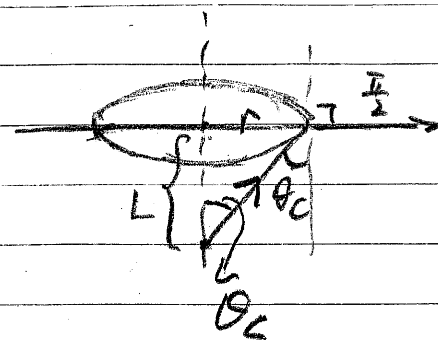
$\therefore \frac{n_1}{n_2} = \frac{\sin\frac{\pi}{2}}{\sin\theta_c} = \frac{1}{\sin\theta_c}$

n_2 must be less than n_1!

11-15. If $\begin{cases} n_2 = 1. \\ n_1 = n \end{cases}$ $\therefore \frac{n_1}{\cancel{n_2}} = \frac{1}{\sin\theta_c} = n$, or $n = \frac{1}{\sin\theta_c}$

11-16. Note

$\sin\theta_c = \frac{1}{n} = \frac{1}{1.33} \approx 0.75$



and $\sin\theta_c = \frac{r}{\sqrt{r^2+L^2}}$

$\therefore \frac{r}{L} = \tan\theta_c = \frac{\sin\theta_c}{\cos\theta_c} = \frac{0.75}{\sqrt{1-(0.75)^2}} = 1.14$ [If $\theta_i > \theta_c$ No light gets out, you have total internal Reflection]

$\therefore r = 1.14 \times 50\,cm = 0.57\,m$

Diameter $= 2r = 1.14\,m$