FORMULAE FOR WEEK 10

Spring Mass oscillator (Mechanical)

$$\frac{1}{2}kA^2 = \frac{1}{2}kx^2 + \frac{1}{2}m\left(\frac{\Delta x}{\Delta t}\right)^2$$

$$x = A\cos\omega t \qquad \omega = \sqrt{\frac{k}{M}}$$

Electrical Oscillator

$$\frac{Q^2}{2C} = \frac{q^2}{2C} + \frac{1}{2}L\left(\frac{\Delta q}{\Delta t}\right)^2$$

$$q = Q\cos\omega t \qquad \omega = \frac{1}{\sqrt{LC}}$$

$\sim$ $\mathcal{E} = \mathcal{E}_m \sin\omega t$

Resistor $i = \frac{\mathcal{E}_m}{R}\sin\omega t \qquad \langle P_w \rangle = \frac{\mathcal{E}_m^2}{2R}$

Capacitor $i_c = \mathcal{E}_m C\omega \cos\omega t \qquad \langle P_W \rangle = 0$

Inductor $i_L = -\frac{\mathcal{E}_m}{\omega L}\cos\omega t \qquad \langle P_W \rangle = 0$

L - C - R series circuit

$$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

$$\tan\theta = \frac{\omega L - \frac{1}{\omega C}}{R}$$

FIELD EQNS

$$\sum_c \vec{E}\cdot\Delta\vec{A} = \frac{1}{\varepsilon_0}\sum Q_i \qquad \sum_c \vec{B}\cdot\Delta\vec{A} = 0$$

$$\sum_c \vec{B}\cdot\Delta\vec{l} = \mu_0 \sum I_i \qquad \sum_c \vec{E}_{NC}\cdot\Delta\vec{l} = -\frac{\Delta\Phi_B}{\Delta t}$$

$$i_D = \varepsilon_0 \frac{\Delta \phi_E}{\Delta t} \qquad \phi_E = \vec{E} \cdot \Delta \vec{A}$$

Maxwell's EQNS.

$$\sum_c \vec{E} \cdot \Delta \vec{A} = \frac{1}{\varepsilon_0} \Sigma \Delta q$$

$$\sum_c \vec{B} \cdot \Delta \vec{A} = 0$$

$$\sum_c \vec{B} \cdot \Delta \vec{l} = \mu_0 \Sigma I_c + \mu_0 \varepsilon_0 \frac{\Delta \phi_E}{\Delta t}$$

$$\sum_c \vec{E}_{NC} \cdot \Delta \vec{l} = - \frac{\Delta \phi_B}{\Delta t}$$

Week 10

10-1) When we pull M to $x = A$, we give the system potential energy. Then, upon releasing the mass, the energy is converted into kinetic energy by the force of the spring, then back into potential energy stored in the spring. The energy equation is written:

$$E_{tot} = \tfrac{1}{2} kx^2 + \tfrac{1}{2} mv^2 = \tfrac{1}{2} k(A)^2 \quad \rightarrow (1)$$

In the circuit, we first charge the capacitor with potential V by closing the switch S_1. Then, we open switch S_1 and release the energy stored in C by closing S_2. The potential then drives a current in the lower circuit, converting energy stored in the capacitor to energy stored in the magnetic field of the inductor. The Inductor then uses this energy to drive a new current that is then stored in the capacitor as a potential between the plates. The energy equation for this system is:

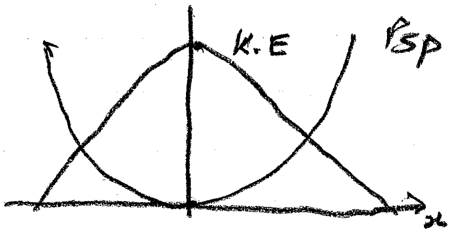
$$E_{tot} = \tfrac{1}{2} CV^2 + \tfrac{1}{2} L i^2$$

But $C = \frac{Q}{V}$ so $E_{tot} = \frac{1}{2}\frac{Q^2}{C}$

$$\tfrac{1}{2} Q^2/C = \tfrac{1}{2} q^2/C + \tfrac{1}{2} L i^2 = \overbrace{\frac{1}{2}\frac{q^2}{C}}^{U_E} + \overbrace{\frac{1}{2}\left(\frac{\Delta q}{\Delta t}\right)^2}^{U_B} \quad \rightarrow (2)$$

From Eq. (1) we learnt that x will vary as

$x = A \cos \omega t$, where $\omega = \sqrt{\frac{k}{M}}$ because



potential energy changes to kin Energy and vice versa

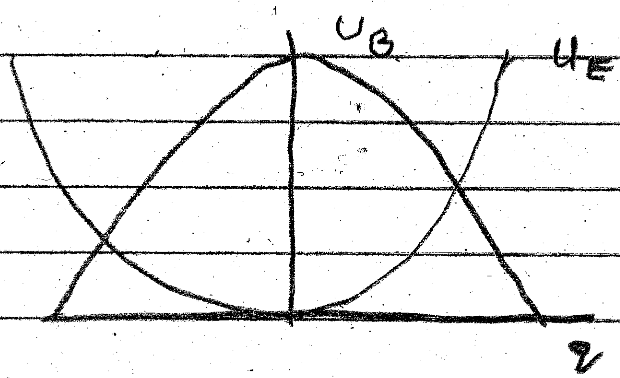
10-1 Eq (2) is the Exact equivalent if we identify

$q \longleftrightarrow x$

$\frac{1}{C} \longleftrightarrow k$ (spring const.)

$L \Leftrightarrow M$ (Mass)

so now Energy stored in E-field in the Capacitor changes into energy stored in B-field in the inductor and vice versa



E collapses generating B and B collapses generating E.

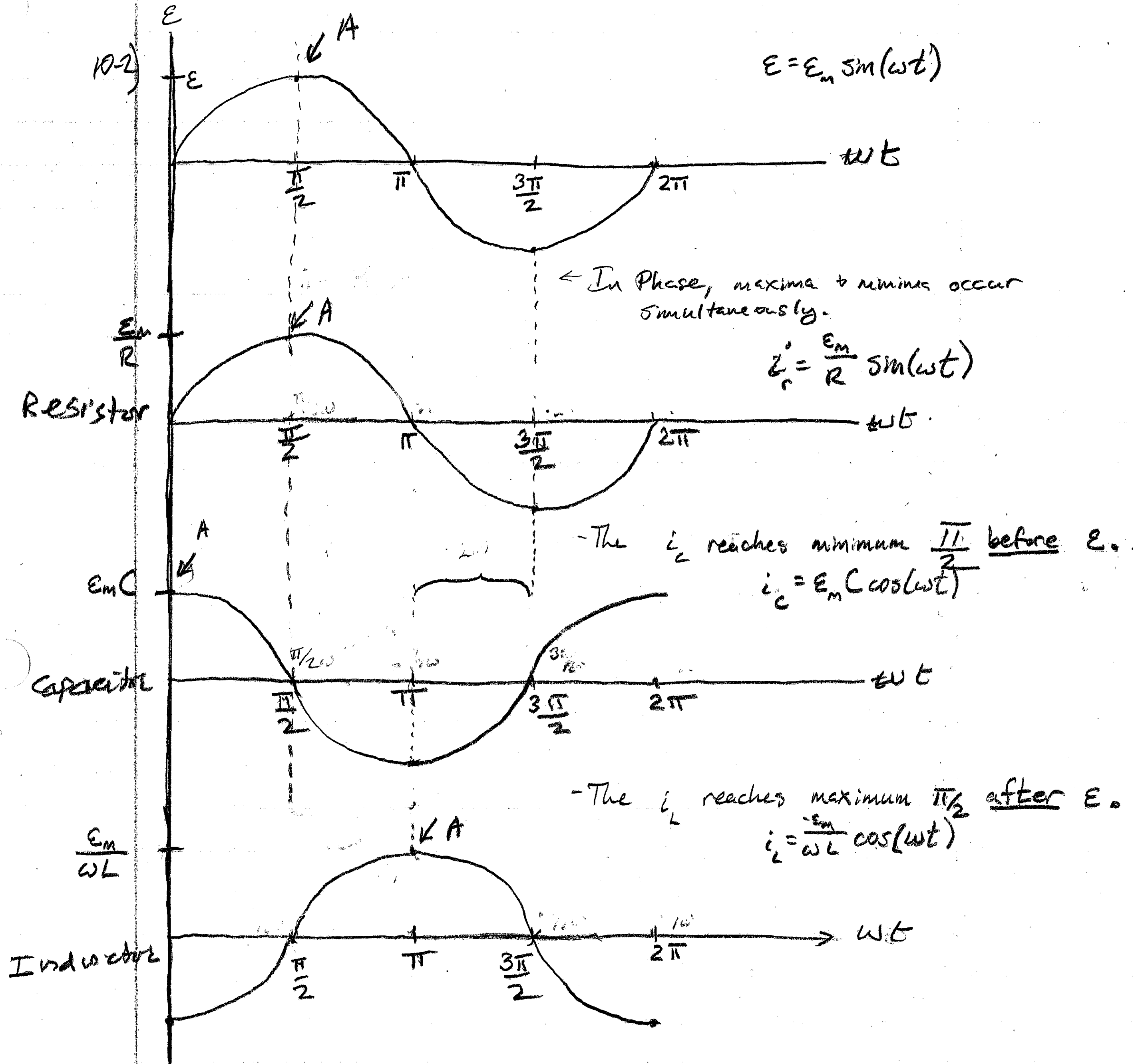
and therefore q varies as

$$q = Q \cos \omega t$$

where

$$\omega = \sqrt{\frac{1}{LC}}$$

So Eq (1) and (2) represent the same basic physical phenomenon — LINEAR HARMONIC OSCILLATIONS — EQ 1 - mechanical EQ 2 — Electrical.



Look carefully at the points labelled A

Recall that in RC circuit Current precedes Potential

in LC circuit Potential precedes Current

10.3) For our circuit elements, the current is given as:

$$i_R = \frac{\varepsilon_m}{R}\sin(\omega t), \quad i_C = \varepsilon_m C\omega\cos(\omega t), \quad i_L = \frac{-\varepsilon_m}{\omega L}\cos(\omega t)$$

If we double ω (frequency), and the

i) current increases by a factor of two, we see that it must be a capacitor since the Amplitude is proportional to ω.

ii) current reduces by a factor of two, we see that it must be an inductor since the current amplitude is inversely proportional to ω.

iii) current stays the same, we must have a resistor since the current amplitude of a resistor does not depend on ω.

10-4) $\mathcal{E} = 150 \sin(\omega t)$ Volts $= A$

1.) $V_{rms} = \frac{A}{\sqrt{2}} = \frac{150}{\sqrt{2}} \simeq 106.1 \; V_{rms}$

2.) $I = \frac{\mathcal{E}}{R} = 1.5 \sin(\omega t)$ Amps

$I_{rms} = \frac{A}{\sqrt{2}} \simeq 1.06$ Amps

3.) Energy absorbed per second in resistor is given by:

$$P = I_{rms} V_{rms} = \frac{V_{rms}^2}{R} = I_{rms}^2 R = 112.5 \text{ Watts}$$

$$\left[\langle \sin^2 \omega t \rangle = \frac{1}{2} \text{ so } \left\langle \frac{\mathcal{E}_m^2 \sin^2 \omega t}{R} \right\rangle = \frac{\mathcal{E}_m^2}{2R} \right]$$

10-5) $V_{rms} = \frac{A}{\sqrt{2}}$ so $A_{max} = \sqrt{2} \cdot V_{rms\,max} \simeq 162$ Volts

$A_{min} = \sqrt{2} \cdot V_{rms\,min} \simeq 155$ Volts

10-6) $i_m = i_L = \left| \frac{-\mathcal{E}_m}{\omega L} \right|$ at maximum.

$10^{-3} = \frac{5}{(2\pi \cdot 600) L}$ so $L \simeq 1.33$ H

NONE Because

$\mathcal{E} = \mathcal{E}_m \sin \omega t$

$i = -\frac{\mathcal{E}_m}{\omega L} \cos \omega t$

Average over cycle $\langle \sin \omega t \cos \omega t \rangle = 0$

10-7) At maximum,

$$i_m = i_c = |e_m C \omega|$$

$$10^{-3}A = |5V C (2\pi \cdot 600 s^{-1})|$$

$$\therefore C \approx 5.31 \times 10^{-8} F$$

10-8) By looking at the circuit, we see there is an inductor, capacitor and resistor in series. An inductor is going to resist a change in current, so we see that as $\omega \to \infty$, the voltage is oscillating too quickly to allow the inductor to let any current pass. Also, if $\omega \to 0$, we have a DC circuit, and as we all know from lecture, a capacitor doesn't allow any charge to flow through it so at DC (constant voltage) the current will be zero.

<u>Given</u> $i_m = \dfrac{\mathcal{E}_m}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}}$.

if $\omega \to \infty$ $(\omega L >> R)$ $\quad i_m \to \dfrac{\mathcal{E}_m}{\omega L}$. INDUCTOR CUTS-OFF CURRENT.

if $\omega \to 0$ $(\frac{1}{\omega C} >> R)$. $\quad i_m \to \mathcal{E}_m \omega C$ Capacitor Cuts-OFF Current.

Maximum i_m when $\omega L = \dfrac{1}{\omega C}$

$$i_m = \frac{\mathcal{E}_m}{R}.$$

10-9 $\quad i = i_m \sin \omega t$

$V = \mathcal{E}_m \sin(\omega t + \phi)$.

Power $\quad P(t) = i\,V$.

$$= i_m \mathcal{E}_m \sin(\omega t) \sin(\omega t + \phi)$$

$$= i_m \mathcal{E}_m \left[\sin^2 \omega t \cos\phi + \sin \omega t \cos \omega t \sin\phi\right]$$

average over a cycle

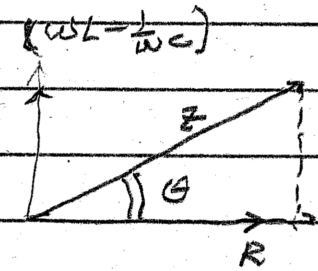
$$\langle \sin^2 \omega t \rangle = \frac{1}{2}$$

$$\langle \sin \omega t \cos \omega t \rangle = 0$$

So

$$\langle P \rangle = \frac{i_m \mathcal{E}_m \cos\phi}{2}$$

$$i_m = \frac{\mathcal{E}_m}{Z}$$



and $\quad \cos\phi = \frac{R}{Z}$.

So $\quad \langle P \rangle = \frac{\mathcal{E}_m^2}{2Z} \cos\phi$ $\qquad Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$

$$= \frac{\mathcal{E}_m^2}{2R} \cos^2\phi$$

10-10) Maxwell was concerned because we can see that if we have a change in the flux Φ_B it causes an non-Coulomb electric field.

Since we believe in the basic symmetry of nature It then stands to reason that if we were to have a change in the flux of E, we should generate a $\vec{B}$ field. However, looking at equation (1) we see that there is no term to describe a $\vec{B}$ field caused by a change in flux of $\vec{E}$, so the equation appears - and indeed is - incomplete. Maxwell proposed the existence of a displacement current $i_D = \varepsilon_0 \left(\frac{\Delta \Phi_E}{\Delta t}\right)$ to "complete" what are now called Maxwell's equations.

10-11) A conduction current involves a movement of mobile electrons inside a conducting material due to an applied electric field. It is a measure of the total charge flowing per second through some area.

$$I_c = \frac{\Delta q}{\Delta t}$$

A displacement current is generated by changing the flux of $\vec{E}$

$$i_D = \varepsilon_0 \frac{\Delta \Phi_E}{\Delta t}$$

I_c exists only in a conductor

i_D exists in Vacuum.