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PHYS 122 | **Exam II** | **November 3, 2011**
Prof. S. M. Bhagat

Name: SOLUTION

(Sign in ink, print in pencil)

Notes

1) There are four (6) problems in this exam. Please make sure that your copy has all of them.
2) Please show your work indicating clearly what formula you used and what the symbols mean. Just writing the answer will not get you full credit. In stating vectors give both magnitude and direction.
3) Write your answers on the sheet provided.
4) Do not forget to write the units
5) Do not hesitate to ask for clarification at any time during the exam. You may buy a formula at the cost of one point.

Take Care! God Bless You!

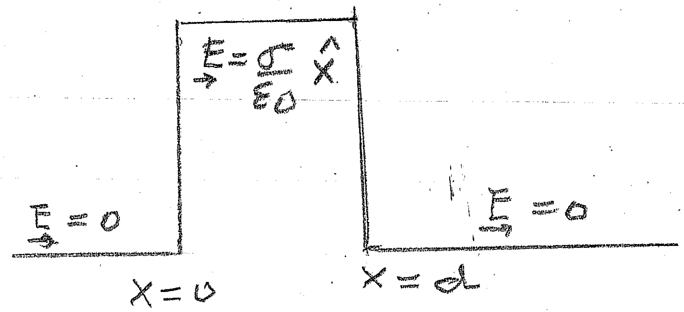
$$k = 9\times 10^9 \frac{N\cdot m^2}{C^2}$$

$$\varepsilon_0 = 9x10^{-12}\frac{F}{m}$$

Mass of proton	$m_p = 1.6x10^{-27}kg$
Mass of electron	$m_e = 9x10^{-31}kg$
Elementary Charge	$e = 1.6x10^{-19}C$

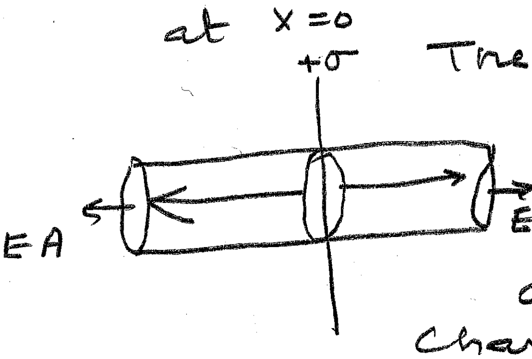
NO CALCULATORS!

Prob1 Write down Gauss's Law for an $\underline{E}$ – field and use it to show how you would generate the $\underline{E}$ – field shown below: (6,10)



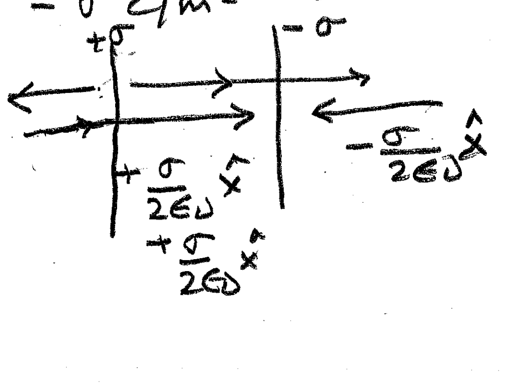
The total flux of $\vec{E}$ through a closed surface is determined solely by the charges enclosed by the surface: $\sum_c \vec{E}\cdot\Delta\vec{A} = \frac{1}{\epsilon_0}\sum Q_i$

To create the field above start with a sheet carrying σ c/m² and located at $x=0$



The $\vec{E}$-field is perpendicular to the sheet so we choose a cylinder to calculate $\sum_c \vec{E}\cdot\Delta\vec{A} = EA + EA = 2EA$ and this must equal $\frac{1}{\epsilon_0}\times$ charge on disk of area A so

$2EA = \frac{1}{\epsilon_0}\sigma A$ $\quad \vec{E} = \frac{\sigma}{2\epsilon_0}\hat{x}$ on the right and $-\frac{\sigma}{2\epsilon_0}\hat{x}$ on the left. Next, put sheet with $-\sigma$ c/m² at $x = d$. we will get $-\frac{\sigma}{2\epsilon_0}\hat{x}$ and $+\frac{\sigma}{2\epsilon_0}\hat{x}$



with both sheets in place $\vec{E} = 0$ for $x < 0$ or $x > d$ and between the sheets both add to give

$$\vec{E} = \frac{\sigma}{\epsilon_0}\hat{x}$$

Prob 2a Under stationary conditions, where would the extra charge on a conducting object reside? Why? (4)

Under stationary conditions mobile charges inside must be at rest so $E_{inside} \equiv 0$. By Gauss' law $\sum q_i = 0$ so any extra charge must reside on the surface.

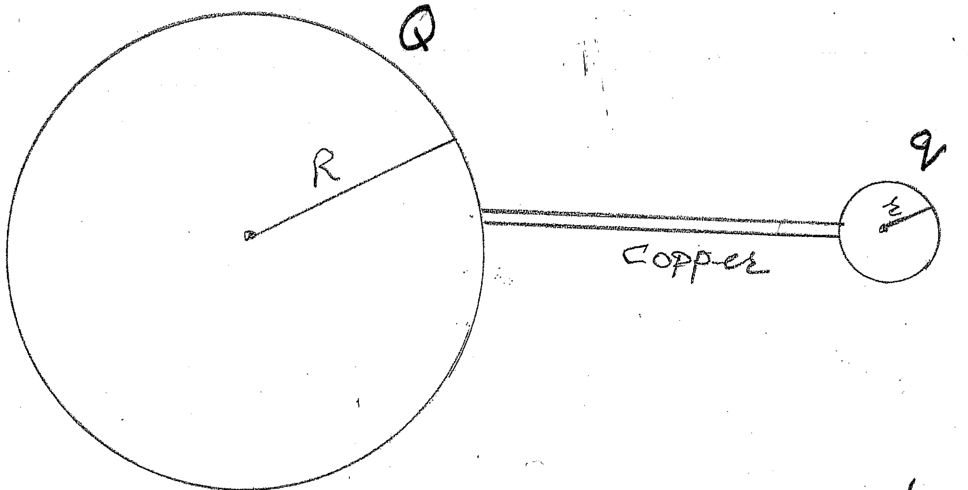
Prob 2b Why is the surface of a conductor an equipotential? (4)

If charges are not to move $\vec{E}_{surf}$ must be perpendicular to surface. $\Delta V = -\vec{E} \cdot \Delta\vec{s}$ so if $\vec{E} \perp \Delta\vec{s}$

$\Delta V = 0$.

Prob2c Two conducting spheres are connected by a copper rod and carry charge. If R = 10r, what is the ratio of the $\underrightarrow{E}$ – fields, E(R) and E(r) on the surfaces of the spheres? Why?

(8)



By problem 2a,b charges must be on surface, say Q on R & q on r, and surface must be equipotential

$$V(R) = \frac{Q}{4\pi\varepsilon_0 R} = \frac{q}{4\pi\varepsilon_0 r} = V$$

E field on Surf. of R : $E(R) = \dfrac{Q}{4\pi\varepsilon_0 R^2} = \dfrac{V}{R}$

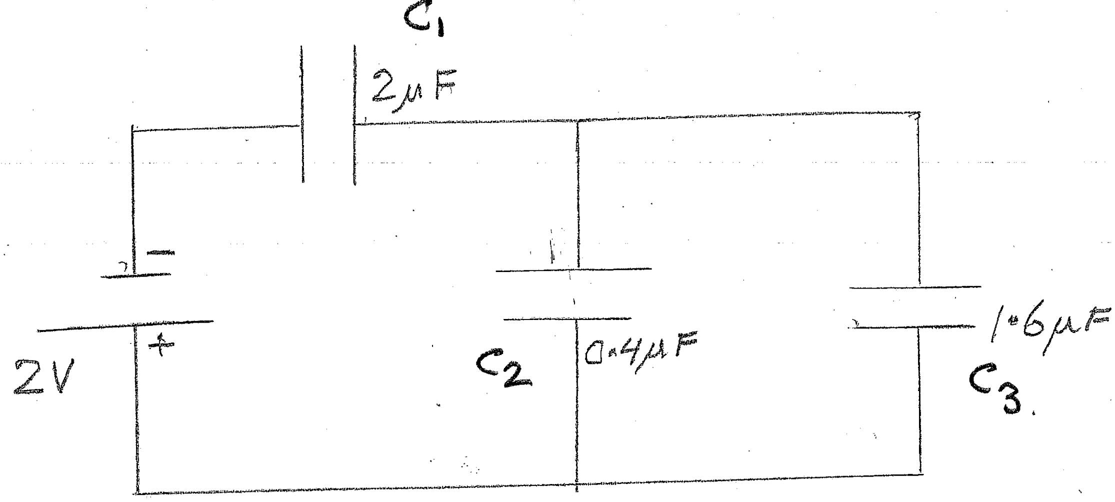
on r $E(r) = \dfrac{q}{4\pi\varepsilon_0 r^2} = \dfrac{V}{r}$

$$\frac{E(r)}{E(R)} = \frac{R}{r} = 10.$$

Prob 3a In order to charge a capacitor, one requires $U_E = \frac{Q^2}{2C}$ Joules of work, where does this energy go? Why? (6)

The energy is stored in the $\vec{E}$-field between the plates of the Capacitor. Once there is charge on the plates, the space between them is filled by an $\vec{E}$-field.

Prob 3b In the circuit shown, which capacitor has (i) the highest, (ii) the lowest charge? Why? (10)



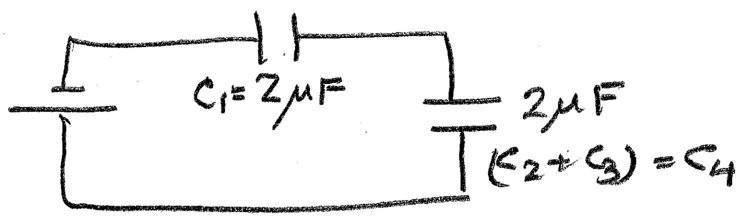
$C = \dfrac{Q}{V}$ hence in series Q being common & V's adding

$$\frac{1}{C_s} = \Sigma \frac{1}{C_i}$$

in parallel V is common and Q's add

so $C_p = \Sigma C_i$

The circuit reduces to



Since C_1 & C_4 in series.

$Q_1 = Q_4$

But $Q_4 = Q_2 + Q_3$

So Q_1 is largest!

In parallel V is common so

$V_2 = V_3$.

$Q_2 = C_2 V_2 = 0.4 V_2$

$Q_3 = C_3 \cdot V_2 = 1.6 V_2$

So Q_2 is smallest.

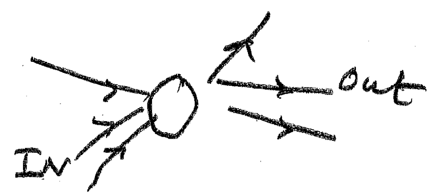
Prob 4a Write down the Physical Bases for Kirchhoff's rules.

(3,3)

Loop Rule: Potential at any point is unique, hence total change of potential on a closed loop must be zero: $\sum_{\text{Loop}} \Delta V = 0$

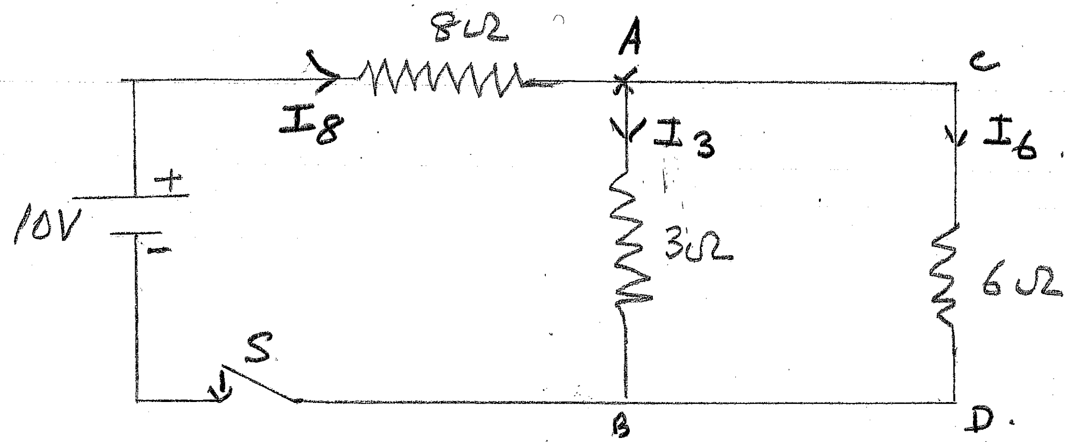
JUNCTION RULE CURRENT IS FLUX OF CHARGE, CHARGE IS CONSERVED, SO AT A JUNCTION:

$$\sum I_{out} = \sum I_{in}$$



Prob 4b For the circuit below, when the switch is closed, which resistor has (i) the highest, (ii) the lowest current? Why?

(10)



$I = \frac{V}{R}$

Once switch is closed currents flow as shown:

At Junction A

JUNCTION RULE → $I_3 + I_6 = I_8$ so I_8 is largest.

By loop rule potential drops on 6Ω & 3Ω must be equal $V_{CD} - V_{BA} = 0$!

$$I_3 = \frac{V}{3} \text{ amp}$$

$$I_6 = \frac{V}{6} \text{ amp}$$

so I_6 is least.

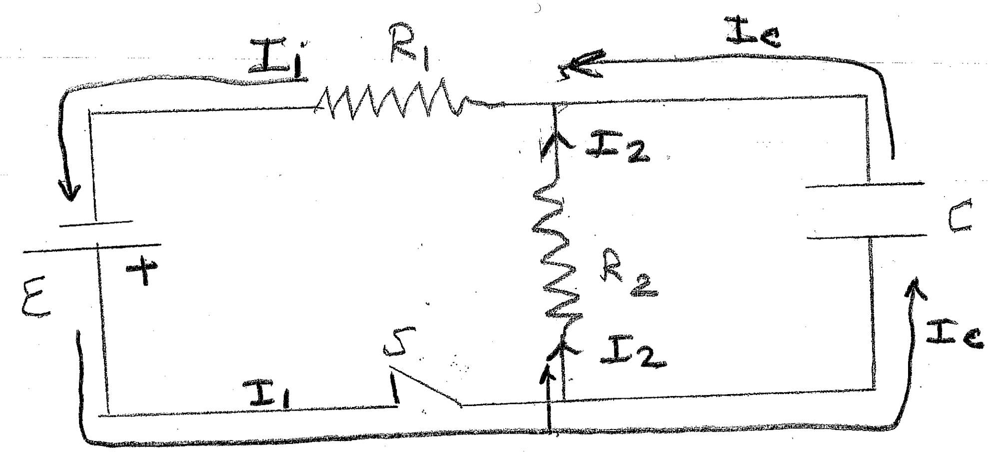
Prob 5a Why does the time constant for a RC circuit depend on both R and C?

(5)

The process involves transferring charge from Battery to Capacitor plates and current must go through R. The larger the R, the smaller the current, the longer it will take. The charge on C is $Q = CV_c$ so the larger the C the greater Q is required to build up to V_c and the longer it will take.

Prob 5b In the circuit shown what are (a) the currents (i) just after the switch is closed (ii) a long time later? (b) potential across C a long time later? Why? (15)

$R_1 = R_2 = 10\,\Omega$
$\varepsilon = 10V$



When S is closed currents flow from Battery plates to plates of C. No Current inside C.

Again Junction Rule requires

$$I_2 + I_c = I_1.$$

at $t = 0^+$ there is no charge on C

$V_c = 0$ so $V_{R_2} = 0$. [Parallel]

$I_2 = 0$

$$I_c = I_1 = \frac{\mathcal{E}}{R_1} = \frac{10}{10} = 1\text{amp.}$$

at $t \to \infty$ capacitor is fully charged

$I_c = 0$

$$I_1 = I_2 = \frac{\mathcal{E}}{R_1 + R_2} = \frac{10}{20} = 0.5\text{amp.}$$

$$V_c = I_2 R_2 = 0.5 \times 10 = 5\,V.$$

Prob 6a How do you distinguish between an $\underset{\rightarrow}{E}$ – field and a $\underset{\rightarrow}{B}$ – field? (5)

In an $\underset{\rightarrow}{E}$ - field a stationary charge experiences a force. $\underset{\rightarrow}{F_E} = q\underset{\rightarrow}{E}$.

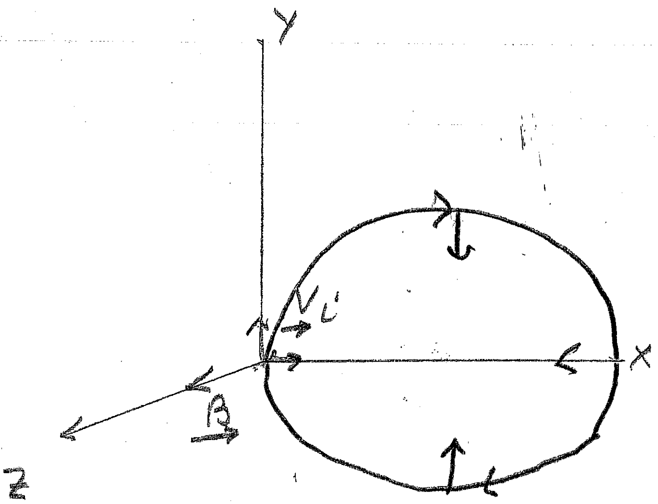
In a $\underset{\rightarrow}{B}$ - field a moving charge experiences a force which's always perpendicular to its velocity

$$\underset{\rightarrow}{F_B} = q\left[\underset{\rightarrow}{v} \times \underset{\rightarrow}{B}\right].$$

Prob 6b A deuteron $[q = 1.6\times10^{-19}\,c,\ m = 3.2\times10^{-27}\,kg]$ with a velocity of $\underrightarrow{Vi}\,\hat{y}$ is introduced in a region where $\underrightarrow{B} = B\,\hat{z}$.

(i) Show that the deuteron will move in a circular orbit in the xy-plane (6)

(ii) If you double its velocity, what happens to its angular velocity? (5)



When deuteron is first introduced force on it is $\underrightarrow{F_B} = q\,[\underrightarrow{V_i}\times\underrightarrow{B}]$ along $+\hat{x}$. This causes $\underrightarrow{V_i}$ to turn and $\underrightarrow{F_B}$ turns always remaining $\perp$ to $\underrightarrow{V_i}$. Since no work is done, magnitude of $\underrightarrow{V}$ does not change,

$$\underrightarrow{F_B} = -qvB\hat{r}$$

So it provides centripetal force

$$\underrightarrow{F_c} = -\frac{Mv^2}{r}\hat{r}$$

The orbit is a circle in xy-plane since $\underrightarrow{F_B}$ is $\perp$ to $\underrightarrow{B}$ also.

Setting $F_B = F_c$

$$\frac{V}{r} = \frac{qB}{M}.$$

So angular velocity is independent of V! No change on doubling V.