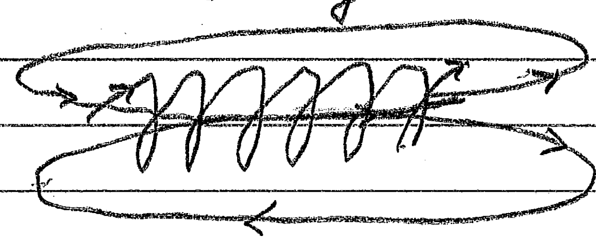
SOLUTIONS — 13

<u>S-36</u> A bar magnet attracts a piece of Iron because the $\vec{B}$ field produced by a bar magnet increases as one goes closer to the magnet and the piece of Iron reduces its potential energy ($U = -\vec{\mu} \cdot \vec{B}$) as it approaches the bar magnet. Here, $\vec{\mu}$ is the magnetic moment of the piece of Iron because the domains in it are aligned by $\vec{B}$.

<u>S-37</u> $\sum_c \vec{B} \cdot \Delta\vec{A} = 0$

Implies that the Elementary Generators of the $\vec{B}$-field must be magnetic dipoles which have no size so that the so-called "source" and "sink" are at the same point in space. Another way of understanding this is that such a dipole will produce a field whose lines form closed loops with no open ends anywhere. Imagine a Solenoid



<u>S-38</u> A Coulomb $\vec{E}$ field is generated by a stationary charge. Hence

5-39 $\quad \sum_C \underset{\to}{B} \cdot \Delta \underset{\to}{l} = \mu_0 \sum I_L \quad (1)$

$$\sum_C \underset{\to}{E_{NC}} \cdot \Delta \underset{\to}{l} = -\frac{\Delta \phi_B}{\Delta t} \quad (2)$$

Maxwell's concern was indeed very basic as it was informed by the realisation that according to the fundamental symmetry of nature if a time varying flux of $\underset{\to}{B}$ generates an $\underset{\to}{E}$ field, there must exist a term which affirms that the time varying flux of $\underset{\to}{E}$ will in turn generate a $\underset{\to}{B}$ field. Indeed, as we know now $\underset{\to}{E}$ and $\underset{\to}{B}$ are two aspects of a single fundamental field the Electromagnetic field.

5-40 Conduction current involves flow of charge in a conductor

$$I_c = \frac{\Delta q}{\Delta t}$$

Displacement current arises when the flux of $\underset{\to}{E}$ varies with time (Nothing is flowing)

$$I_d = \varepsilon_0 \frac{\Delta \phi_E}{\Delta t} \qquad \phi_E = \underset{\to}{E} \cdot \underset{\to}{A}$$

→ I_d exists in a vacuum, I_c does not!

I_d is non-zero between the plates of a capacitor during charging or discharging

Coulomb $\underset{\rightarrow}{E} = \frac{Q}{4\pi\epsilon_0 r^2}\hat{r}$

Coulomb $\underset{\rightarrow}{E}$ lines start at +ive charges and end at −ive charges leading to Gauss' law

$$\sum_c \underset{\rightarrow}{E} \cdot \underset{\rightarrow}{\Delta A} = \frac{1}{\epsilon_0}\sum Q_i$$

Total flux of $\underset{\rightarrow}{E}$ through a closed surface is determined solely by the enclosed charges.

A non-coulomb $\underset{\rightarrow}{E_{NC}}$ appears in every loop surrounding a region where flux of $\underset{\rightarrow}{B}$ field is changing with time. $\underset{\rightarrow}{E_{NC}}$ field lines form closed loops hence total flux through a closed surface

$$\sum_c \underset{\rightarrow}{E_{NC}} \cdot \underset{\rightarrow}{\Delta A} \equiv 0.$$

and the circulation of $\underset{\rightarrow}{E_{NC}}$ around a closed loop $\sum_c \underset{\rightarrow}{E_{NC}} \cdot \underset{\rightarrow}{\Delta l}$ is determined by the time rate of change of flux of $\underset{\rightarrow}{B}$, $(\Phi_B = \underset{\rightarrow}{B} \cdot \underset{\rightarrow}{A})$

$$\sum_c \underset{\rightarrow}{E_{NC}} \cdot \underset{\rightarrow}{\Delta l} = -\frac{\Delta \Phi_B}{\Delta t}$$

The "minus" sign on the right is crucial. It ensures that the sense of $\underset{\rightarrow}{E_{NC}}$ is such as to always <u>oppose</u> the <u>change</u> in the <u>flux</u> of $\underset{\rightarrow}{B}$

since $\vec{E}$ varies with time. No I_c ever appears between the plates of a capacitor.

To be totally precise, the only reason I_d is called a current is because it will give rise to a a $\vec{B}$ field via Ampere's law

S-41 If $E = cB$, $\eta_E = \eta_B$ because $c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$

Thus,

$$\eta_E = \frac{1}{2}\epsilon_0 E^2 = \frac{1}{2}\epsilon_0 c^2 B^2 = \frac{1}{2}\epsilon_0 \cdot \frac{1}{\mu_0 \epsilon_0} B^2$$

$$= \frac{B^2}{2\mu_0}.$$

S-42 The speed of light is $c = 3\times10^8$ m/s so light takes almost no time to travel to you. Sound travels at 330m/s in air so if you hear the thunder 5secs later, it must have come from $330\times5 = 1650$ m.

S-43 $\epsilon_0 = 9\times10^{-12}$ F/m $\mu_0 = 4\pi\times10^{-7}$ H/m

capacitance

$$\frac{F}{m} \rightarrow \frac{Q}{VL}$$

Inductance

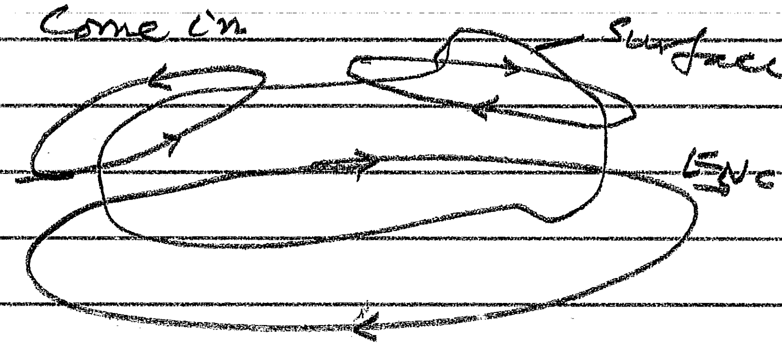
$$\frac{H}{m} \quad \frac{VT^2}{QL}$$

$$\boxed{L = -\frac{\mathcal{E}}{\frac{\Delta i}{\Delta t}} \qquad C = Q/V}$$

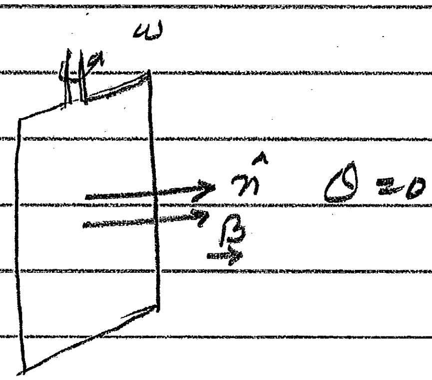
$$\mu_0\epsilon_0 \quad \frac{\cancel{Q}}{\cancel{V}L}\cdot\frac{\cancel{V}T^2}{\cancel{Q}L} = \frac{T^2}{L^2}$$

$$\frac{1}{\sqrt{\mu_0\epsilon_0}} = \frac{L}{T} \rightarrow \text{dimensions of vel.}$$

S-44/45 As discussed above flux of $\vec{E}_{NC}$ through a closed surface is **zero** because E_{NC} field lines form closed loops so for any closed surface the same number of lines go out as come in.



S-45/46 To make it work like a generator rotate the coil at an angular velocity $\vec{\omega} = \omega\hat{y}$ starting when $\hat{n} \parallel \vec{B}$

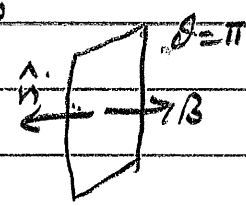


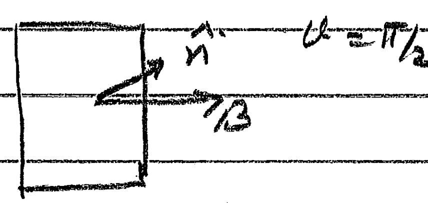
At $t = 0$ $\phi_B = BA$

$\frac{1}{4}$ cycle later $\phi_B = 0$

$\frac{1}{2}$ cycle later

$\phi_B = -BA.$

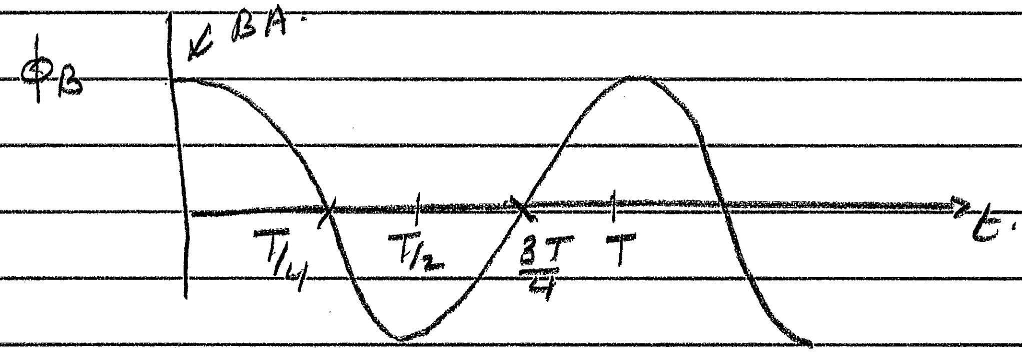




and so on.

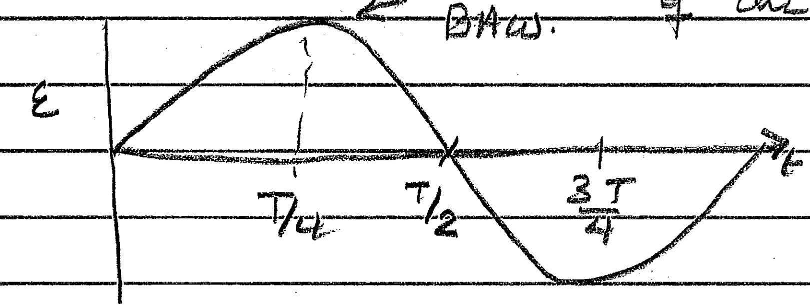
Indeed $\phi_B = BA\cos\omega t$ $\qquad \theta = \omega t$

Period $T = \frac{2\pi}{\omega}$

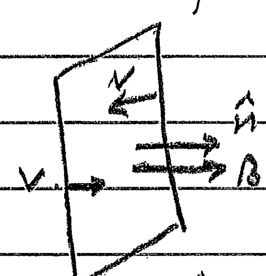


Since ϕ_B is changing with time an Emf $\mathcal{E} = \sum_C \vec{E}_{NC}\cdot\Delta\vec{l}$ will appear in the coil given by

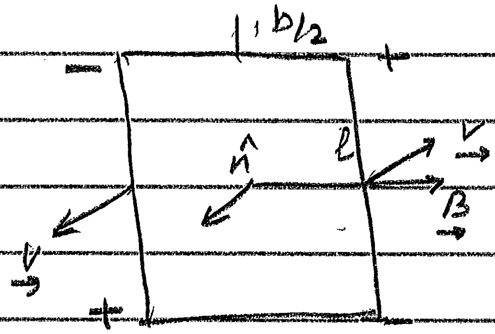
$$\mathcal{E} = -\frac{\Delta\phi_B}{\Delta t} \rightarrow \text{negative of the slope of the above curve}$$



and one can see that when ϕ_B is maximum, slope is zero while when ϕ_B is zero slope is largest so emf is largest when flux is zero and zero when flux is maximum. You can also see this by looking for the motional Emf in the coil.



$\hat{n} \parallel \vec{B}$ No force on the electrons Emf = 0.
$\vec{v} \parallel \vec{B}$



$\therefore v = \frac{b}{2}\omega.$

$\hat{n} \perp \underset{\to}{B}$ $\underset{\to}{v} \perp \underset{\to}{B}$, maximum force on electrons
motional emf maximum

$$\text{emf} = 2vBl = 2\frac{b}{2}Bl\omega = BA\omega.$$

S-47 $U_B = \frac{1}{2}LI^2$. This energy is stored in the $\underset{\to}{B}$ field which is established when there is a current in the inductor. For example take a solenoid. Establish a current I in it

$$L = \mu_0 n^2 V \qquad B = \mu_0 n I$$

$$U_B = \frac{1}{2}LI^2 = \frac{1}{2}\mu_0 n^2 V I^2$$

$$= \frac{1}{2\mu_0}[\mu_0 n I]^2 V = \frac{B^2}{2\mu_0} V$$

so energy density in B

$$\eta_B = \frac{B^2}{2\mu_0}$$

Week 13

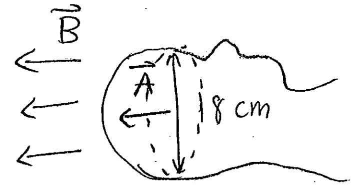
Chapter 25

25.9

Magnetic flux is given by $\Phi_B = \vec{B} \cdot \vec{A}$

If we assume the field and the patient will be aligned such that the flux is maximized, that is, such that

$\theta = 0° \Rightarrow \cos\theta = 1$, then



$\Phi_B = BA\cos\theta = BA$

$B = 1.6T$; estimate head diameter as 18cm :

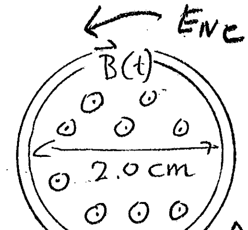
$A = \pi r^2 = \pi\left(\frac{0.18m}{2}\right)^2 = 0.0254\ m^2$

$\Rightarrow \Phi_B = 1.6T(0.0254\ m^2) = 0.041\ Wb$

25.15 $\mathcal{E} = \sum \vec{E}_{NC} \cdot \Delta\vec{\ell} = -A\frac{\phi_B}{\Delta t}$

EMF from a changing flux through a wire loop is given by Faraday's law :

$\mathcal{E} = -N\frac{\Delta\Phi_B}{\Delta t}$; w/ $\Phi_1 = \vec{B} \cdot \vec{A}$



axis of coil ∥ field $\Rightarrow \cos\theta = 1$.

area is constant, change in flux is from change in $\vec{B}$:

$\vec{B} = B\hat{z}$
→ since B reduces emf or ENC must produce B out of page

$\mathcal{E} = -NA\frac{\Delta B}{\Delta t} = -(1000)\pi\left(\frac{0.02m}{2}\right)^2 \frac{(0.10T - 0)}{10 \times 10^{-3}s} = 3.1\ V$

25.27

Intensity is given by $I = \frac{1}{2} c \epsilon_0 E_0^2$ (eqn 25.16)

where E_0 is the E-field amplitude;

Intensity is also power per area: $I = \frac{P}{A}$

a point source of EM radiation will give off waves in a spherically symmetric way, so the field strength some distance away will be the same at all points on a sphere of that radius; ∴ A is the area of a sphere, $A = 4\pi r^2$

$$\Rightarrow \quad I = \frac{P}{4\pi r^2} = \frac{1}{2} \epsilon_0 c E_0^2$$

solve this for radius:

$$P = \frac{1}{\cancel{2}} \epsilon_0 c E_0^2 (\overset{2}{\cancel{4}}\pi r^2)$$

$$r^2 = \frac{P}{2\pi \epsilon_0 c E_0^2} \Rightarrow r = \sqrt{\frac{P}{2\pi \epsilon_0 c E_0^2}}$$

(a) so for $E_0 = 100$ V/m and $P = 10$ W:

$$r_{100} = \sqrt{\frac{10\,W}{2\pi(8.85\times10^{-12}\,C^2/Nm^2)(3\times10^8\,m/s)(100\,V/m)^2}}$$

$r_{100} = 0.25$ m

(b) for $E_0 = 0.010$ V/m

$$r_{.010} = \sqrt{\frac{10\,W}{2\pi(8.85\times10^{-12}\,C^2/Nm^2)(3\times10^8\,m/s)(0.010\,V/m)^2}}$$

$r_{0.010} = 2500$ m

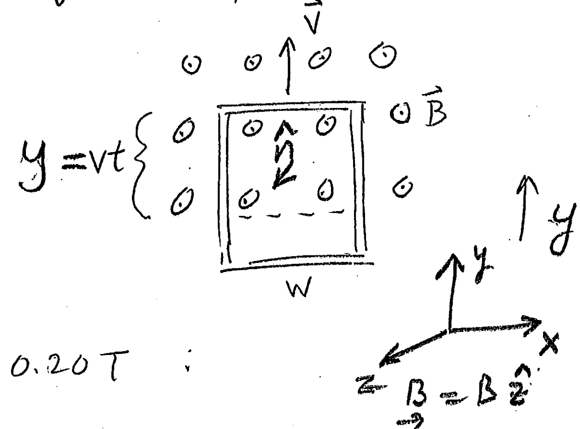
25.53

first we need the EMF induced:

$$\mathcal{E} = -\frac{\Delta\Phi_m}{\Delta t}$$

here the field is constant, and its the change in area that gives a change in flux; the width of the loop is constant, but the length of the loop in the field is changing with time; using the basic definition of velocity:

$$\frac{\Delta A}{\Delta t} = w\frac{\Delta x}{\Delta t} = wv$$



note the area immersed will increase w/ time for +y motion:

for $w = 5.0$ cm ; $v = 50$ m/s ; $B = 0.20$ T :

$$|\mathcal{E}| = Bwv = 0.20\,T\,(0.050\,m)(50\,m/s)$$

$$\mathcal{E} = 0.50\ V$$

now to get the current, we know $R = 0.10\,\Omega$, just use Ohm's law:

$$I = \frac{\mathcal{E}}{R} = \frac{0.50\ V}{0.10\ \Omega} = 5.0\ A$$

for the direction, since $\vec{B}$ and $\hat{n}$ are out of the page, and A is increasing, the change in flux is out of the page; ∴ the induced EMF leads to a $\vec{B}$-field that points into the page; using right-hand rule implies a clockwise current.