PHYS 122 Exam III December 4, 2009

Prof. S. M. Bhagat

Name: Solution

(Sign in ink, print in pencil)

Notes

1) There are four (4) problems in this exam. Please make sure that your copy has all of them.
2) Please show your work indicating clearly what formula you used and what the symbols mean. Just writing the answer will not get you full credit. In stating vectors give both magnitude and direction.
3) Write your answers on the sheets provided.
4) Do not forget to write the units
5) Do not hesitate to ask for clarification at any time during the exam. You may buy a formula at the cost of one point.

Take Care! God Bless You!

$$k_e = 9x10^9 \frac{N \cdot m^2}{C^2}, \mu_0 = 4\pi 10^{-7} H/m$$

$$\varepsilon_0 = 9x10^{-12} \frac{F}{m}$$

Mass of proton $m_p = 1.6x10^{-27} kg$

Mass of electron $m_e = 9x10^{-31} kg$

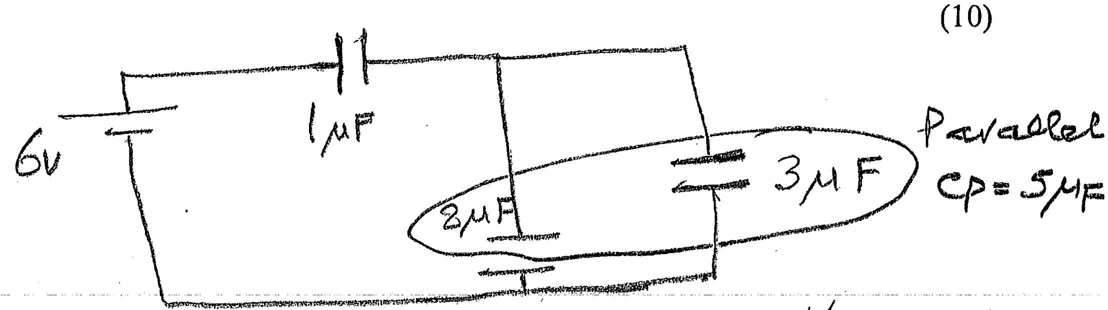
Elementary Charge $e = 1.6x10^{-19} C$

Magnetic Mom. of El.: $\mu_e = 9.27x10^{-24} \frac{N-m}{T}$

Problem 1a

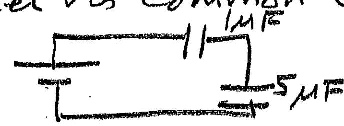
In the circuit below which capacitor has i) the most charge, ii) the least charge? Why?

(10)



In Series Q is common, V's add so $\frac{1}{C_s} = \Sigma \frac{1}{C_i}$

In parallel V is common Q's add so $C_p = \Sigma C_i$

Circuit is 

series $Q_1 = Q_5$

But $Q_5 = Q_2 + Q_3$

so Q_1 largest.

Since 2μF & 3μF in parallel $V_2 = V_3$

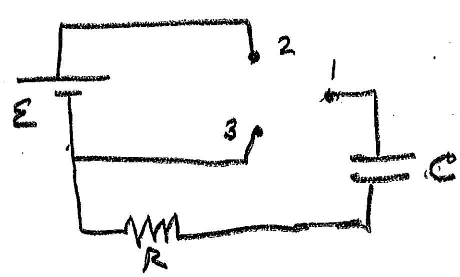
$Q_2 = 2 V_2$ so Q_2 is smallest $(Q = CV)$.

Problem 1b

Why does the characteristic time for an RC circuit (T=RC) depend on both R and C?

(10)

The process involves transferring charge to (charging) or from (discharging) the capacitor plates. The charge must go through R so R controls rate of charge flow. Larger the R slower the flow rate, longer it will take. C controls the amount of charge needed to establish the potential V/C. The larger the C, the larger the charge required and the longer it will take.



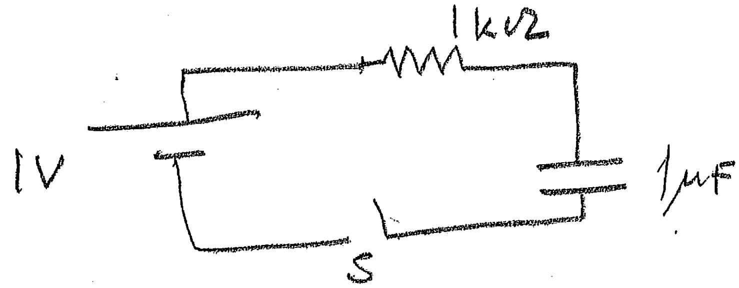
[1→2 charging

late 1–3 discharging]

Problem 1c

What is the current inside the capacitor immediately after the switch is closed?

(5)



Zero – No current flows inside C at any time

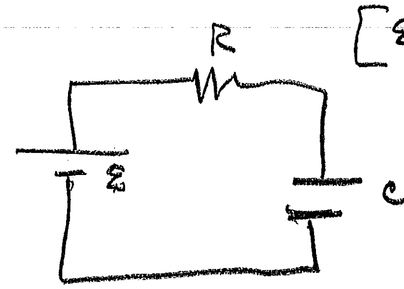
Problem 2a

What are the physical bases of Kirchhoff's Rules? (5)

Loop Rule arises b/c Force is Conservative and so potential at any point is unique. Total change of potential going around any closed loop must be zero

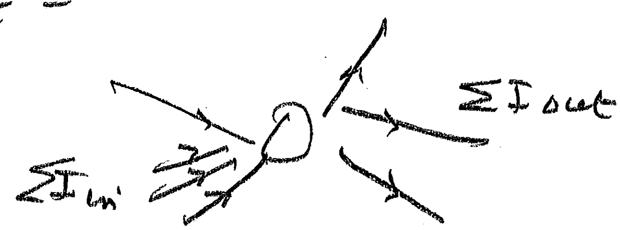
$$\sum_{Loop} \Delta V = 0$$

$[\mathcal{E} - IR - \frac{q}{C} = 0]$



Junction Rule: arises b/c current is flux of charge and charge is conserved so total current out of a junction must equal total current into it

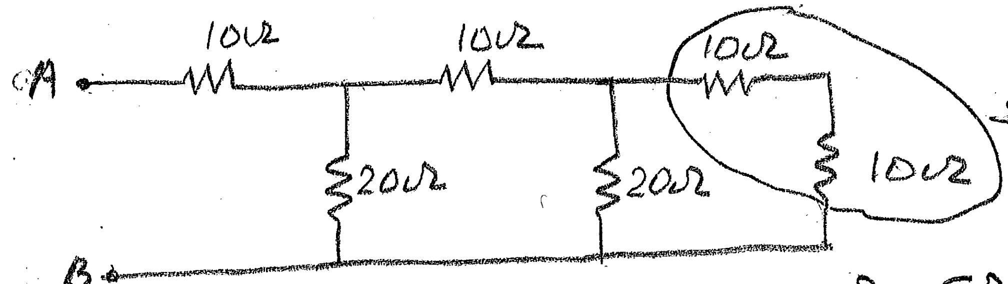
$$\sum I_{out} = \sum I_{in}$$



Problem 2b

Calculate R_{AB}. If you apply 10V across AB what are the currents in the resistors?

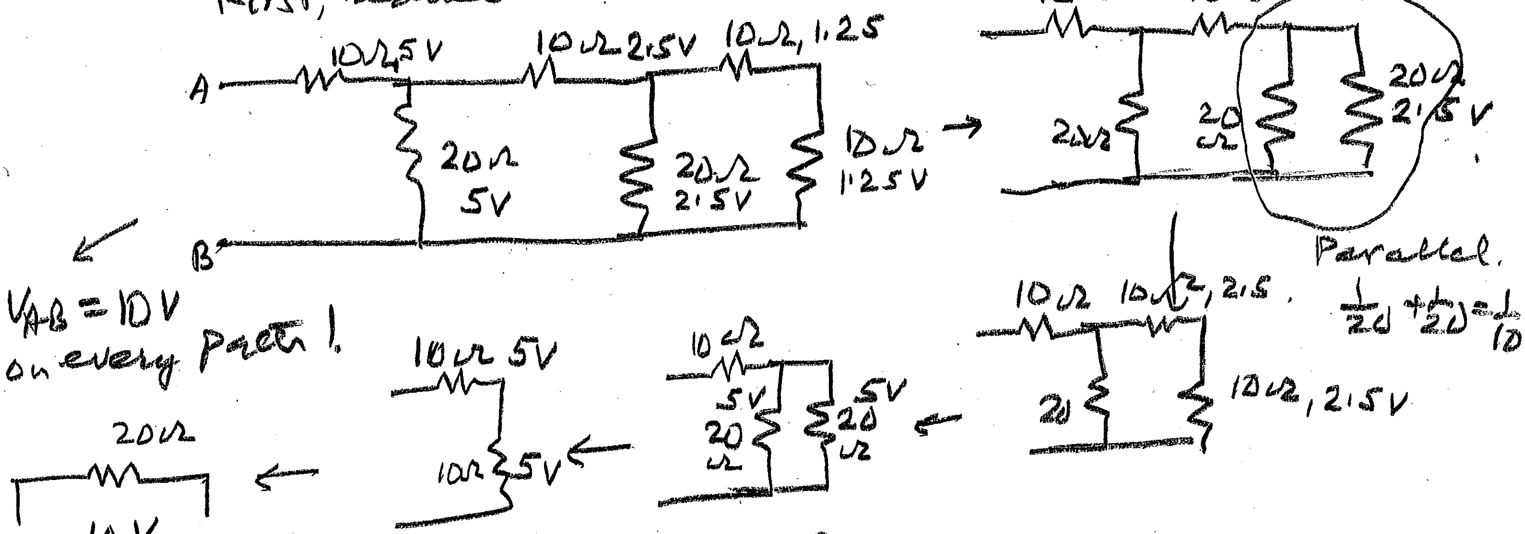
(20)



(i) In series I is common, V's added $R_S = \Sigma R_i$

(ii) In parallel V is " I's add $\frac{1}{R_P} = \Sigma \frac{1}{R_i}$

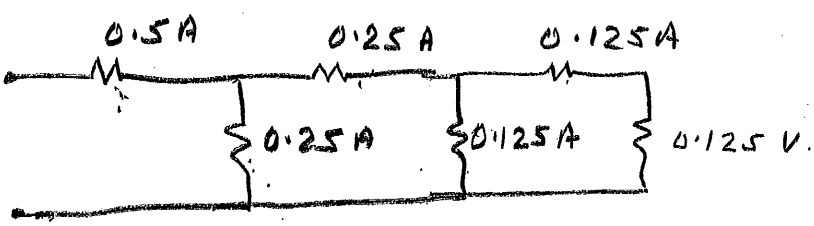
First, reduce the ladder



$R_{AB} = 20\Omega$

Now apply 10V and use rules (i) & (ii), the resulting voltage drops are drawn, in series V's add in paral they are common.

To calculate currents use $I = \frac{V}{R}$ in each case.



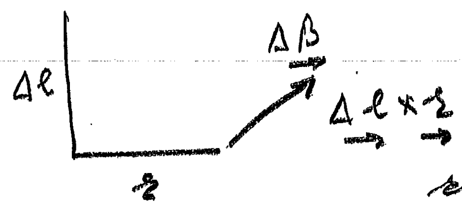
ensure that junction Rule is obeyed

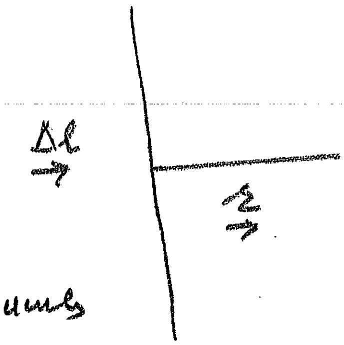
Problem 3a

We are told that a current I flowing in a conductor $\underset{\rightarrow}{\Delta \ell}$ creates a $\underset{\rightarrow}{B}$ field at a point $\underset{\rightarrow}{r}$ given by

$$\underset{\rightarrow}{\Delta B} = \frac{\mu_0}{4\pi} I \frac{\underset{\rightarrow}{\Delta l} \times \underset{\rightarrow}{r}}{r^3}$$

Show that the $\underset{\rightarrow}{B}$ field circulates around the current. (10)



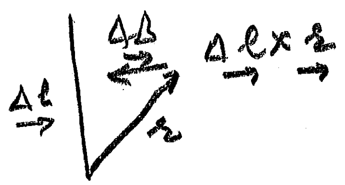


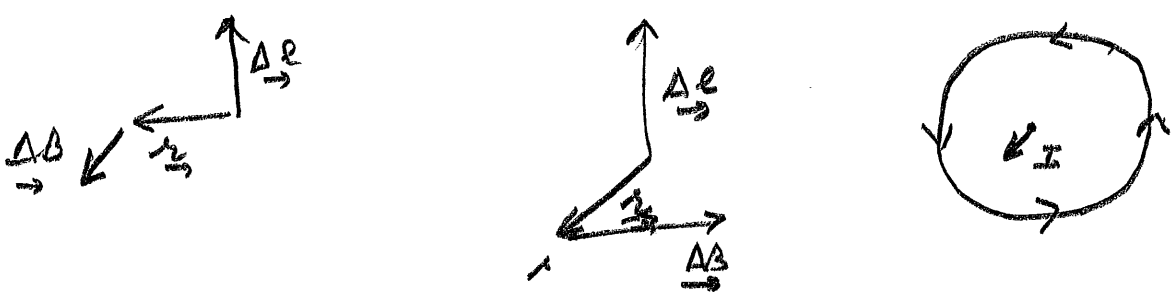
rh hand rule.

$\underset{\rightarrow}{\Delta l} \parallel$ Thumb

$\underset{\rightarrow}{r} \parallel$ Fingers

$\underset{\rightarrow}{\Delta l} \times \underset{\rightarrow}{r} \perp$ Palm.





$\underset{\rightarrow}{\Delta B}$ circulates around current

Problem 3b

Show that parallel currents attract and antiparallel currents repel one another. (10)

Current-current forces arises b/c

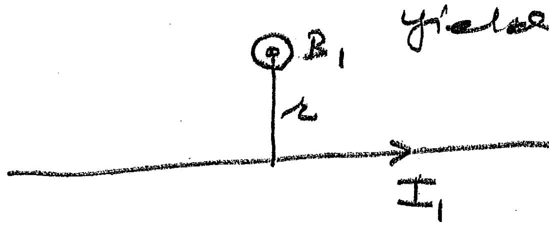
i) A current generates a $\vec{B}$ field that circulates around it

$$\vec{B} = \frac{\mu_0 I}{2\pi r}\hat{\phi}$$

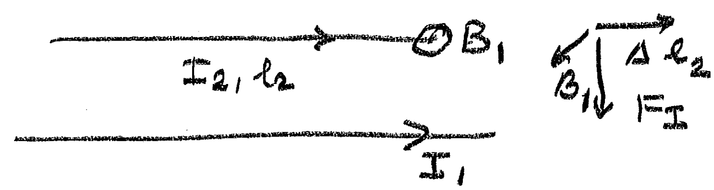
and (ii) A current experiences a force in a $\vec{B}$ field.

$$\vec{F}_I = I\, \Delta\vec{l} \times \vec{B}.$$

Start with I_1 yields $\vec{B}_1 = \frac{\mu_0 I_1}{2\pi r}\hat{z}$

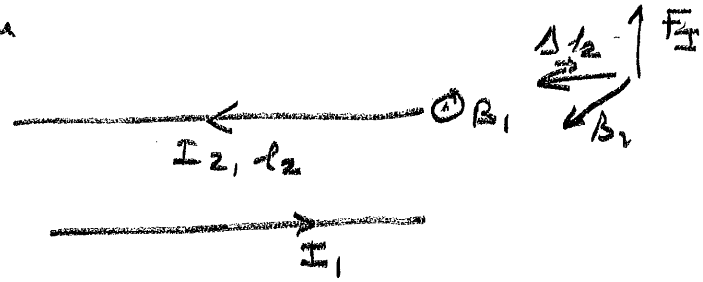


||- Case



$$\vec{F}_{I_2, I_1} = -\frac{\mu_0 I_1}{2\pi r} I_2 l_2 \hat{y}.$$

Anti-parallel case



$$\vec{F}_{I_2, I_1} = \frac{+\mu_0 I_1 I_2 l_2}{2\pi r}\hat{y}.$$

Problem 4a

What is the difference between a coulomb $\underset{\rightarrow}{E}$-field and a non-Coulomb $\underset{\rightarrow}{E}$-field?

(5, 5)

A coulomb $\underset{\rightarrow}{E}$ is generated by a stationary charge $\quad \underset{\rightarrow}{E} = \dfrac{Q}{4\pi\varepsilon_0 r^2}\hat{r}$

hence $\quad \sum_c \underset{\rightarrow}{E} \cdot \underset{\rightarrow}{\Delta A} = \frac{1}{\varepsilon_0}\sum Q_i$ (Gauss' law).

A non-Coulomb $\underset{\rightarrow}{E}_{NC}$ appears in every loop surrounding a region where flux of $\underset{\rightarrow}{B}$ is changing with time. Lines of $\underline{E}_{NC}$ close on themselves so $\sum_c \underset{\rightarrow}{E}_{NC} \cdot \underset{\rightarrow}{\Delta A} = 0$ and the circulation of $\underset{\rightarrow}{E}_{NC}$ around a closed loop is determined by the time rate of change of $\phi_B = \underset{\rightarrow}{B} \cdot \underset{\rightarrow}{A}$ inside the loop:

$$\sum \underset{\rightarrow}{E}_{NC} \cdot \underset{\rightarrow}{\Delta l} = -\frac{\Delta \phi_B}{\Delta t}$$

The minus sign on the right ensures that the sense of $\underset{\rightarrow}{E}_{NC}$ is such as to oppose the change in ϕ_B.

Problem 4b

Why is the total flux of the $\underset{\rightarrow}{B}$-field through a closed surface always equal to zero?

(5)

Firstly, $\underset{\rightarrow}{B}$ field lines form closed loops (no beginning or end), Secondly, elementary generators of $\underset{\rightarrow}{B}$ are magnetic dipoles which have no spatial extent (+ & − coincide) so

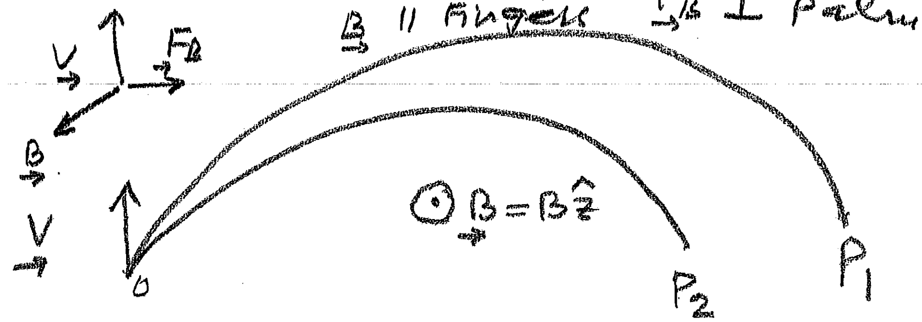
$$\sum_c \underset{\rightarrow}{B} \cdot \underset{\rightarrow}{\Delta A} = 0$$

Problem 4c

Shown are the paths of the particles in a mass spectrometer. Both have same initial velocity $\underset{\rightarrow}{v} = v\hat{y}$. For the $\underset{\rightarrow}{B}$-field shown, what is the sign of the charge (+ive or -ive) on the particles. In case I both particles have same charge but different masses, where will the larger mass land, P_1 or P_2. Why? In case II both have same mass but different charges, where will the larger charge land? Why? (15)

$q\underset{\rightarrow}{v} \parallel$ Thumb

$\underset{\rightarrow}{B} \parallel$ fingers $\quad \underset{\rightarrow}{F_B} \perp$ Palm



As shown, for charges to turn right at the origin, force F_B must be along $+\hat{x}$, since $\underset{\rightarrow}{F_B} = q\,[\underset{\rightarrow}{v} \times \underset{\rightarrow}{B}]$ we need $q\underset{\rightarrow}{v}$ along $+\hat{y}$, hence q must be <u>positive</u>.

The radius of the orbit $R = \dfrac{MV}{qB}$ b/c $\underset{\rightarrow}{F_B} = -qvB\hat{r}$ provides the centripetal force $\underset{\rightarrow}{F_c} = -\dfrac{MV^2}{R}\hat{r}$.

So $\quad R_1 = \dfrac{M_1 V}{q_1 B}$, $\quad R_2 = \dfrac{M_2 V}{q_2 B}$.

If $M_1 > M_2$, $q_1 = q_2$ $\quad R_1 > R_2$ $\quad M_1$ lands at P_1

If $M_1 = M_2$, $q_1 > q_2$ $\quad R_1 < R_2$ $\quad q_1$ lands at P_2