PHYS 122 **EXAM 1** **October 1, 2009**
Prof. S.M. Bhagat

Name: SOLUTION

(Sign in ink, print in pencil)

Notes

1. There are six (6) problems in this exam. Please make sure that your copy has all of them.
2. Please show your work, indicating clearly what formula you used and what the symbols mean. Just writing the answer will not get you full credit. In stating vectors, give both magnitude and direction.
3. Write your answers on the sheets provided.
4. Do not forget to write the units.
5. Do not hesitate to ask for clarification at any time during the exam. You may buy a formula at the cost of one point.

Take Care! God Bless You!

$$\mu_0 = 4\pi x 10^{-7}\ \tfrac{T-m}{A}$$

$$\varepsilon_0 = 9\times10^{-12}\ F/m$$

Mass of proton	$m_p = 1.6\times10^{-27}\ kg$
Mass of electron	$m_e = 9\times10^{-31}\ kg$
Elementary Charge	$e = 1.6\times10^{-19}\ C$

Problem 1a

A mass is acted upon by a force

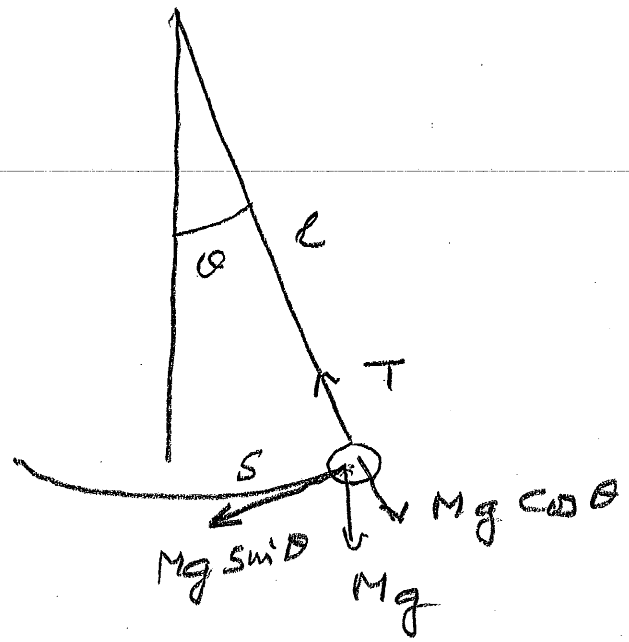
$$\underset{\rightarrow}{F} = Cx\hat{x},$$

where C is a positive quantity. Will it exhibit linear harmonic oscillations? Justify your answer. (6)

No, it will not exhibit linear harmonic oscillations. To get linear harmonic oscillations you need a restoring force, that is force vector must be opposite to the displacement vector so that it is always acting to bring the mass back to its equilibrium position at $x=0$.

Problem 1b

In the experiment on a simple pendulum why do you need the amplitude to be small so it will show linear harmonic oscillations? (10)



The forces acting on the mass are Weight $-Mg\,\hat{y}$ and tension T along the string. Resolve the forces along string giving $Mg\cos\theta$ and along the arc

$$\vec{F}_\tau = -Mg\sin\theta\,\hat{\tau}$$

where $\hat{\tau}$ is the unit vector along the tangent. $\vec{F}_\tau$ is the force which will cause the oscillation. It is indeed opposite to the displacement but it is not linear in θ. However, if

$$\theta \ll 1$$
$$\sin\theta = \theta$$

and the force becomes

$$\vec{F}_\tau = -Mg\,\theta\,\hat{\tau}$$
$$= -\frac{MgS}{l}\,\hat{\tau}$$

where S = arc length and l = length of pendulum.

Now $\vec{F}_\tau$ is proportional to displacement and opp. to it and gives LH oscns.

Problem 2a

What is a travelling wave? (6)

A deviation (D) from equilibrium which is a function of both position (x) and time (t) in such a way that x and t appear in the function in the combination

$$(x \mp vt)$$

If so D cannot be stationary. It will travel as a wave with velocity

$$\underset{\sim}{v} = \pm v \hat{x}$$

Problem 2b

As written below, there are quantities missing on the right side of the equation

$$D = \ldots \sin(x - vt)$$

where
D is a physical quantity (displacement, pressure, …)
x is a length
V a velocity
And t a time

Justify the quantities you insert and in each case explain their physical significance (10)

D is a physical qty and has dimensions, Sine function is dimensionless so we must introduce A such that

$$D = A \sin(x - Vt)$$

where A will have dimensions of D and determine the nature of the wave (displacement, …)

Since Sine is dimensionless, its argument must be dimensionless so we must introduce a length λ such that

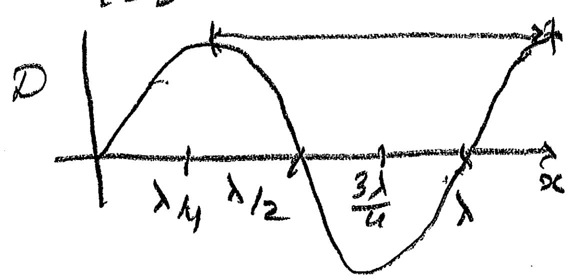
$$D = A \sin \frac{(x - Vt)}{\lambda}$$

Since Sine is periodic with period 2π it is convenient to write

$$D = A \sin \frac{2\pi(x - Vt)}{\lambda}.$$

Then A has significance of amplitude, largest value of D.

Plot D as a fn of x at t=0



and we see that λ is the repeat distance, "Wave length"

Problem 3a

A wave is written as

$$\underset{\rightarrow}{y} = 0.05(\sin 6.28x \cos 12.56t)\hat{y}$$
$$= A \sin (kx \cos \omega t)\hat{y}$$

where lengths are in meters and times in secs. Is this wave

(i) longitudinal or transverse?

(ii) Travelling or stationary? Calculate its

i) amplitude

ii) wavelength

iii) frequency. (3,3,3,4,3)

(i) Wave is Transverse because $\underset{\rightarrow}{A} \perp \hat{x}$

(ii) It is stationary because it has nodes at $x = 0$ and every $\lambda/2$.

i) amplitude $A = 0.05m.$

(ii) Wave length $k = \frac{2\pi}{\lambda} = 6.28 m^{-1}$

$$\lambda = \frac{2\pi}{6.28} = 1 m.$$

(iii) frequency $\omega = 2\pi f = 12.56$ rad/s

$$f = \frac{\omega}{2\pi} = 2 Hz$$

Problem 4a

What is sound? (6)

Any mechanical wave whose frequency lies between 20 Hz and 20 kHz can be called sound as our ears detect it.

Problem 4b

The velocity of sound in a gas is written as

$$v = \sqrt{\frac{\gamma k_B T}{m}}$$

Why is there a γ in this equation? (10)

Sound is a displacement wave. If displacement is a function of position, there is a volume (V) change. If V changes, pressure (P) must change and we need to know the relationship between P and V. Since freq. of sound is greater than 20 Hz, the variations are very rapid and therefore there is no exchange of heat with the surroundings

$\boxed{V\uparrow P\downarrow}$ $DQ \equiv 0$ means process is adiabatic and the PV relationship is

$$PV^{\gamma} = \text{const}, \text{ with } \gamma = \frac{c_p}{c_v}$$

$DQ \equiv 0$

Problem 4c

What is the speed of sound on the moon? (The moon has no atmosphere) (4)

There is no sound on the
moon.

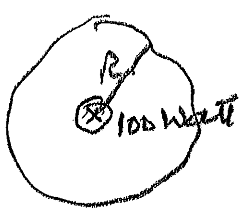
Problem 5a

What is light? (6)

Light is a transverse Electromagnetic wave whose speed in vacuum is $3\times10^8 m/s$ and whose wavelengths in vacuum are $400nm < \lambda_0 < 800nm$.

Problem 5b

You are sitting 2m away from a 100 watt bulb (efficiency for light emission 2.5%). What is the amplitude of the E-field in the electromagnetic wave entering your eye? Why?
(10)



Intensity at 2 meters

$$I = \frac{P}{4\pi R^2} = \frac{2.5}{4\pi \times 2^2} \text{ Watt}/m^2$$

Intensity of EM wave of amplitude E_m is

$$I = \frac{1}{2}\epsilon_0 c E_m^2 = \frac{1}{2}\times 9\times10^{-12}\times 3\times10^8\times E_m^2$$

$$E_m^2 = \frac{2\times2.5\times10^4}{4\times\pi\times4\times9\times3}(N/c)^2$$

$$E_m \sim 6\ N/c$$

Problem 6a

Show that when light from two identical but incoherent sources arrives at a detector, the total intensity is just twice of that due to one source. (14)

Emission of light involves enormous # of uncorrelated events so phase is random in time. Two waves

$E_1 = E_m \sin(kx - \omega t + \phi_1)$; $E_2 = E_m \sin(kx - \omega t + \phi_2)$

$(I_1 = \frac{1}{2} \epsilon_0 c E_m^2)$

so $E = E_1 + E_2 = 2E_m \cos\left(\frac{\phi_1 - \phi_2}{2}\right) \sin\left(kx - \omega t + \frac{\phi_1 + \phi_2}{2}\right)$

So wave has amplitude $2E_m \cos\left(\frac{\phi_1 - \phi_2}{2}\right)$ and intensity

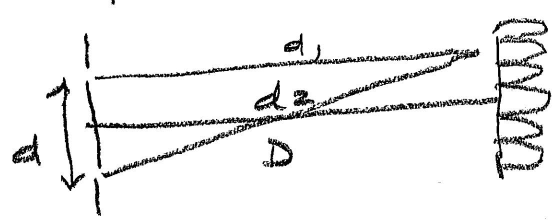
$I = \frac{1}{2} \epsilon_0 c \, 4E_m^2 \cos^2\left(\frac{\phi_1 - \phi_2}{2}\right)$

But if sources are incoherent $(\phi_1 - \phi_2)$ is random in time so we need to average $\langle I \rangle = \frac{1}{2} \epsilon_0 4E_m^2 \left\langle \cos^2\left(\frac{\phi_1 - \phi_2}{2}\right)\right\rangle$. But $\left\langle \cos^2\left(\frac{\phi_1 - \phi_2}{2}\right)\right\rangle = \frac{1}{2}$ so $\langle I \rangle = \frac{1}{2} \epsilon_0 4E_m^2 \cdot \frac{1}{2} = 2I_1$

Problem 6b

If the two sources in problem 6a were coherent what would you observe? Why? (6)

If the two sources are coherent $(\phi_1 - \phi_2)$ is fixed (taken to be zero), $I = \frac{1}{2} \epsilon_0 c \, 4E_m^2 \cos^2\left(\frac{\phi_1 - \phi_2}{2}\right)$



but $(\phi_1 - \phi_2)$ now varies as $(d_1 - d_2)$ when d_1 is distance travelled by one wave and d_2 by the other. This produces equal intensity, equally spaced INTERENCE MAXIMA on a screen. The dark spots in between are also equally spaced at

$\frac{D\lambda}{d}$.