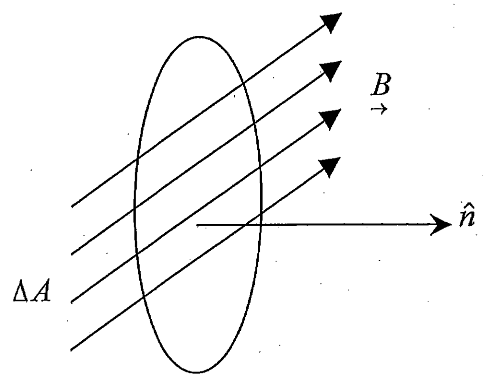
NON-COULOMB $\vec{E}$ FIELD

(INDUCTION).

We begin by considering a uniform $\vec{B}$-field represented by a set of parallel lines. Next, imagine an area $\Delta \vec{A}$ whose normal

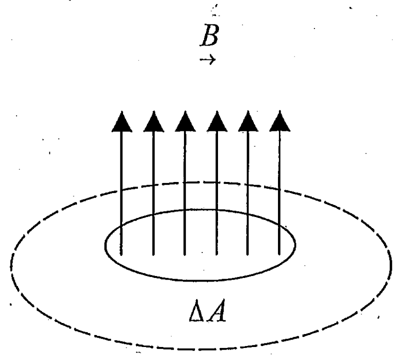


is along $\hat{n}$. Then, as in the case of $\vec{E}$, we define flux of $\vec{B}$ through $\Delta\vec{A}$ to be

$$\Delta\Phi_B = \vec{B}\cdot\Delta\vec{A} = B\Delta A\,Cos(\vec{B},\hat{n})$$

As before, flux is maximum when $\vec{B} \parallel \hat{n}$ ($\perp$ to area) and zero if $\vec{B}\perp\hat{n}$ ($\vec{B}$ lines lie in plane of ΔA).

<u>Faraday's</u> discovery was that if Φ_B varies with time, that is, $\dfrac{\Delta\Phi_B}{\Delta t} \neq 0$, there will be an *emf* ε induced in every closed loop surrounding the region where $\Delta\Phi_B$ is changing. For example, if in the picture $\vec{B}$ varies with the time, or angle of $\vec{B}$ with $\hat{n}$



<u>Note</u>: Flux of $\vec{B}$ can vary with time due to 3 variations:
i) $\vec{B}$ is a function of t.
ii) $\Delta\vec{A}$ is a function of t.
(iii) Angle between $\vec{B}$ and $\hat{n}$ is a function of t.

varies with time, $\dfrac{\Delta\Phi_B}{\Delta t} \neq 0$, and an *emf* will appear in the closed loop, represented by the dotted line, surrounding ΔA. We know that if there is an *emf* in a circuit (closed loop) there must be an $\vec{E}$-field present at every point of the circuit such that $\varepsilon = \Sigma_c \vec{E}\cdot\Delta\vec{l}$

Where the sum Is over the closed loop and $\Delta \vec{l}$ represents displacement along the loop.

Next, we add Lenz's principle to Faraday's discovery to write

$$\Sigma_c \vec{E} \cdot \Delta \vec{l} = -\frac{\Delta \Phi_B}{\Delta t}$$

The minus sign on the right is crucial, it implies that the *emf* or equivalently $\vec{E}$-field, is such that it opposes the change in the flux of $\vec{B}$ which generates the *emf.*

Another remarkable point to note is that the $\vec{E}$-field generated by a time varying Φ_B is represented by lines which close on themselves – there is no beginning and no end. This is totally different from the $\vec{E}$-field implied by Coulomb's law: $\vec{E} = \frac{q}{4\pi\varepsilon_0 r^2}\hat{r}$ which "started" at +ive charges and "ended" at –ive charges.

We call the $\vec{E}$-field generated by the time varying flux of $\vec{B}$ a NON-COULOMB $\vec{E}$-FIELD:

$$\Sigma_c \vec{E}_{NC} \cdot \Delta \vec{l} = -\frac{\Delta \Phi_B}{\Delta L}.$$

Notice that for $\vec{E}_{NC}$ Gauss' law will always give

$$\Sigma_c \vec{E}_{NC} \cdot \Delta \vec{A} = 0.$$

Total flux of $\vec{E}_{NC}$ through a closed surface is always equal to zero.

<u>Note</u>: A charge q placed in $\vec{E}_{NC}$ experiences a force

$$\vec{F}_E = q\,\vec{E}_{NC}$$

exactly as for a Coulomb $\vec{E}$-field.

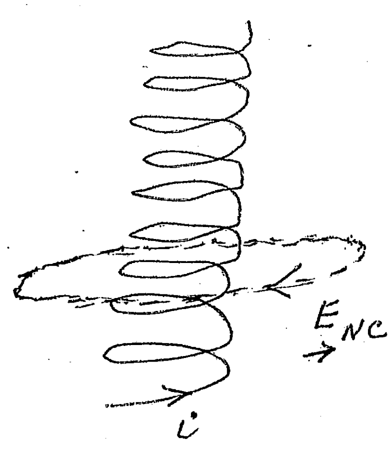
<u>APPLICATIONS</u>

Calculate the $\vec{E}_{NC}$ for a solenoid at a distance r from its axis when the flux of $\vec{B}$ is varied by time variation of the current in the solenoid. That is, $\frac{\Delta i}{\Delta t} \neq 0 \Rightarrow \frac{\Delta B}{\Delta t} \neq 0 \Rightarrow \frac{\Delta \Phi_B}{\Delta t} = 0$. Consider a solenoid wound on a tube of radius a. If there are n turns per meter and the current flow is as shown, there is a uniform $\vec{B}$ inside it

$$\vec{B} = \mu_0 n i\,\hat{y}$$

and

$$\frac{\Delta \vec{B}}{\Delta t} = \mu_0 n \frac{\Delta i}{\Delta t}\hat{y}$$



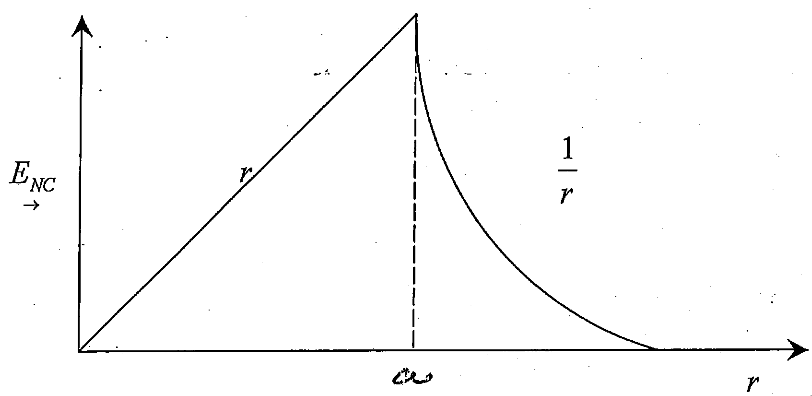
The problem has cylindrical symmetry about axis of solenoid, $\vec{E}_{NC}$ at r is a function of r only and must be azimuthal. Let i increase with time. Then flux of $\vec{B}$ along $+\hat{y}$ is increasing with time. Take a circular loop. As shown direction of $\vec{E}_{NC}$ must be clockwise (as viewed from above) to oppose increase of Φ_B.

Next,

If $r<a$ $E_{NC}2\pi r = -\mu_0 n\pi r^2 \frac{\Delta i}{\Delta t}$ *[LOOP INSIDE SOLENOID]*

If $r>a$ $E_{NC}2\pi r = -\mu_0 n\pi a^2 \frac{\Delta i}{\Delta t}$ *[LOOP OUTSIDE SOLENOID]*

Magnitude



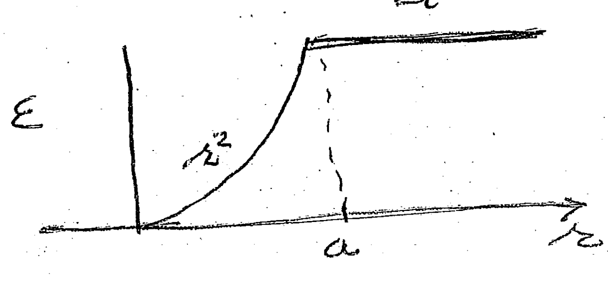
$E_{NC} = -\frac{\mu_0 n r}{2}\frac{\Delta i}{\Delta t}$ $r<a$

$E_{NC} = -\mu_0 n \frac{a^2}{2r}\frac{\Delta i}{\Delta t}$ $r>a$

It is useful to ask what is the E-mf in these two loops $\mathcal{E} = \sum \vec{E}_{NC}\cdot\Delta\vec{l}$

$r<a$ $\mathcal{E} = E_{NC}\cdot 2\pi r = -\frac{\mu_0 n\pi r^2\Delta i}{\Delta t}$

$r>a$ $\mathcal{E} = -\mu_0 n\pi a^2\frac{\Delta i}{\Delta t}$



Inside $\mathcal{E}$ increases as r^2
outside it is const. for all r!