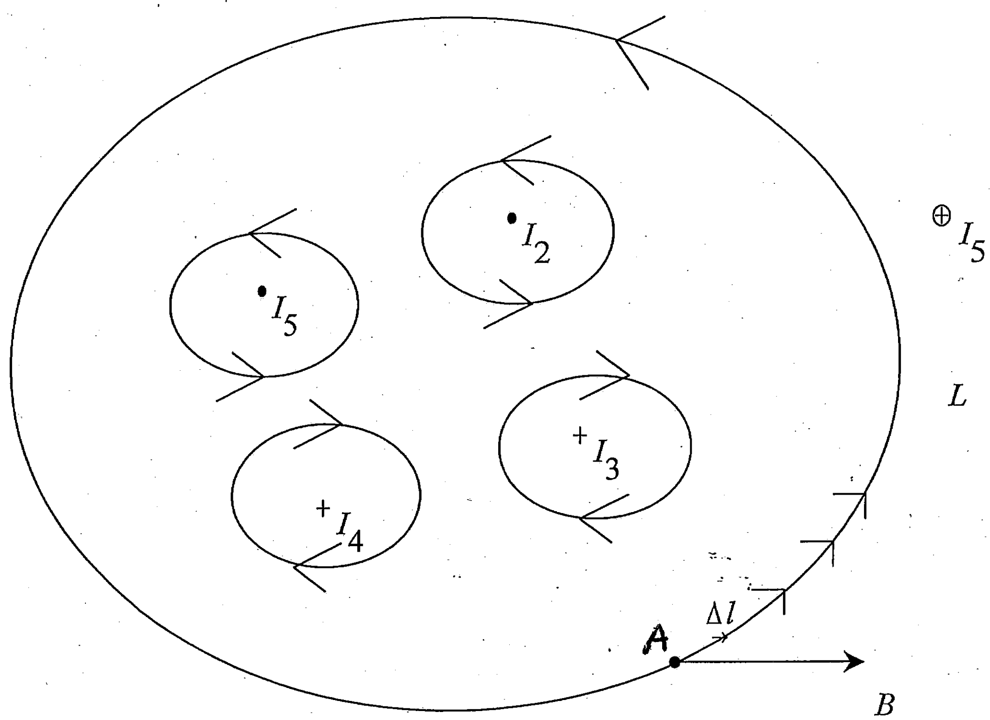
Ampere's Law - Applications

Consider the situation shown schematically in the diagram. Currents $I_1, I_2, I_3, I_4, I_5$ are flowing out of $(\cdot)$ or $(+)$ into the paper. The corresponding $\underset{\rightarrow}{B}$-fields swirl around their sources as shown. The main point is that the $\underset{\rightarrow}{B}$-field lines circulate around the currents. Choose a closed loop $(L)$. Start at $A$, measure $B$, choose small stop $\underset{\rightarrow}{\Delta l}$ along loop. Calculate Dot product (component of $\underset{\rightarrow}{B}$ along $\underset{\rightarrow}{\Delta l}$ multiplied by $\Delta l$)

$$\underset{\rightarrow}{B} \cdot \underset{\rightarrow}{\Delta l} = B \Delta l \cos(\underset{\rightarrow}{B}, \underset{\rightarrow}{\Delta l}).$$

If $\underset{\rightarrow}{B} \perp \underset{\rightarrow}{\Delta l}$, $\underset{\rightarrow}{B} \cdot \underset{\rightarrow}{\Delta l} = 0$.




Rpeat this calculation at every step as shown. $\underset{\rightarrow}{B} \cdot \underset{\rightarrow}{\Delta l}$

Write out the sum

$$\Sigma_c \underset{\rightarrow}{B} \cdot \underset{\rightarrow}{\Delta l};$$

$c$: closed loop.

This sum is called circulation of $\underset{\rightarrow}{B}$ around closed loop and Ampere says that it is determined solely by currents threading through the surface on which the loop is drawn & only currents within the loop contribute, i.e. exclude $I_5$. The mathematical Equation is:

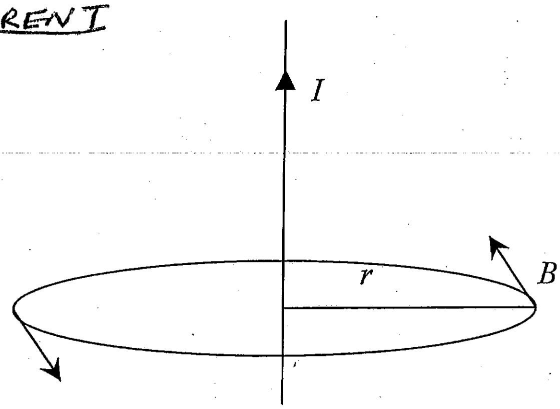
$$\Sigma_c \underset{\rightarrow}{B} \cdot \underset{\rightarrow}{\Delta l} = \mu_0 \Sigma I_i, \mu_0 = 4\pi \times 10^{-7} \frac{T-m}{A}$$

In words, circulation of $\vec{B}$ around a closed loop is proportional to the algebraic sum of the currents threading the surface on which the loop is drawn.

Note: As in case of Gauss' Law, Ampere's law gives circulation but not $\vec{B}$. To get $\vec{B}$ you need high symmetry!

Applications

1.) SINGLE CURRENT




Single wire with current $I$, there is cylindrical symmetry so $\vec{B}$ can be a function of $r$ only & must encircle $I$. [ $\vec{B}$ and $\vec{\Delta \ell}$ are parallel to one another ]
Appropriate loop is circle of radius $r$ centered on the wire

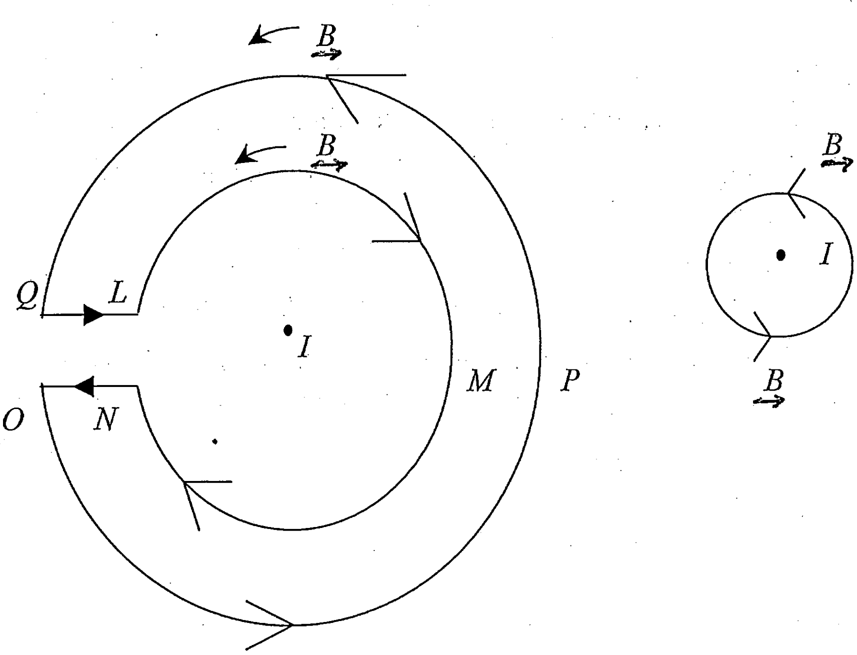
$$B \cdot 2\pi r = \mu_0 I$$

$$\text{so, } \vec{B} = \frac{\mu_0 I}{2\pi r}\hat{\varnothing}$$

as claimed previously

2.) Next, we begin by showing that if current is outside the loop it contributes nothing to the circulation. Choose $L\ M\ N\ O\ P\ Q$ with $I$ at the center of the circles of radii $r_1$, and $r_2$

I is out of paper.



$$\underset{\rightarrow}{B}(r_1) = \frac{\mu_0 I}{2\pi r_1}\hat{\varnothing}$$

$\underset{\rightarrow}{B} \perp \underset{\rightarrow}{\Delta l}$

$$\underset{\rightarrow}{B}(r_2) = \frac{\mu_0 I}{2\pi r_2}\hat{\varnothing}$$

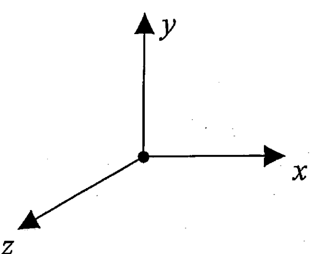
$$\sum_{c} \underset{\rightarrow}{B}\cdot\underset{\rightarrow}{\Delta l} = \frac{\mu_0 I}{2\pi r_1}\cdot 2\pi r_1 + O + \frac{\mu_0 I}{2\pi r_2}\cdot 2\pi r_2 + O$$

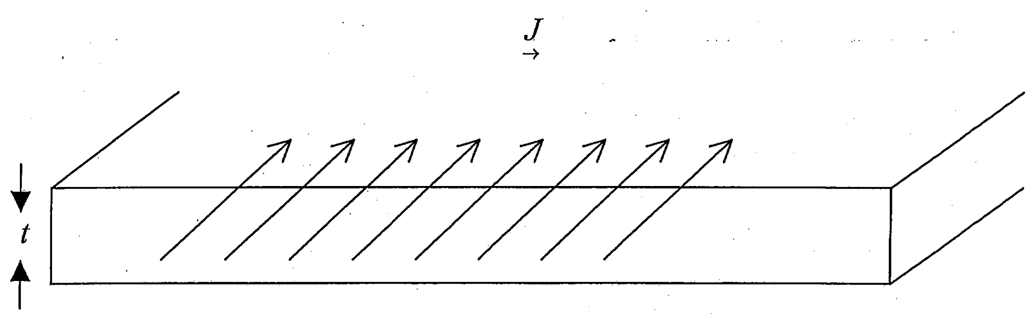
$$= L\rightarrow M\rightarrow N + N\rightarrow O + O\rightarrow P\rightarrow Q + Q\rightarrow L$$

$$= O$$

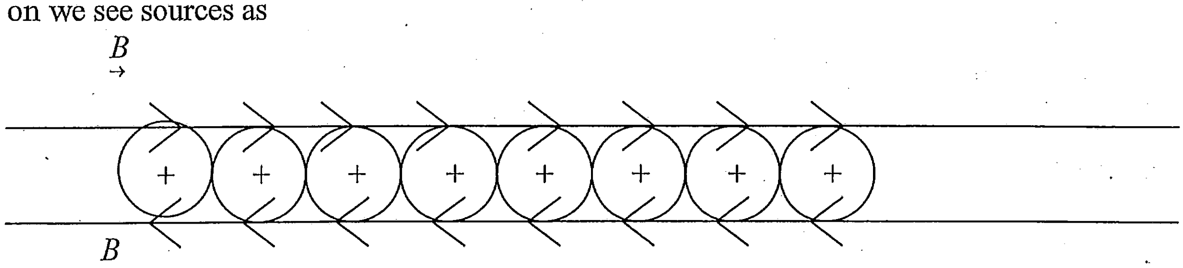
[The first term is -ive] $\underset{\rightarrow}{B}$ and $\underset{\rightarrow}{\Delta l}$ are opposite to one another

3.)

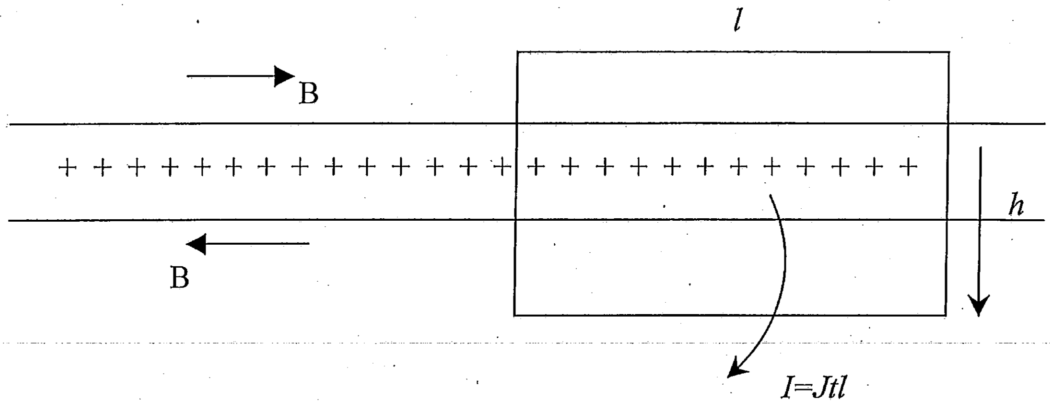



At $y$=0 there is a Current sheet of thickness $t$ carrying current density $\underset{\rightarrow}{J} = -J\hat{z}$. Looking at it end-on we see sources as



and we see that $y$-components of $\underset{\to}{B}$ cancel out. $\underset{\to}{B} \| \hat{x}$ survives. Let us take loop of width $l$ and height $h$.

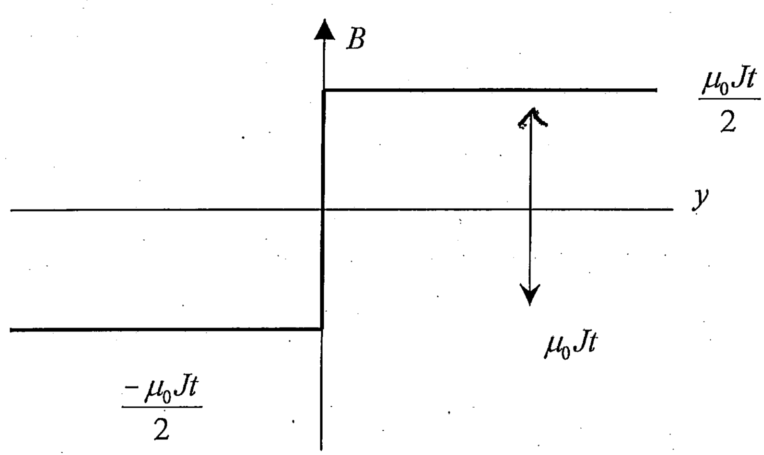



$$\Sigma_c \underset{\to}{B} \bullet \underset{\to}{\Delta l} = Bl + 0 + Bl + 0$$
$$= 2Bl$$
$$= \mu_0 Jtl$$

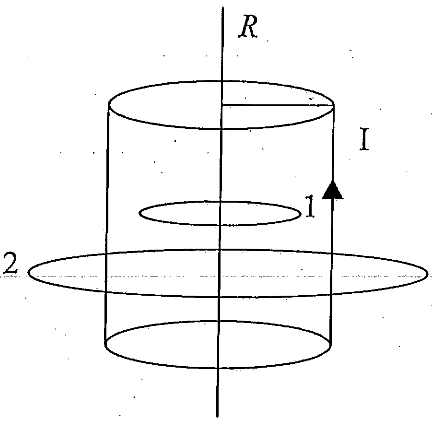
so,

$$B = \frac{\mu_0 Jt}{2} \;\&\; \underset{\to}{B} = \frac{\mu_0 Jt}{2}\hat{x} \qquad y > 0$$
$$= \frac{-\mu_0 Jt}{2}\hat{x} \qquad y > 0$$

That is, $\underset{\to}{B}$-field will jump by $\mu_0 Jt$ on crossing the current sheet from y<0 to y>0.

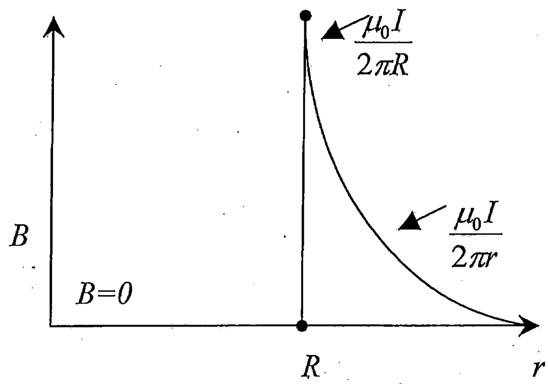


4.) Hollow Cylindrical Conductor- Radius $R$, carries uniform current. We want $\underset{\rightarrow}{B}$ at a distance $r$ from its axis. Since there is a cylindrical symmetry we should use circles centered on the axis for our closed loop.
For $r<R$. use loop 1.



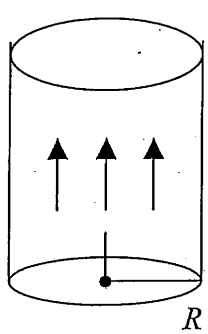

$B \cdot 2\pi r = 0$. No Current threads through loop 1.
for $r>R$
~~so~~, $B \cdot 2\pi r = \mu_0 I$, the entire current threads loop 2.

so, $\underset{\rightarrow}{B} = \frac{\mu_0 I}{2\pi r}\hat{\varnothing}$



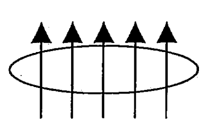

Note: if cylinder has wall thickness $t$: $I = J \cdot \cancel{2\pi t}\ 2\pi R t$ and field at surface would be $\mu_0 Jt$.
Again field would jump by $\mu_0 Jt$ on crossing a current sheet.

5.) SOLID CYLINDRICAL CONDUCTOR – with uniform current




Define $J = \frac{I}{\pi R^2}$

Now for $r<R$ $I = J\pi r^2$



$B \cdot 2\pi r = \mu_0 J\pi r^2$

$\underset{\rightarrow}{B} = \frac{\mu_0 Jr}{2}\hat{\varnothing}$ $r < R$

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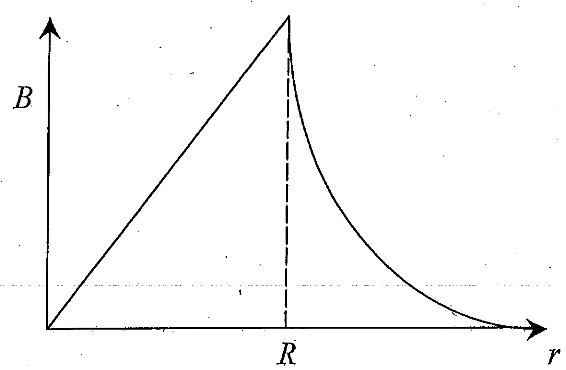
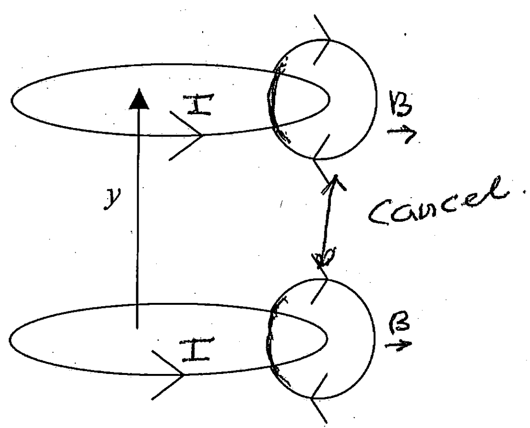
For $>R$, entire $I$ contributes $\underset{\rightarrow}{B} = \frac{\mu_0 I}{2\pi r}\hat{\varnothing}$




6.) Solenoid: Tightly wound, small radius, length much larger than radius:

N turns, L long, $n = \frac{N}{L}$ =# of turns per meter.

Look at two neighboring turns




r-component cancels
$B_y$ inside survives.

Long-narrow solenoid. [B field lines must come out of top, loop around and enter at bottom with no breaks or bends allowed].
$\underset{\rightarrow}{B} \approx 0$ just outside.

Take loop as shown
$$Bl = \mu_0 nIl$$
$$B = \mu_0 nI$$

For case shown $\underset{\rightarrow}{B} = \mu_0 nI\hat{y}$