- **Theme Music:** **Jerry Lee Lewis**

  Whole Lotta Shakin' Goin' On

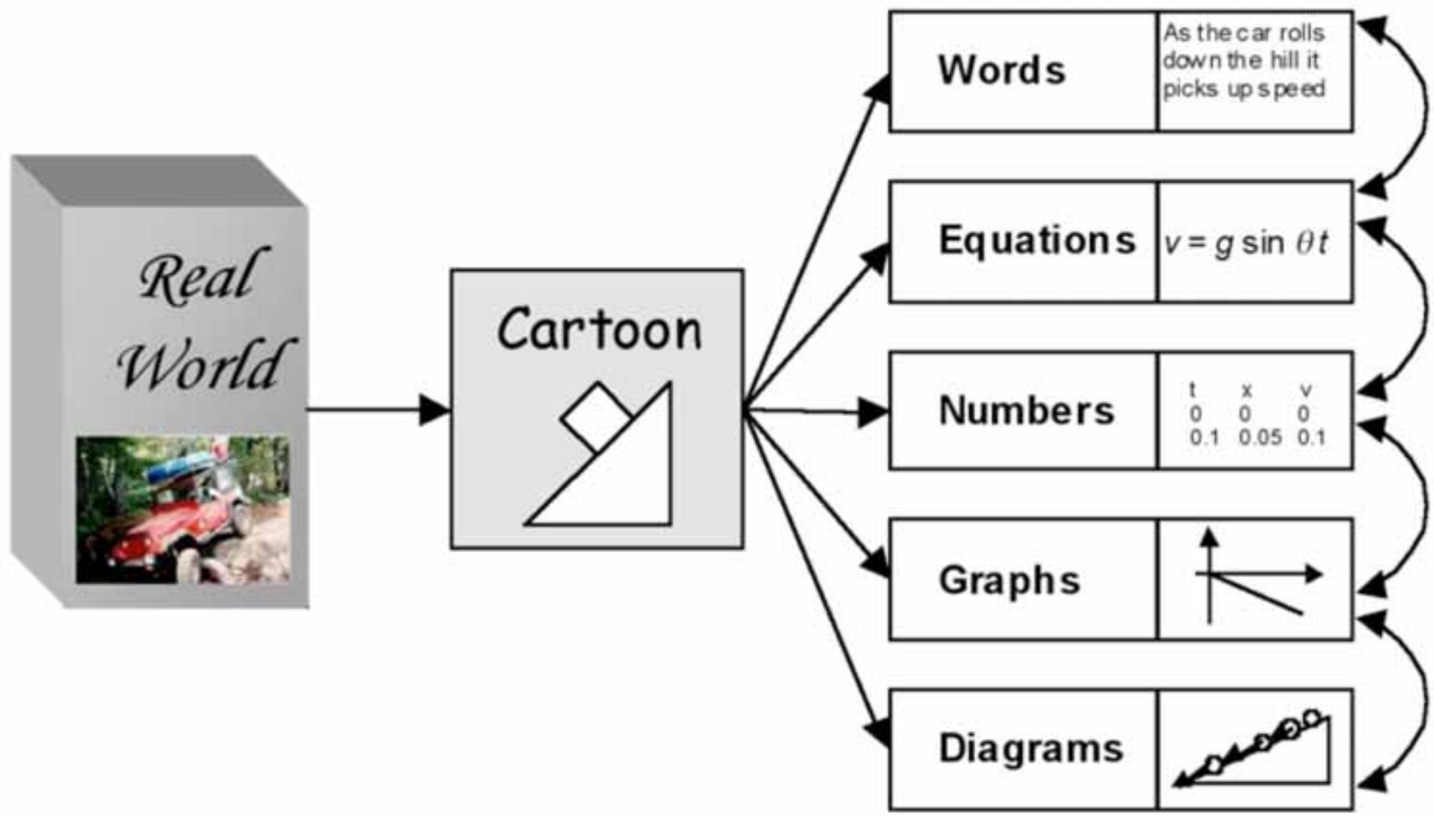
- **Cartoon:** **Bill Watterson**

  Calvin & Hobbes

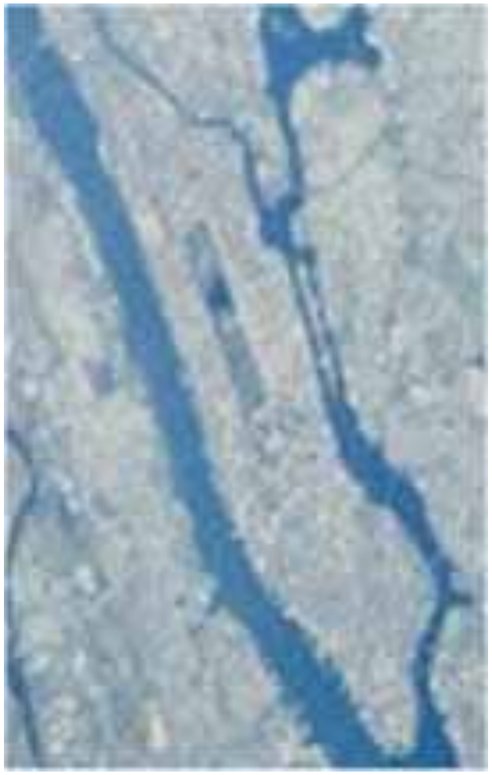
Outline

- Modeling and the use of equations
- Mass on a spring: Doing the math
  - Recap
  - sin and cos of time
- Energies
  - Recap of work and energy
  - The energy balance of the mass and spring

In physics we make a lot of use of multiple representations to help reify a concept



The most useful map may not be the most accurate map.

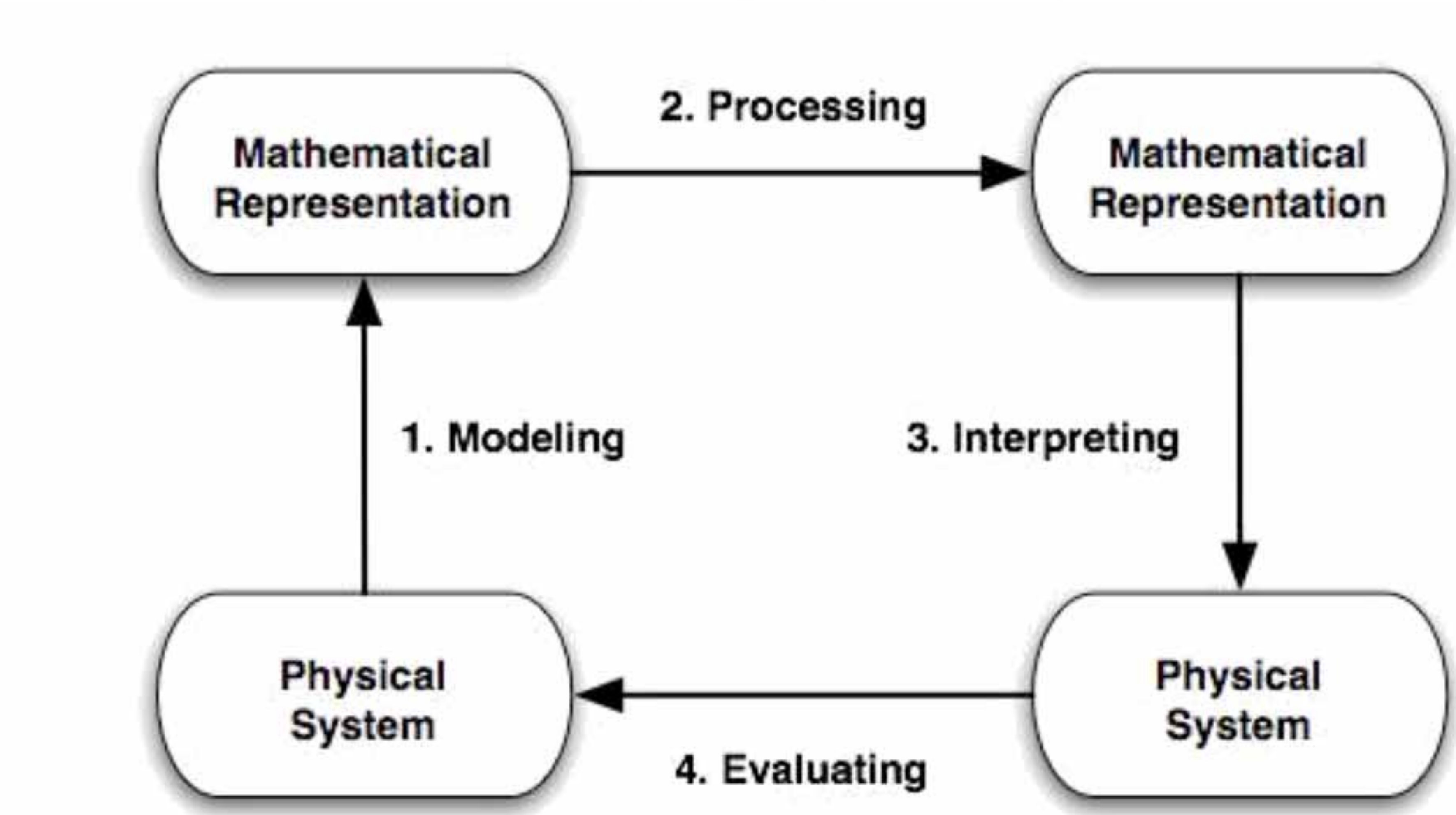


NYC –
The Real Thing
(Landsat photo)

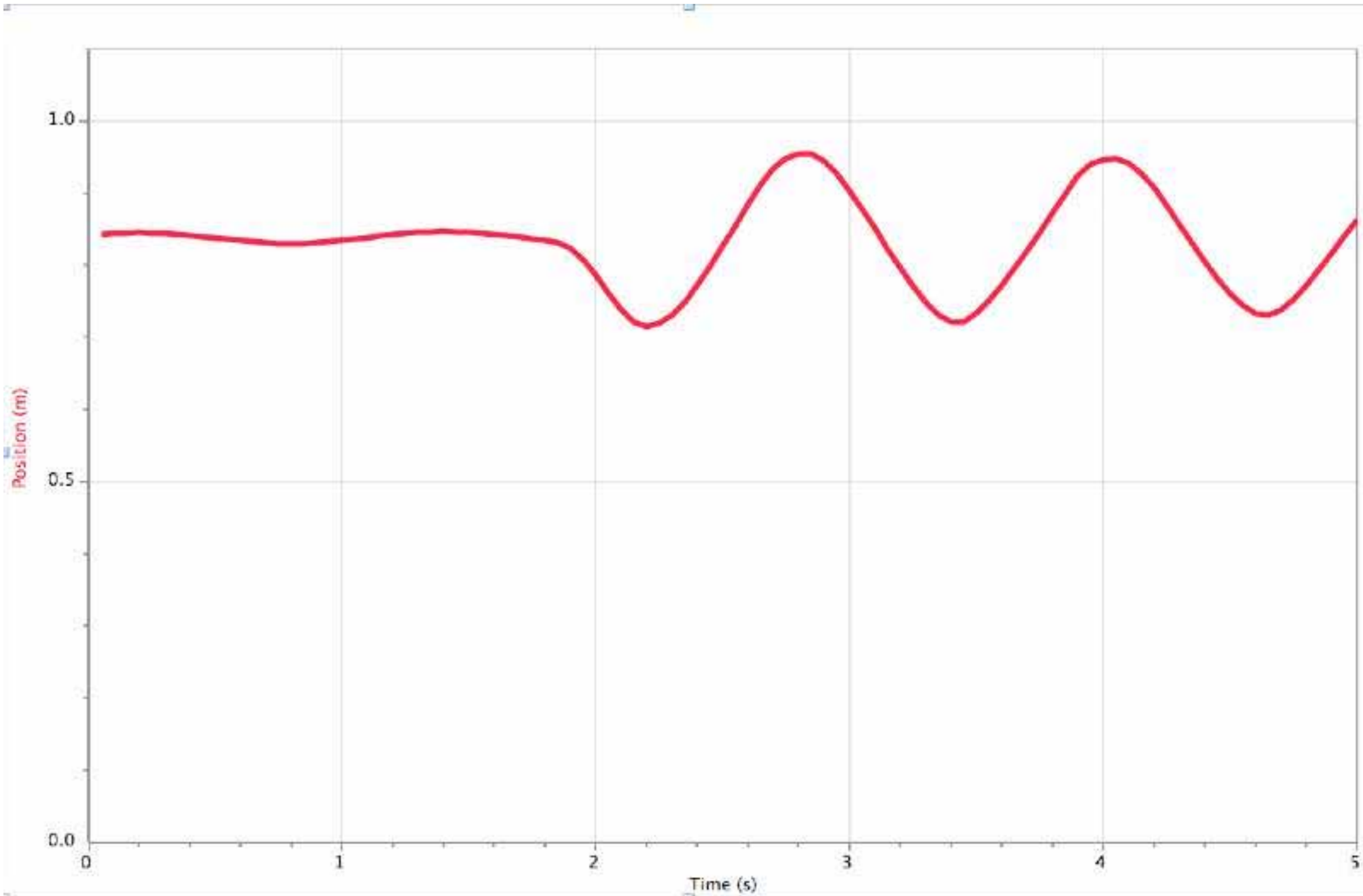
NYC –
Subway map
(c. 1948)

NYC –
Subway map
(current)

Unpacking our use of equations in physics: modeling a physical system



The Data (hanging mass on spring): graphical representation



Building a solution

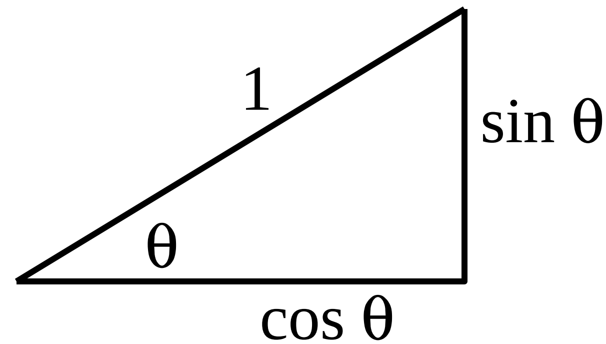
- We can't just use, say $x = \cos t$ because as we've seen, the units are all wrong.
  - The argument of cos must be an angle — therefore dimensionless.
  - The displacement, *x*, must be a length, while cos is a ratio and therefore dimensionless.
- This works better – if we choose our axes correctly.

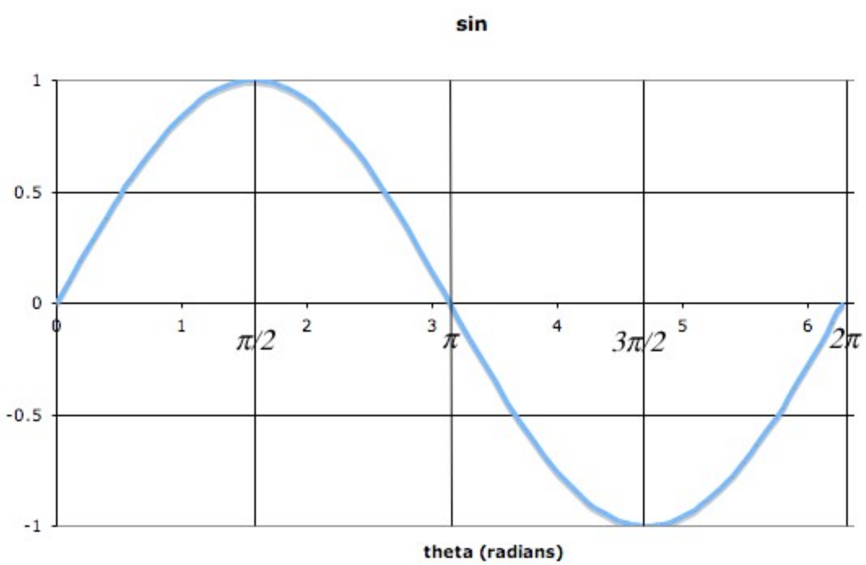
$$x = A\cos\omega_0 t$$

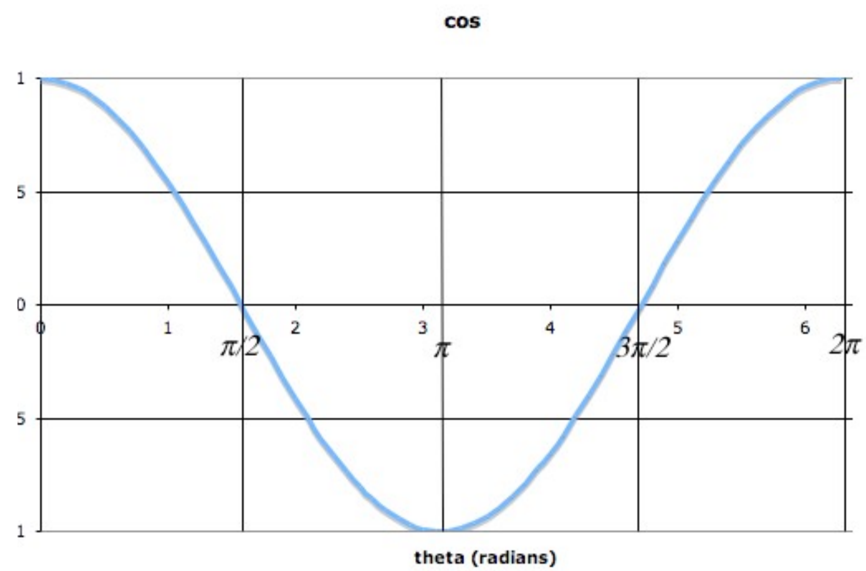
$$v = -\omega_0 A\sin\omega_0 t$$

Graphs: sin(θ) vs cos(θ)

- Which is which? How can you tell?
- The two functions sin and cos are derivatives of each other (slopes), but one has a minus sign.
  Which one?
  How can you tell?





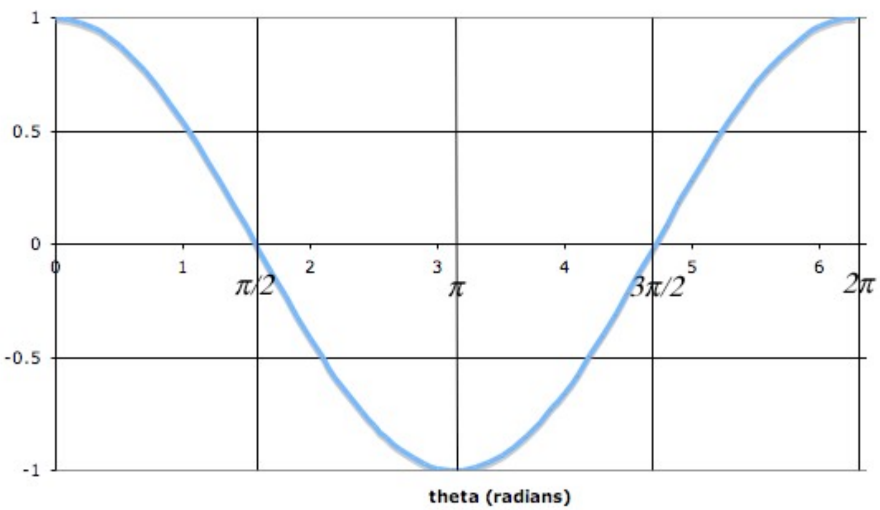


Graphs: $\sin(\theta)$ vs $\sin(\omega_0 t)$

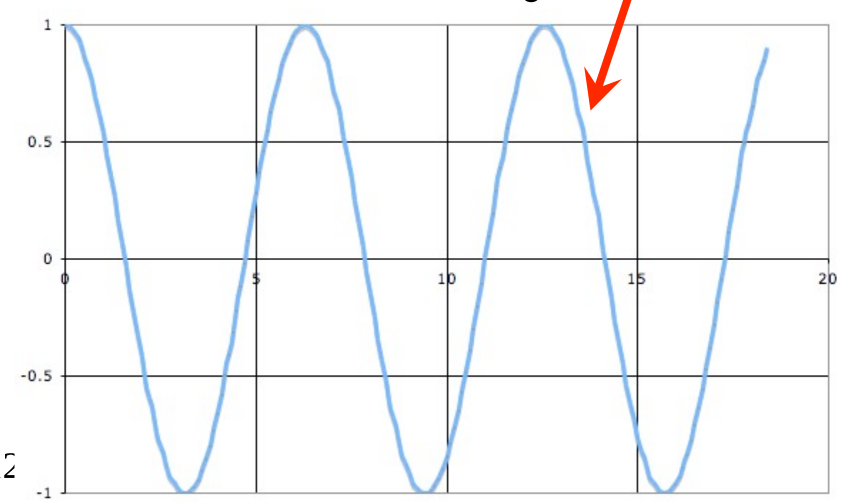
- For angles, $\theta = 0$ and $\theta = 2\pi$ are the same so you only get one cycle.
- For time, t can go on forever so the cycles repeat.

What does changing ω_0 do to this graph?

$\cos(\theta)$



$\cos(\omega_0 t)$



Interpreting the Result

- We'll leave it to our friends in math to show that these results actually satisfy the N2 equations.
- What do the various terms mean?
  - A is the maximum displacement — the *amplitude* of the oscillation.
  - What is ω_0? If T is the *period* (how long it takes to go through a full oscillation) then

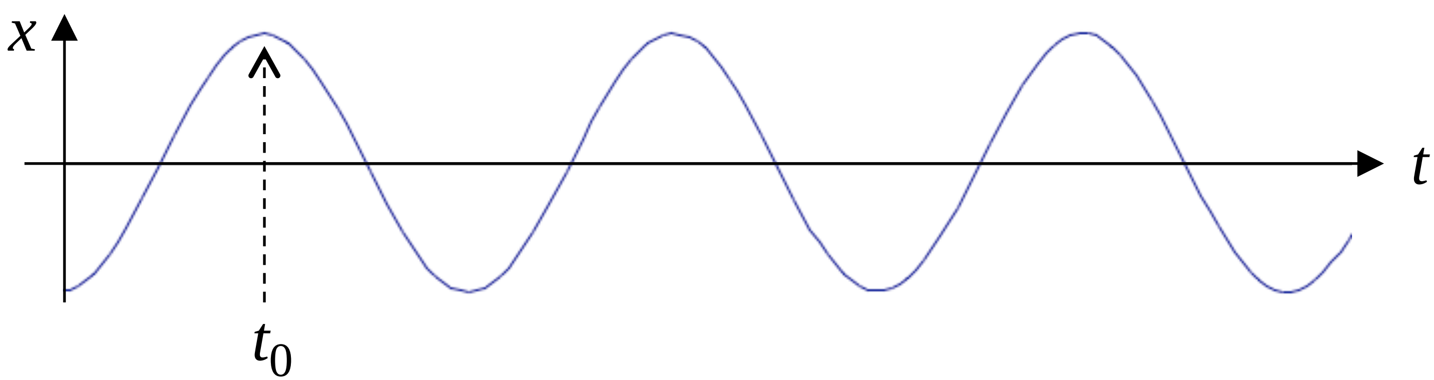
$$\omega_0 t : 0 \rightarrow 2\pi$$

$$t \quad : 0 \rightarrow T$$

$$\omega_0 T = 2\pi \quad \Rightarrow \quad \omega_0 = \frac{2\pi}{T}$$

Interpreting the Result

- What about the starting point?
  Using cos means you always start at a peak when $t = 0$. That might not always be true.



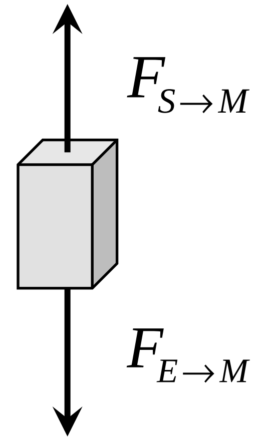
$$x(t) = A\cos(\omega_0(t - t_0))$$

$$= A\cos(\omega_0 t - \omega_0 t_0) = A\cos(\omega_0 t - \phi)$$

Foothold ideas: Mass on a spring

- There is an equilibrium (balance) point where the mass can stay without moving.
- Whichever way the mass moves, the force is in the direction of pushing it back to its equilibrium position.
- When it gets back to its equilibrium, it's still moving so it overshoots.

Summary with Equations: Mass on a spring (forces)



$$a = \frac{1}{m}F^{net} \qquad F^{net} = -kx$$

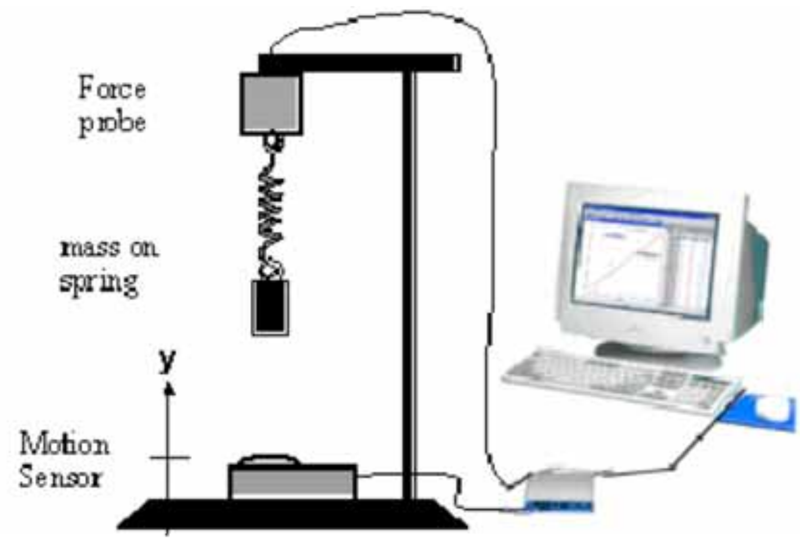
Measured from where?

$$a = -\omega_0^2 x \qquad \omega_0^2 = \frac{k}{m}$$

$$x(t) = A\cos(\omega_0 t - \phi)$$

$$\omega_0 = \frac{2\pi}{T}$$

Interpret!



Foothold Ideas: Energy and Work

- We can rewrite N2 to focus on the part of the forces that change the object's speed.
- Define Kinetic Energy and Work

$$KE = \tfrac{1}{2} mv^2 \qquad \text{Work} = \vec{F}^{net} \cdot \Delta\vec{r}$$

- Rewrite N2 by taking the dot product with the displacement (to select part of force acting along the motion)

$$\Delta\left(\tfrac{1}{2} mv^2\right) = \vec{F}^{net} \cdot \Delta\vec{r}$$

Potential Energy

- For some kinds of forces (gravity, springs) the work done by that force can be brought to the other side and treated as a kind of energy. {*Conservative forces*}

$$U_g = mgh \qquad U_{spr} = \tfrac{1}{2}k\left(\Delta l\right)^2$$

- For some kinds of forces (friction, air resistance) you can't do this. {*Non-conservative forces*}

Energy Conservation Equations

- If there are no non-conservative forces (friction, viscosity, drag)

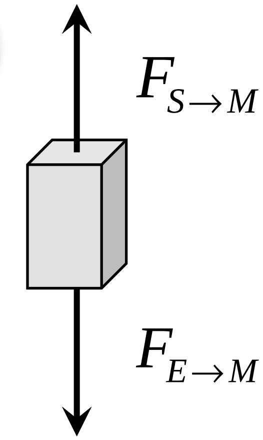
$$\tfrac{1}{2}mv_i^2 + U(\vec{r}_i) = \tfrac{1}{2}mv_f^2 + U(\vec{r}_f)$$

$$\Delta\left(\tfrac{1}{2}mv^2 + U\right) = 0$$

- If there are non-conservative forces:

$$\Delta\left(\tfrac{1}{2}mv^2 + U\right) = \vec{F}^{net}_{non-cons} \cdot \Delta\vec{r}$$

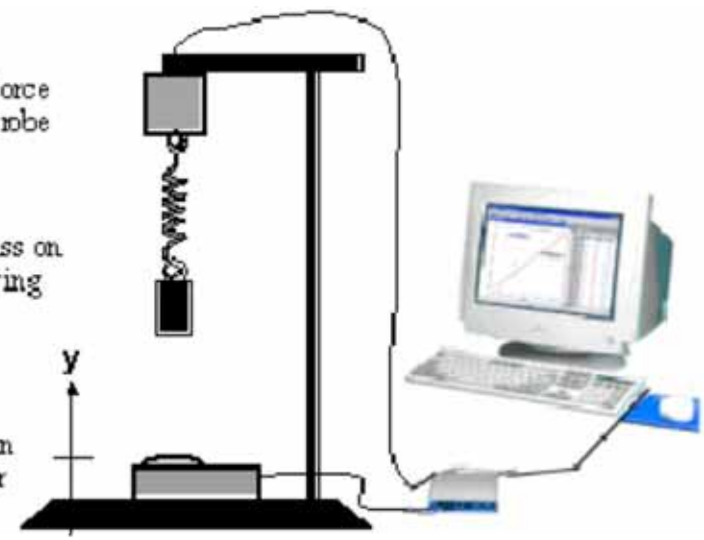
Summary with Equations: Mass on a spring (Energy)



Measured from where?

$$E = \tfrac{1}{2}mv^2 + mgh + \tfrac{1}{2}k\left(\Delta l\right)^2$$

$$E_i = E_f$$



Why is the Mass on a Spring Important?

- Although the mass on a spring ("harmonic oscillator") seems of little real interest, it really is important.
- These are the properties that make it work.
  - A deviation from equilibrium is associated with creation of a cause that tends to push the system back towards equilibrium.
  - Once it reaches equilibrium, the cause disappears so it overshoots the equilibrium point.
- Any system that has these properties (and there are lots) will have the same math as the harmonic oscillator, so the HO will be a good analogy.