32. **REASONING** The rotational analog of Newton's second law is given by Equation 9.7,  $\Sigma \tau = I\alpha$ . Since the person pushes on the outer edge of one pane of the door with a force **F** that is directed perpendicular to the pane, the torque exerted on the door has a magnitude of *FL*, where the lever arm *L* is equal to the width of one pane of the door. Once the moment of inertia is known, Equation 9.7 can be solved for the angular acceleration  $\alpha$ .

The moment of inertia of the door relative to the rotation axis is  $I = 4I_P$ , where  $I_P$  is the moment of inertia for one pane. According to Table 9.1, we find  $I_P = \frac{1}{3}ML^2$ , so that the rotational inertia of the door is  $I = \frac{4}{3}ML^2$ .

**SOLUTION** Solving Equation 9.7 for  $\alpha$ , and using the expression for *I* determined above, we have

$$\alpha = \frac{FL}{\frac{4}{3}ML^2} = \frac{F}{\frac{4}{3}ML} = \frac{68 \text{ N}}{\frac{4}{3}(85 \text{ kg})(1.2 \text{ m})} = \boxed{0.50 \text{ rad/s}^2}$$

36. **REASONING** The force applied to the baton creates a net torque that gives the rod an angular acceleration, according to Newton's second law as expressed for rotational motion  $(\Sigma \tau = I\alpha)$ . From the data given, we will calculate the moment of inertia I and the angular acceleration  $\alpha$ . Newton's second law, then, will allow us to obtain the net torque  $\Sigma \tau$ . From the net torque, we will be able to determine the force, since the magnitude of the torque in this case is just the magnitude of the force times the lever arm of 2.0 cm.

**SOLUTION** The momentum of inertia of the baton is the sum of the individual moments of inertia of its parts. For a thin rod of length L rotating about an axis perpendicular to the rod at its center, the moment of inertia is  $I_{\text{rod}} = \frac{1}{12} M_{\text{rod}} L^2$ . For each ball (assumed to be very small compared to the length of the rod), the moment of inertia is  $I_{\text{ball}} = M_{\text{ball}} (L/2)^2$ . The total moment of inertia, then, is

$$I_{\rm beton} = I_{\rm rod} + 2I_{\rm ball} = \frac{1}{12} M_{\rm rod} L^2 + 2M_{\rm ball} \left(\frac{1}{2}L\right)^2 = \frac{1}{12} M_{\rm rod} L^2 + \frac{1}{2} M_{\rm ball} L^2$$

Using Equation 8.4 from the equations of kinematics for rotational motion, we can determine the angular acceleration as

$$\alpha = \frac{\omega - \omega_0}{t}$$

where  $\omega$  and  $\omega_0$  are, respectively, the final and initial angular velocities. Since only one force of magnitude F creates the torque and since Equation 9.1 gives the magnitude of the torque as the magnitude of the force times the lever arm  $\ell$ , Newton's second law becomes  $\Sigma \tau = F\ell = I\alpha$ . Substituting our expressions for the moment of inertia and angular acceleration of the baton into the second law gives

$$F\ell = I_{\text{baton}} \alpha = \left(\frac{1}{12} M_{\text{rod}} L^2 + \frac{1}{2} M_{\text{ball}} L^2\right) \left(\frac{\omega - \omega_0}{t}\right)$$

## 46. REASONING

a. The kinetic energy is given by Equation 9.9 as  $KE_R = \frac{1}{2}I\omega^2$ . Assuming the earth to be a uniform solid sphere, we find from Table 9.1 that the moment of inertia is  $I = \frac{2}{5}MR^2$ . The mass and radius of the earth is  $M = 5.98 \times 10^{24}$  kg and  $R = 6.38 \times 10^6$  m (see the inside of the text's front cover). The angular speed  $\omega$  must be expressed in rad/s, and we note that the earth turns once around its axis each day, which corresponds to  $2\pi \, \text{rad/day}$ .

b. The kinetic energy for the earth's motion around the sun can be obtained from Equation 9.9 as  $KE_R = \frac{1}{2}I\omega^2$ . Since the earth's radius is small compared to the radius of the earth's orbit  $(R_{\text{orbit}} = 1.50 \times 10^{11} \text{ m})$ , see the inside of the text's front cover), the moment of inertia in this case is just  $I = MR_{\text{orbit}}^2$ . The angular speed  $\omega$  of the earth as it goes around the sun can be obtained from the fact that it makes one revolution each year, which corresponds to  $2\pi$  rad/year.

## **SOLUTION**

a. According to Equation 9.9, we have

$$KE_{R} = \frac{1}{2}I\omega^{2} = \frac{1}{2}(\frac{2}{5}MR^{2})\omega^{2}$$

$$= \frac{1}{2}\left[\frac{2}{5}(5.98 \times 10^{24} \text{ kg})(6.38 \times 10^{6} \text{ m})^{2}\right]\left[\left(\frac{2\pi \text{ rad}}{1 \text{ day}}\right)\left(\frac{1 \text{ day}}{24 \text{ h}}\right)\left(\frac{1 \text{ h}}{3600 \text{ s}}\right)\right]^{2}$$

$$= 2.57 \times 10^{29} \text{ J}$$

b. According to Equation 9.9, we have

$$KE_{R} = \frac{1}{2}I\omega^{2} = \frac{1}{2}(MR_{\text{orbit}}^{2})\omega^{2}$$

$$= \frac{1}{2}(5.98 \times 10^{24} \text{ kg})(1.50 \times 10^{11} \text{ m})^{2} \left[ \left(\frac{2\pi \text{ rad}}{1 \text{ yr}}\right) \left(\frac{1 \text{ yr}}{365 \text{ day}}\right) \left(\frac{1 \text{ day}}{24 \text{ h}}\right) \left(\frac{1 \text{ h}}{3600 \text{ s}}\right) \right]^{2}$$

$$= \boxed{2.67 \times 10^{33} \text{ J}}$$

53. SSM WWW REASONING Let the two disks constitute the system. Since there are no external torques acting on the system, the principle of conservation of angular momentum applies. Therefore we have  $L_{\rm initial} = L_{\rm final}$ , or

$$I_{\rm A}\omega_{\rm A} + I_{\rm B}\omega_{\rm B} = (I_{\rm A} + I_{\rm B})\omega_{\rm final}$$

This expression can be solved for the moment of inertia of disk B.

**SOLUTION** Solving the above expression for  $I_B$ , we obtain

$$I_{\rm B} = I_{\rm A} \left( \frac{\omega_{\rm final} - \omega_{\rm A}}{\omega_{\rm B} - \omega_{\rm final}} \right) = (3.4 \text{ kg} \cdot \text{m}^2) \left[ \frac{-2.4 \text{ rad/s} - 7.2 \text{ rad/s}}{-9.8 \text{ rad/s} - (-2.4 \text{ rad/s})} \right] = \boxed{4.4 \text{ kg} \cdot \text{m}^2}$$

57. **SSM REASONING** Let the space station and the people within it constitute the system. Then as the people move radially from the outer surface of the cylinder toward the axis, any torques that occur are internal torques. Since there are no external torques acting on the system, the principle of conservation of angular momentum can be employed.

SOLUTION Since angular momentum is conserved,

$$I_{\rm final}\omega_{\rm final}=I_0\omega_0$$

Before the people move from the outer rim, the moment of inertia is

$$I_0 = I_{\text{station}} + 500 m_{\text{person}} r_{\text{person}}^2$$

or

$$I_0 = 3.00 \times 10^9 \text{ kg} \cdot \text{m}^2 + (500)(70.0 \text{ kg})(82.5 \text{ m})^2 = 3.24 \times 10^9 \text{ kg} \cdot \text{m}^2$$

If the people all move to the center of the space station, the total moment of inertia is

$$I_{\text{final}} = I_{\text{station}} = 3.00 \times 10^9 \text{ kg} \cdot \text{m}^2$$

Therefore,

$$\frac{\omega_{\text{final}}}{\omega_0} = \frac{I_0}{I_{\text{final}}} = \frac{3.24 \times 10^9 \text{ kg} \cdot \text{m}^2}{3.00 \times 10^9 \text{ kg} \cdot \text{m}^2} = 1.08$$

This fraction represents a percentage increase of 8 percent.