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PHYSICS 121 | Prof. S. J. Gates
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Section I. Multiple Choice Questions

Each question in this section is worth eight (8) points. You should **NOT** take more than two minutes per question. If you do, it is advisable to continue on to the next question!

(1.) (a.) Since $a = v^2/R$ and $v = \frac{2\pi R}{T}$ implies $a = 4\pi^2 \frac{R}{T^2}$, we find $a = 4\pi^2 \times \frac{4m}{(4\,s)^2} = \pi^2 \frac{m}{s^2} = 9.87 \frac{m}{s^2}$.

(2.) (a.) As with the problem on the practice exam, the velocities and heights are related by $(V_1)^2 = 2\,g\,H_1$ and $(V_2)^2 = 2\,g\,H_2$ so that $\frac{H_1}{H_2} = \frac{(V_1)^2}{(V_2)^2}$ $= [(2\,\frac{m}{s})^2]\,/\,[(6\,\frac{m}{s})^2] = 1/9$.

(3.) (b.) Since both start from rest $W_1 = \frac{1}{2}\,m_1\,(V_1)^2$ and $W_2 = \frac{1}{2}\,m_2\,(V_2)^2$ and since $W_1 = W_2$ it follows that $3\,kg\,(V_1)^2 = 27\,kg\,(V_2)^2$ or $V_1 = 3\,V_2$.

(4.) (b.) Here we have the data; $\omega_0 = 2\pi\ \frac{\text{rad}}{s}$, $\omega_f = 4\pi\ \frac{\text{rad}}{s}$, $\theta_0 = 0$ and $\theta_f = 6\pi$ rad and one of the basic angular kinematic equations is $(\omega_f)^2 = (\omega_0)^2 + 2\ \alpha\ (\Delta\theta)$. Putting in the numbers yields $\alpha = \pi\ \frac{\text{rad}}{s^2}$, also since $\omega = \omega_0 + \alpha\ t = 2\pi\ \frac{\text{rad}}{s} + \left(\pi\ \frac{\text{rad}}{s^2}\right)\ t$ we see that the value of $t = 2\ s$ works.

(5.) (d.) We can find the work done by using $W = \Delta KE = \frac{1}{2}\,m\,[0\ -\ (v_0)^2]$ $= -\frac{1}{2}\ (2000\ kg)\ (6\frac{m}{s})^2 = -36000\ J$. The time over which the collision occurs is $\Delta t = 0.12\ s$. So the power delivered is $\Big|W\Big|/\Delta t = 300{,}000\ J$.

Section II. Analytical Questions

Problem (1.)

From Newton's second law it follows that $m\ a = F$ and the problem gives the force and acceleration. Therefore, it must be so that

$$m\,(9\,\tfrac{m}{s^4}\,t^2) \;=\; 36\,(\tfrac{N}{s^3}\,t^3) \quad\rightarrow\quad m \;=\; 4\,t\,\frac{kg}{s}$$

and we see that the mass depends on the time t. The problem also states that the bucket can only hold 16 kg when full. So we have

$$16\,kg \;=\; 4\,t\,\frac{kg}{s} \quad\rightarrow\quad t \;=\; 4\,s$$

Problem (2.)

This is a momentum conservation problem. The momenta for ball #1 and ball #2 before the collision are given by

Momentum	x-comp.	y-comp.
$\vec{p}_1$	$10\,kg\,(7\,\frac{m}{s})$	0
$\vec{p}_2$	0	$5\,kg\,(14\,\frac{m}{s})$
$\vec{P}^{Before}_{tot}$	$70\,(\frac{kg\,m}{s})$	$70\,(\frac{kg\,m}{s})$

After the collision the two balls are stuck together so that $M_t = 15\ kg$ but we don't know the x-component of velocity V'_x nor the y-component of velocity V'_y

Momentum	x-comp.	y-comp.
$\vec{P}^{After}_{tot}$	$15\,kg\,V'_x$	$15\,kg\,V'_y$

Momentum conservation implies that

$$15\,kg\,V'_x = 70\,(\frac{kg\,m}{s}) \quad , \quad 15\,kg\,V'_y \;=\; 70\,(\frac{kg\,m}{s}) \quad ,$$
$$V'_x = \frac{14}{3}\,(\frac{m}{s}) \quad , \quad V'_y \;=\; \frac{14}{3}\,(\frac{m}{s}) \quad ,$$

and to find the angle we note

$$tan\theta \;=\; \frac{V'_y}{V'_x} \;=\; \frac{(14/3)}{(14/3)} \;=\; 1 \quad\rightarrow\quad \theta \;=\; 45^o.$$

Problem (3.)

In general for a planet going around the Sun with speed V and at radius R, we have

$$\frac{G\,M_S}{R^2} \;=\; \frac{V^2}{R} \quad\rightarrow\quad G\,M_S \;=\; V^2\,R$$

but also we have $V = R\,\omega$, so that

$$G\,M_S \;=\; (\omega)^2\,R^3$$

If we call Earth planet # 2 and Mercury planet # 1 then

$$G\,M_S \;=\; (\omega_1)^2\,(R_1)^3 \quad \& \quad G\,M_S \;=\; (\omega_2)^2\,(R_2)^3$$

or equivalently

$$(\omega_1)^2\,(R_1)^3 \;=\; (\omega_2)^2\,(R_2)^3$$

so finally

$$\begin{aligned}
(\omega_1)^2 &= \frac{(R_2)^3}{(R_1)^3}\,(\omega_2)^2 \;=\; \frac{(1.5\,\times\,10^{11})^3}{(5.8\,\times\,10^{10})^3}\,(2\,\times\,10^{-7}\,\frac{\text{rad}}{s})^2 \\
&= \frac{(15\,\times\,10^{10})^3}{(5.8\,\times\,10^{10})^3}\,(2\,\times\,10^{-7}\,\frac{\text{rad}}{s})^2 \;=\; \frac{(15)^3}{(5.8)^3}\,(2\,\times\,10^{-7}\,\frac{\text{rad}}{s})^2 \\
&= (2.59)^3\;(2\,\times\,10^{-7}\,\frac{\text{rad}}{s})^2 \quad\rightarrow \\
\omega_1 &= [\,\sqrt{(2.59)^3}\,]\,\times\,2\,\times\,10^{-7}\,\frac{\text{rad}}{s} \;=\; 4.16\,\times\,2\,\times\,10^{-7}\,\frac{\text{rad}}{s} \\
&= 8.32\times\,10^{-7}\,\frac{\text{rad}}{s}
\end{aligned}$$

Problem (4.)

We first can find the linear velocity

$$V \;=\; \frac{120\,cm}{10\,s} \;=\; 12\,\frac{cm}{s} \;=$$

and the angular speed and linear speed are related by $V = R\,\omega$ so that $\omega = V/R$ or

$$\omega \;=\; \frac{12}{4}\,\frac{\text{rad}}{s} \;=\; \;=\; 3\,\frac{\text{rad}}{s}$$

Problem (5.)

Even though this problem takes place on an incline, its solution proceeds exactly as on a flat surface. The problem gives that the kinetic energy is changed by 2 J so we have the equation

$$2\,J \;=\; \Delta KE \;=\; W$$

where the last part follows from the Work-Energy Theorem. But the work is also given by $W = F\ s\ cos\theta = -F\ s$ since the angle between the friction and the motion is 180^o. So it also the case that

$$\begin{aligned} 2\,J = \;& -\,F\,\times\,(10\,m) \;\;\rightarrow\;\; F \;=\; -\,\tfrac{1}{5}\,kg\,\tfrac{m}{s^2} \\ \rightarrow\;\; M\,a \;&=\; -\,\tfrac{1}{5}\,kg\,\tfrac{m}{s^2} \;\;\rightarrow\;\; (6\,kg)\,a \;=\; -\,\tfrac{1}{5}\,kg\,\tfrac{m}{s^2} \\ \rightarrow\;\; a \;&=\; -\,\frac{1}{30}\,\frac{m}{s^2} \end{aligned}$$

(a.) One of the basic kinematic equations reads $(v_f)^2 = (v_0)^2 + 2\ a\ \Delta x$ and when it comes to rest $v_f = 0$ so

$$\begin{aligned} 0 \;&=\; (15\,\tfrac{m}{s})^2 \;+\; 2\,(-\tfrac{1}{30}\,\tfrac{m}{s^2})\,\Delta x \;\;\rightarrow \\ \Delta x \;&=\; (15)^3\,m \;=\; 3,375\,m \end{aligned}$$

(b.) Another one of the basic kinematic equations reads $v_f = v_0 + a\ t$, so that we have

$$\begin{aligned} 0 \;&=\; 15\,\tfrac{m}{s} + (-\tfrac{1}{30}\,\tfrac{m}{s^2})\,t \;\;\rightarrow \\ t \;&=\; 2(15)^2\,s \;=\; 450\,s \end{aligned}$$

Problem (6.)

This problem simply uses conservation of energy. If we assume that the mass begins from rest, then we must have before it starts to swing downward

$$E_{Before} \;=\; KE_{Before} \;+\; PE_{Before} \;=\; 0 \;+\; m\,g\,H$$

and at the bottom of its swing

$$E_{After} \;=\; KE_{After} \;+\; PE_{After} \;=\; \tfrac{1}{2}\,m\,V^2 \;+\; 0$$

so that the conversation law implies $V^2 \;=\; 2\,g\,H$. The centripetal acceleration is given by $a_C = V^2/R$ so we find

$$\begin{aligned} a_C \;&=\; \frac{2\,g\,H}{R} \;\;\rightarrow\;\; H \;=\; \frac{a_C}{2\,g}\,R \\ H \;&=\; \frac{20\,\frac{m}{s^2}}{2\,(9.8\,\frac{m}{s^2})}\,[\,2.5\,m] \;=\; 1.02\,[\,2.5\,m] \;=\; 2.55\,m \end{aligned}$$