Department of Physics
University of Maryland
College Park, Maryland

PHYSICS 121 — Prof. S. J. Gates
Fall 2002 — Exam I Solutions — Oct. 4, 2002

Section I. Multiple Choice Questions

Each question in this section is worth eight (8) points. You should **NOT** take more than two minutes per question. If you do, it is advisable to continue on to the next question!

(1.) (a.) Since the kinematic equation for the ball is $v_f = -80\,\frac{m}{s} + (-16\,\frac{m}{s^2})\,t$, we see that as t increases the speed (given by $\left|v_f\right|$) increases.

(2.) (d.) A scalar and a vector cannot be added.

(3.) (b.) For an object thrown, the horizontal velocity never changes from the initial horizontal velocity. The initial horizontal velocity is just $16\,cos(60^o)\frac{m}{s} = 8\,\frac{m}{s}$.

(4.) (c.) (2.46) $(9.8\,\frac{m}{s^2})$ This follows from a result given in a lecture. The acceleration of gravity g on Earth's surface is equal to $g = 9.8\,\frac{m}{s^2} = G\,[M_E/(R_E)^2]$ but for Jupiter

$$g_J = G\Big(\frac{M_J}{(R_J)^2}\Big) = G\Big(\frac{314.5 M_E}{(11.31\,R_E)^2}\Big)$$
$$= \Big(\frac{314.5}{(11.31)^2}\Big)G\Big(\frac{M_E}{(R_E)^2}\Big) = 2.46\,g$$

(5.) (d.) All of the situations described in (a.), (b.) and (c.) can occur by making a choice of coordinates.

Section III. Analytical Questions

Problem (1.)

A bug is located at $(1m,\,2m)$ at $t = 0s$. The same bug is observed to be located at $(2m,\,-3m))$ at $t = 3s$. Finally it is observed to be located at $(-3m,\,-1m)$ at $t = 3.5s$.

(a.) The average velocity between $t = 0s$ and $t = 3s$ is given by

$$\overline{v}_x = \Big(\frac{2 - 1}{3 - 0}\Big)\frac{m}{s} = 0.66\,\frac{m}{s} \quad , \quad \overline{v}_y = \Big(\frac{-3 - 2}{3 - 0}\Big)\frac{m}{s} = -\,1.66\,\frac{m}{s}$$

(b.) The average velocity between $t = 3s$ and $t = 3.5s$, is given by

$$\overline{v}_x = \Big(\frac{-3 - 2}{3.5 - 3}\Big)\frac{m}{s} = 10\,\frac{m}{s} \quad , \quad \overline{v}_y = \Big(\frac{-1 + 3}{3.5 - 3}\Big)\frac{m}{s} = 4\,\frac{m}{s}$$

(c.) The average average acceleration between $t = 3s$ and $t = 3.5s$ may be defined by

$$\overline{a}_x = \Big(\frac{10 - 0.66}{3.5 - 3}\Big)\frac{m}{s^2} = 18.68\,\frac{m}{s^2} \quad , \quad \overline{a}_y = \Big(\frac{4 + 1.66}{3.5 - 3}\Big)\frac{m}{s^2} = 11.32\,\frac{m}{s^2} \quad .$$

Problem (2.)

The kinematic position equations for cars #1 and cars #2 are given by

$$x_{f,1} = \frac{1}{2}\,(0.6)\,(9.8\tfrac{m}{s^2})\,t^2 \quad , \quad x_{f,2} = 45\,m + \frac{1}{2}\,(0.3)\,(9.8\tfrac{m}{s^2})\,t^2$$

If there is a tie then

$$\begin{aligned}
&\frac{1}{2}\,(0.6)\,(9.8\tfrac{m}{s^2})\,t^2 = 45\,m + \frac{1}{2}\,(0.3)\,(9.8\tfrac{m}{s^2})\,t^2 \quad \rightarrow \\
&\frac{1}{2}\,(0.3)\,(9.8\tfrac{m}{s^2})\,t^2 = 45\,m \quad \rightarrow \\
&\frac{3\,(98)}{200}\,(\tfrac{m}{s^2})\,t^2 = 45\,m \quad \rightarrow \quad t = \Big(\sqrt{\frac{3000}{98}}\Big)\,s = 5.5\,s
\end{aligned}$$

The kinematic velocity equations for cars #1 and cars #2 are given by

$$\begin{aligned}
&v_{f,1} = (0.6)\,(9.8\tfrac{m}{s^2})\,(5.5\,s) \quad , \quad v_{f,2} = (0.3)\,(9.8\tfrac{m}{s^2})\,(5.5\,s) \\
&v_{f,1} = 32.5\,\frac{m}{s} \quad , \quad v_{f,2} = 16.2\,\frac{m}{s}
\end{aligned}$$

Problem (3.)

The kinematic equation for the y-velocity of the ball has $v_{f,y} = 0$ at the highest point of the ball's flight. Therefore

$$0 = 196\,\frac{m}{s} - (9.8\,\tfrac{m}{s^2})\,t \quad \rightarrow \quad t = \Big(\frac{196}{9.8}\Big)\,\frac{m}{s} = 20\,s$$

is the time it takes to reach that point. This implies that the ball is in the air for 40 s. This is impossible.

Problem (4.)

This problem is solved by setting up a table as was done in many of the homework problems

Vector	x-component	y-component
$\vec{d}_1$	$0\,m$	$-3\,m$
$\vec{d}_2$	$-\,4\,m$	$0\,m$
$\vec{d}_3$	$0\,m$	$10\,m$
$\vec{d}_4$	$-\,2\,m$	$0\,m$
$\vec{d}_5$	$0\,m$	$8\,m$
$\vec{d}_6$	$2\,m$	$0\,m$
$\vec{d}_1\,+\,...\,+\,\vec{d}_6$	$-\,4\,m$	$15\,m$

So magnitude of the total displacement vector is

$$\left|\vec{d}_1\,+\,...\,+\,\vec{d}_6\right| \;=\; \sqrt{16+225}\;m \;=\; \sqrt{241}\;m \;=\; 15.52\,m$$

and the angle above the y-axis in the third quadrant is just

$$tan(\theta) \;=\; \frac{15}{4} \quad\rightarrow\quad \theta \;=\; tan^{-1}\Big(\frac{15}{4}\Big) \;=\; 75.1^o$$

Problem (5.)

Let the tension in the string on the left be denoted by T_1 and name the angle θ_1 between the vertical and the first string. Let the tension in the string on the right be denoted by T_2 and name the angle θ_2 between the vertical and the second string. Finally let the tension in the vertical string be named T_3.

(a.) For the 5 kg mass after drawing a force diagram, Newton's second law gives the first two equations

$$m_1\, a_{1,x} \;=\; 0 \quad , \quad m_1\, a_{1,y} \;=\; T_3 \;-\; m_1\, g$$

For the 50 kg mass after drawing a force diagram, Newton's second law gives the third and fourth equation

$$m_2\, a_{2,x} \;=\; -\, T_2 \;+\; \mu\, N_f \quad , \quad m_2\, a_{1,y} \;=\; N_f \;-\; m_2\, g$$

From the place where the strings are tied

$$\begin{aligned} T_1\, cos(\theta_1) \;+\; T_2\, cos(\theta_2) &=\; T_3 \\ -\, T_1\, sin(\theta_1) \;+\; T_2\, sin(\theta_2) &=\; 0 \end{aligned}$$

(b.) Since we have an equilibrium, it follows that *all* accelerations are equal to zero. So from the second equation above we see $T_3 = m_1 g$ and from the fourth equation $N_f = m_2 g$. Using this information, the third equation implies $T_2 \;=\; \mu\, m_2\, g$ and using the conditions where the strings are tied gives

$$\begin{aligned} T_1\, cos(\theta_1) \;+\; \mu\, m_2\, g\, cos(\theta_2) &=\; m_1\, g \\ -\, T_1\, sin(\theta_1) \;+\; \mu\, m_2\, g\, sin(\theta_2) &=\; 0 \end{aligned}$$

We multiply the first of these by $sin(\theta_1)$, multiply the second by $cos(\theta_1)$ and then add the two equations together

$$\begin{aligned} T_1\, cos(\theta_1)\, sin(\theta_1) \;+\; \mu\, m_2\, g\, cos(\theta_2)\, sin(\theta_1) &=\; m_1\, g\, sin(\theta_1) \\ -\, T_1\, sin(\theta_1)\, cos(\theta_1) \;+\; \mu\, m_2\, g\, sin(\theta_2)\, cos(\theta_1) &=\; 0 \\ \hline \mu\, m_2\, g\, cos(\theta_2)\, sin(\theta_1) \;+\; \mu\, m_2\, g\, sin(\theta_2)\, cos(\theta_1) &=\; m_1\, g\, sin(\theta_1) \\ \mu\, m_2\, g \Big[cos(\theta_2)\, sin(\theta_1) \;+\; sin(\theta_2)\, cos(\theta_1) \Big] &=\; m_1\, g\, sin(\theta_1) \end{aligned}$$

$$\mu \;=\; \frac{m_1\, sin(\theta_1)}{m_2 \Big[cos(\theta_2)\, sin(\theta_1) \;+\; sin(\theta_2)\, cos(\theta_1) \Big]}$$

$$\mu \;=\; \frac{sin(\theta_1)}{10 \Big[sin(\theta_1 \;+\; \theta_2) \Big]}$$

(c.) We already know $T_3 = (5\ kg)\ (9.8\ \frac{m}{s^2}) = 49\ N$. For T_2 and T_1 we find

$$T_2 \;=\; \frac{(49\, N)\, sin(\theta_1)}{\Big[sin(\theta_1 \;+\; \theta_2) \Big]} \quad , \quad T_1 \;=\; \frac{(49\, N)\, sin(\theta_2)}{\Big[sin(\theta_1 \;+\; \theta_2) \Big]}$$

Problem (6.)

The radius of the Moon must be $R = (3.48)/2 \times 10^6\ m = 1.74 \times 10^6\ m$. The circumference of the Moon is then given by

$$C \;=\; 2\,\pi\,R$$

and if it takes time T to completely go around then the speed is

$$v \;=\; \frac{2\,\pi\,R}{T}$$

and also the acceleration must be

$$a \;=\; \frac{v^2}{R} \;=\; \frac{4\,\pi^2\ R}{T^2}$$

This acceleration is to be set equal to the acceleration of gravity at the Moon surface

$$\begin{aligned}
\frac{1}{6}\,g &= \frac{4\,\pi^2\ R}{T^2} \\
T^2 &= \frac{24\,\pi^2\ R}{g} \\
T &= \pi\,\sqrt{\frac{24\,R}{g}} \\
&= (3.14)\,\sqrt{\frac{24\,(1.74\ \times\ 10^6\,)}{9.8}}\ s \;=\; 6485\,s \;=\; 1.8\,hrs.
\end{aligned}$$