**Department of Physics**
**University of Maryland**

Physics 121 Fall 2002
Homework Assignment # 8

Problem Solutions

5)For this problem we recall the result

$$\overline{\alpha} \;=\; \frac{\Delta\,\omega}{\Delta\,t}$$

as well we know $1\,\text{rev} \;=\; 2\,\pi\,\text{rad}$ and $1\;min = 60\;s$. It follows that

$$\begin{aligned}
\omega_f \;&=\; 210\,\frac{\text{rev}}{min} \;=\; 210\,\frac{2\,\pi\,\text{rad}}{60\,s} \;=\; 21.9\,\frac{\text{rad}}{s}\\
\omega_0 \;&=\; 480\,\frac{\text{rev}}{min} \;=\; 480\,\frac{2\,\pi\,\text{rad}}{60\,s} \;=\; 50.3\,\frac{\text{rad}}{s}\\
\rightarrow\quad \Delta\,\omega &= [\;21.9\;-\;50.3\;]\,\frac{\text{rad}}{s} \;=\; -\;28.4\,\frac{\text{rad}}{s}\\
\Delta\,t &= \;74\,(60)\,s \;=\; 4440\,s
\end{aligned}$$

So that finally

$$\overline{\alpha} \;=\; \frac{-\,28.4\,\frac{\text{rad}}{s}}{4440\,s} \;=\; -\,6.4\,\times\,10^{-3}\,\frac{\text{rad}}{s^2}$$

The magnitude is just $6.4\,\times\,10^{-3}\,\frac{\text{rad}}{s^2}$.

12) At $t$ seconds, the angular positions for the two people are respectively given by

$$\theta_1 \;=\; \omega_1\,t \;\;,\;\; \theta_2 \;=\; \omega_2\,t$$

For them to meet the sum of their two angular positions must equal $2\,\pi\,\text{rad}$ and this leads to the equation

$$\begin{aligned}
\omega_1\,t \;+\;\omega_2\,t \;&=\; 2\,\pi\,\text{rad}\\
t \;&=\; \frac{2\,\pi\,\text{rad}}{\omega_1\;+\;\omega_2}\\
t \;&=\; \frac{2\,\pi\,\text{rad}}{1.7\,\times\,10^{-3}\,\frac{\text{rad}}{s}\;+\;3.4\,\times\,10^{-3}\,\frac{\text{rad}}{s}} \;=\; 1200\,s
\end{aligned}$$

14) In class we saw why the formula

$$s \;=\; r\,\theta \quad\rightarrow\quad \theta \;=\; \frac{s}{r}$$

is valid. This problem requires that we make use of this twice. We are given the numerical data

$$s_m = 3.48 \times 10^6\, m \quad , \quad r_m = 3.85 \times 10^8\, m$$
$$s_S = 1.39 \times 10^9\, m \quad , \quad r_S = 1.50 \times 10^{11}\, m$$

so that we can calculate the angle subtended by each as we see them in the sky is given by

$$\theta_m = \frac{3.48 \times 10^6\, m}{3.85 \times 10^8\, m} \quad , \quad \theta_m = \frac{1.39 \times 10^9\, m}{1.50 \times 10^{11}\, m}$$
$$\theta_m = 9.04 \times 10^{-3}\,\text{rad} \quad , \quad \theta_S = 9.27 \times 10^{-3}\,\text{rad}$$

It can be seen that the angle for the sun is a little bit larger than that of the moon. This means that the moon cannot completely “block out” the sun. The areas that the moon and sun block out are respectively given by

$$A_m = \tfrac{1}{4}\pi\,(s_m)^2 \quad , \quad A_S = \tfrac{1}{4}\pi\,(s_S)^2$$
$$A_m = \tfrac{1}{4}\pi\,(r_S\,\theta_m)^2 \quad , \quad A_S = \tfrac{1}{4}\pi\,(r_S\,\theta_S)^2$$

so that the ratio of the areas is given by

$$\frac{A_m}{A_S} = \frac{(r_S\,\theta_m)^2}{(r_S\,\theta_S)^2} = \left(\frac{\theta_m}{\theta_S}\right)^2 = \left(\frac{9.04 \times 10^{-3}}{9.27 \times 10^{-3}}\right)^2$$
$$\frac{A_m}{A_S} = \left(\frac{9.04}{9.27}\right)^2 = 0.951$$

and the moon only blocks out 95.1% of the sun.

22) One of the equations of linear kinematics is given by

$$x_f = x_0 + v_0\,t + \tfrac{1}{2}\,a\,t^2$$

and as discussed in class this can directly be used to derive

$$\theta_f = \theta_0 + \omega_0\,t + \tfrac{1}{2}\,\alpha\,t^2$$

so that upon using the numerical data this becomes

$$\theta_f = 0 + (74.5\,\frac{\text{rad}}{s})\,(4.5\,s) + \tfrac{1}{2}\,(-6.7\,\frac{\text{rad}}{s^2})\,(4.5\,s)^2$$
$$\theta_f = 267\,\text{rad}$$

29) One revolution is $2\,\pi$ rad so we use

$$\overline{\omega} = \frac{\Delta\,\theta}{\Delta\,t} = \frac{2\,\pi\,\text{rad}}{18.9\,s} = 0.33\,\frac{\text{rad}}{s}$$

and the linear speed is related to the angular speed via $\overline{v} = R\,\overline{\omega}$ or equivalently

$$R \;=\; \frac{\overline{v}}{\overline{\omega}} \;=\; \frac{42.6\,\frac{m}{s}}{0.33\,\frac{\text{rad}}{s}} \;=\; 128\,m$$

38) We recall that the centripetal acceleartion $a_c$ is related to the angular speed via $a_c = R\,\omega^2$ so

$$a_{c,1} \;=\; R\,(\omega_1)^2 \quad , \quad a_{c,2} \;=\; R\,(\omega_2)^2$$

and dividing the first by the second we see

$$\begin{aligned}
\frac{a_{c,1}}{a_{c,2}} &= \frac{R\,(\omega_1)^2}{R\,(\omega_2)^2} = \frac{(\omega_1)^2}{(\omega_2)^2} = \Big(\frac{\omega_1}{\omega_2}\Big)^2 \\
\frac{a_{c,1}}{a_{c,2}} &= \Big(\frac{440}{110}\Big)^2 = 16
\end{aligned}$$

39) The centripetal and tangential acceleration are given respectively by

$$a_c \;=\; r\,\omega^2 \quad , \quad a_T \;=\; r\,\alpha$$

and these two components are at right angles to one another so the magnitude of the acceleration is given by the Pythagorean theorem

$$\begin{aligned}
a &= \sqrt{(a_c)^2 \;+\; (a_T)^2} \;=\; \sqrt{(r\,\omega^2)^2 \;+\; (r\,\alpha)^2} \\
&= r\,\sqrt{(\omega^2)^2 \;+\; (\alpha)^2} \;=\; r\,\sqrt{(\omega)^4 \;+\; (\alpha)^2} \\
&= (1.5\,m)\,\sqrt{(14\,\frac{\text{rad}}{s})^4 \;+\; (160\,\frac{\text{rad}}{s^2})^2} \\
&= (1.5)\,\sqrt{(14)^4 \;+\; (160)^2}\,\frac{m}{s^2} \\
&= (1.5)\,\sqrt{(38416) \;+\; (25600)}\,\frac{m}{s^2} \\
&= (1.5)\,\sqrt{(64016)}\,\frac{m}{s^2} \;=\; (1.5)\,253\,\frac{m}{s^2} \;=\; 380\,\frac{m}{s^2}
\end{aligned}$$

40) We need the following three equations for this problem.

$$\omega \;=\; \overline{\omega} \;=\; \frac{\Delta\,\theta}{\Delta\,t} \quad , \quad v_T \;=\; r\,\omega \quad , \quad a_c \;=\; r\,\omega^2$$

The first of these can be substituted into the last two

$$v_T \;=\; r\,\Big(\frac{\Delta\,\theta}{\Delta\,t}\Big) \quad , \quad a_c \;=\; r\,\Big(\frac{\Delta\,\theta}{\Delta\,t}\Big)^2$$

The numerical data are

$$\Delta\,\theta \;=\; 2\,\pi\,\text{rad} \;\; , \;\; \Delta\,t \;=\; 3.16\,\times\,10^7\,s \;\; , \;\; r \;=\; 1.5\,\times\,10^{11}\,m$$

and this leads to

$$\frac{\Delta\,\theta}{\Delta\,t} \;=\; \frac{2\,\pi\,\mathrm{rad}}{3.16\;\times\;10^{7}\,s} \;=\; 1.99\;\times\;10^{-7}\,\frac{\mathrm{rad}}{s}$$

$$v_T \;=\; (1.5\;\times\;10^{11}\,m)\,(1.99\;\times\;10^{-7}\,\frac{\mathrm{rad}}{s}) \;=\; 2.98\;\times\;10^{4}\,\frac{m}{s}$$

$$a_c \;=\; (1.5\;\times\;10^{11}\,m)\,(1.99\;\times\;10^{-7}\,\frac{\mathrm{rad}}{s})^2 \;=\; 5.94\;\times\;10^{-3}\,\frac{m}{s^2}$$

41) Newton's second law implies

$$a_T \;=\; \frac{F}{m} \;=\; \frac{550\,N}{220\,kg} \;=\; 2.5\,\frac{m}{s^2}$$

and we can use one of the kinematical equations to find the tangential speed.

$$v_T \;=\; V_0 \;+\; a_T\,t \;=\; (5\,\frac{m}{s}) \;+\; (2.5\,\frac{m}{s^2})\,(2\,s) \;=\; 10\frac{m}{s}$$

and finally for the centripetal accelearation

$$a_c \;=\; \frac{(v_T)^2}{r} \;=\; \frac{(10\frac{m}{s})^2}{32\,m} \;=\; 3.1\,\frac{m}{s}$$

42) The tangential speed all along the sprocket must be the same and given by

$$v_T \;=\; r\,\omega \;=\; (9\,cm)\,(9.4\,\frac{\mathrm{rad}}{s}) \;=\; 84.6\,\frac{cm}{s}$$

after using the numerical data. At the rear we have the equations

$$a_c \;=\; r_{rear}\,(\omega)^2 \quad , \quad \omega \;=\; \frac{v_T}{r_{rear}}$$

The second one of these can be substituted into the first to yield

$$a_c \;=\; r_{rear}\,\Big(\frac{v_T}{r_{rear}}\Big)^2 \;=\; \frac{(v_T)^2}{r_{rear}}$$

and finally we use the numerical data

$$a_c \;=\; \frac{(84.6\,\frac{cm}{s})^2}{5.1\,cm} \;=\; 1.4\;\times\;10^{3}\,\frac{cm}{s^2}$$