Department of Physics
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Physics 121 Fall 2002
Homework Assignment # 7

Problem Solutions

3) In this problem we are given the numerical data $m = 62\ kg$, $v_0 = 5.5\ \frac{m}{s}$, $\Delta t = 1.65\ s$ and $v_0 = 1.1\ \frac{m}{s}$. It is simple to calculate the change in momentum

$$\Delta p \;=\; m\,[\; v_f \;-\; v_0\;] \;=\; 62\,kg\,[\;-\;1.1\;-\;(-\,5.5)\;]\,\frac{m}{s} \;=\; 272.8\,N\,s$$

and the Impulse-Momentum Theorem states

$$\Delta p \;=\; \overline{F}\,\Delta t \quad\rightarrow\quad \overline{F} \;=\; \frac{\Delta p}{\Delta t} \;=\; \frac{272.8\,N\,s}{1.65\,s} \;=\; 165.3\,N$$

Since the force is a positive number, it acts upward.

4) The numerical data for this problem are $m = 1\ kg$ and $|\vec{v}| = 30\ \frac{m}{s}$. But since the arrows are fired in different directions, we need to treat the different momenta with a spreadsheet.

Momentum	x-component	y-component
$m\,\vec{v}_1$	$30\,kg\,\frac{m}{s}$	0
$m\,\vec{v}_2$	0	$-\,30\,kg\,\frac{m}{s}$
$m\,\vec{v}_1\,+\,m\,\vec{v}_2$	$30\,kg\,\frac{m}{s}$	$-\,30\,kg\,\frac{m}{s}$

This makes and angle of 45^o south of east as seen from

$$tan\theta \;=\; \frac{-\,30\,kg\,\frac{m}{s}}{30\,kg\,\frac{m}{s}} \;=\; -\;1 \quad\rightarrow\quad \theta \;=\; -\;45^o$$

5) We are given the numerical data $m = 0.35\ kg$, $v_0 = 4.0\ \frac{m}{s}$, v_f = - 21 $\frac{m}{s}$. It is simple to calculate the change in momentum

$$\Delta p \;=\; (0.35\,kg)\,[\;-\;21\;-\;4\;]\,\frac{m}{s} \;=\; -\;8.75\,N\,s$$

But the impluse $I = \Delta p = -\,8.75\,N\,s$.

13) We can use one of the kinematic equations to deduce the speed of the ball just before impact.

$$(v_f)^2 = (v_0)^2 + 2\,g\,(y_f - y_0)$$
$$\rightarrow \quad (v_f)^2 = 0 + 2\,(-9.8\,\tfrac{m}{s^2})\,(0 - 1.2\,m)$$
$$v_f = \sqrt{23.5}\,\frac{m}{s} = 4.8\,\frac{m}{s}$$

Now we can use the same equation to find out how fast the ball was moving after the rebound

$$(V_f)^2 = (V_0)^2 + 2\,g\,(Y_f - Y_0)$$

because we know that $V_f = 0$ when $Y_f = 0.7\ m$ so that

$$(0)^2 = (V_0)^2 + 2\,(-9.8\,\tfrac{m}{s^2})\,(0.7\,m - 0)$$
$$\rightarrow \quad (V_0)^2 = 2\,(-9.8\,\tfrac{m}{s^2})\,(0.7\,m) = 13.72\,\frac{m^2}{s^2}$$
$$V_0 = \sqrt{13.72}\,\frac{m}{s} = 3.7\,\frac{m}{s}$$

The impulse I is just equal to $\Delta\,p$ so

$$I = m\,[\,v_{after} - v_{before}\,] = (0.5\,kg)\,[\,3.7 - (-4.8)\,]\,\frac{m}{s} =$$
$$I = (0.5\,kg)\,(8.5\,\tfrac{m}{s}) = 4.2\,N\,s$$

Since the impulse is a positive number, it acts upward.

14) This problem is little tricky. The Impulse-Momentum Theorem can also be written as

$$\overline{F} = \frac{\Delta p}{\Delta t} = \frac{m\,\Delta v}{\Delta t} = \frac{m}{\Delta t}\,\Delta v$$

When a grain of sand is dropped from a height of 2 m, it has a y-component of velocity just before landing of

$$v_y = -\sqrt{2\,g\,H} = -\sqrt{2(9.8\,\tfrac{m}{s^2})\,(2\,m)} = -\,6.26\,\frac{m}{s}$$

After landing on the truck, its y-component of velocity = 0. So that

$$\Delta v_y = [\,0 - (-6.26\,\tfrac{m}{s})\,] = 6.26\,\frac{m}{s}$$

The problem states that the sand hits the truck as a rate of 55 $\frac{kg}{s}$, which tells us

$$\frac{m}{\Delta t} = 55\,\frac{kg}{s}$$

So that finally we conclude

$$\overline{F} = (55\,\tfrac{kg}{s})\,(6.26\,\tfrac{m}{s}) = 344.3\,N$$

19) This is really a momentum conservation problem. Before the bullet is fired the total momentum is zero. This means that afterward we must have

$$0 = m_b v_b + M_{W+G} V_{W+G}$$
$$\rightarrow \quad V_{W+G} = -\frac{m_b}{M_{W+G}} v_b$$

where the numerical data for part (a.) are $m_b = 0.01\ kg$, $v_b = 720\ \frac{m}{s}$ and $M_{W+G} = 51\ kg$

$$V_{W+G} = -\frac{m_b}{M_{W+G}} v_b = -\frac{0.01\,kg}{51\,kg}(720\,\frac{m}{s})$$
$$= -\frac{7.2}{51}\frac{m}{s} = -0.14\,\frac{m}{s}$$

For (b.) we simply change $m_b \rightarrow 5.0 \times 10^{-4}\ kg$ so that

$$V_{W+G} = -\frac{m_b}{M_{W+G}} v_b = -\frac{5 \times 10^{-4}\,kg}{51\,kg}(720\,\frac{m}{s})$$
$$= -7.1 \times 10^{-3}\,\frac{m}{s}$$

20) This problem is solved using conservation of momentum and conservation of energy. Since before they pushed off they were at rest, the total momentum is 0. So we have

$$0 = m_{ed} V_{ed} + M_A V_A \quad \rightarrow \quad V_{ed} = -\Big(\frac{M_A}{m_{ed}}\Big) V_A$$

From many previous problems and energy conservation we know that the two velocities are related to the two height via

$$V_{ed} = \sqrt{2\,g\,H_{ed}} \quad , \quad V_A = \sqrt{2\,g\,H_A}$$
$$\rightarrow \quad (V_{ed})^2 = 2\,g\,H_{ed} \quad , \quad (V_A)^2 = 2\,g\,H_A$$

Now we see

$$H_{ed} = \frac{1}{2\,g}(V_{ed})^2 = \frac{1}{2\,g}\Big(\frac{M_A}{m_{ed}}\Big)^2 (V_A)^2$$
$$= \frac{1}{2\,g}\Big(\frac{M_A}{m_{ed}}\Big)^2 (2\,g\,H_A) = \Big(\frac{M_A}{m_{ed}}\Big)^2 H_A$$
$$H_{ed} = \Big(\frac{120}{78}\Big)^2 (0.65\,m) = 1.5\,m$$

32) This is problem that we can simply solve using a spreadsheet for the components of the momentum. Before the collision we see

Momentum	x-component	y-component
$m_A\,\vec{v}_A$	$(0.025)\,kg\,(5.5)\frac{m}{s}$	0

and afterward we see

Momentum	x-component	y-component
$m_A\,\vec{v}'_A$	$(0.025)\,kg\,(v'_A)\,cos(65^o)$	$(0.025)\,kg\,(v'_A)\,sin(65^o)$
$m_B\,\vec{v}'_B$	$(0.05)\,kg\,(v'_B)\,cos(37^o)$	$-\,(0.05)\,kg\,(v'_B)\,sin(37^o)$

We now set the total x-component of the momentum before the collision equal to the total x-component of the momentum after the collision.

$$\begin{aligned}(0.025)\,kg\,(5.5)\frac{m}{s} &= (0.025)\,kg\,(v'_A)\,cos(65^o) \;+\; (0.05)\,kg\,(v'_B)\,cos(37^o)\\ (5.5)\frac{m}{s} &= (v'_A)\,cos(65^o) \;+\; 2\,(v'_B)\,cos(37^o)\end{aligned}$$

We also set the total y-component of the momentum before the collision equal to the total x-component of the momentum after the collision.

$$\begin{aligned}0 &= (0.025)\,kg\,(v'_A)\,sin(65^o) \;-\; (0.05)\,kg\,(v'_B)\,sin(37^o)\\ 0 &= (v'_A)\,sin(65^o) \;-\; 2\,(v'_B)\,sin(37^o)\end{aligned}$$

At this stage, we have two unknowns v'_A and v'_B and the two equations

$$\begin{aligned}(5.5)\frac{m}{s} &= (v'_A)\,cos(65^o) \;+\; 2\,(v'_B)\,cos(37^o)\\ 0 &= (v'_A)\,sin(65^o) \;-\; 2\,(v'_B)\,sin(37^o)\end{aligned}$$

we multiply the first by $sin(37^o)$, the second $cos(37^o)$ and add them together to find

$$\begin{aligned}(5.5)\,sin(37^o)\,\frac{m}{s} &= (v'_A)\,[\;cos(65^o)\,sin(37^o) \;+\; sin(65^o)\,cos(37^o)\;]\\ (5.5)\,sin(37^o)\,\frac{m}{s} &= (v'_A)\,sin(102^o)\\ v'_A &= \frac{(5.5)\,sin(37^o)\,\frac{m}{s}}{sin(102^o)} \;=\; \frac{(5.5)\,(0.6)\,\frac{m}{s}}{(0.99)} \;=\; 3.4\,\frac{m}{s}\end{aligned}$$

From the second of the two equations that we are trying to solve, we find

$$v'_B = \frac{(5.5)\,sin(65^o)\,\frac{m}{s}}{2\,sin(102^o)} \;=\; \frac{(5.5)\,(0.9)\,\frac{m}{s}}{2\,(0.99)} \;=\; 2.6\,\frac{m}{s}$$

34) To solve this problem we once again use a spreadsheet for the different components of the momentum before the collision

Momentum	x-component	y-component
$m_1\,\vec{v}_1$	0	$-m_A\,V_1$
$m_2\,\vec{v}_2$	$m_B\,V_2\,cos(30^o)$	$m_B\,V_2\,sin(30^o)$
$m_3\,\vec{v}_3$	$-\,m_B\,V_3\,cos(30^o)$	$m_B\,V_3\,sin(30^o)$

The numerical data for this problem are

$$m_A \;=\; 2.5\,\times\,10^{-3}\,kg \quad , \quad m_B \;=\; 4.5\,\times\,10^{-3}\,kg \quad , \quad V_1 \;=\; 575\,\frac{m}{s}$$

Since after the collision all three bullets are at rest,there is no net momentum. Therefore, adding the entries from the x-column leads to

$$0 \;=\; m_B\,V_2\,cos(30^o) \;-\; m_B\,V_3\,cos(30^o) \quad\rightarrow\quad V_2 \;=\; V_3$$

There also no momentum after adding all the entries from the y-column

$$\begin{aligned}
0 \;&=\; -\,m_A\,V_1 \;+\; m_B\,V_2\,cos(30^o) \;+\; m_B\,V_3\,cos(30^o) \\
0 \;&=\; -\,m_A\,V_1 \;+\; 2m_B\,V_2\,cos(30^o) \\
0 \;&=\; -\,(2.5\,\times\,10^{-3}\,kg)\,(575\,\frac{m}{s}) \;+\; 2(4.5\,\times\,10^{-3}\,kg)\,V_2\,(0.86) \\
\rightarrow \quad V_2 \;&=\; \frac{(2.5\,\times\,10^{-3}\,kg)\,(575\,\frac{m}{s})}{2(4.5\,\times\,10^{-3}\,kg)\,(0.86)} \;=\; 319\frac{m}{s}
\end{aligned}$$

40) Momentum conservation gives

$$m\,(-4\,\frac{m}{s}) \;+\; m\,(7\,\frac{m}{s}) \;=\; m\,(v'_A) \;+\; m\,(v'_B)$$

and energy conservation gives

$$\frac{1}{2}\,m\,(-4\,\frac{m}{s})^2 \;+\; \frac{1}{2}\,m\,(7\,\frac{m}{s})^2 \;=\; \frac{1}{2}\,m\,(v'_A)^2 \;+\; \frac{1}{2}\,m\,(v'_B)^2$$

The factors of m can be divided out of each equation and we are left with

$$\begin{aligned}
(-4\,\frac{m}{s}) \;+\; (7\,\frac{m}{s}) \;&=\; (v'_A) \;+\; (v'_B) \\
16\,\frac{m^2}{s^2} \;+\; 49\,\frac{m^2}{s^2} \;&=\; (v'_A)^2 \;+\; (v'_B)^2
\end{aligned}$$

In other words

$$\begin{aligned}
(3\,\frac{m}{s}) \;&=\; (v'_A) \;+\; (v'_B) \\
65\,\frac{m^2}{s^2} \;&=\; (v'_A)^2 \;+\; (v'_B)^2
\end{aligned}$$

If we square the first equation we find

$$\begin{aligned}
9\,\frac{m^2}{s^2} \;&=\; [\;(v'_A)^2 \;+\; (v'_B)^2\;] \;+\; 2\,v'_A\,v'_B \\
9\,\frac{m^2}{s^2} \;&=\; [\;65\,\frac{m^2}{s^2}\;] \;+\; 2\,v'_A\,v'_B \\
-\,56\,\frac{m^2}{s^2} \;&=\; 2\,v'_A\,v'_B \\
-\,28\,\frac{m^2}{s^2} \;&=\; v'_A\,v'_B
\end{aligned}$$

Now we multiply the same first equation by v'_A

$$\begin{aligned}
(3\,\tfrac{m}{s})\,v'_A &= (v'_A)^2 \;+\; v'_B\,v'_A \\
(3\,\tfrac{m}{s})\,v'_A &= (v'_A)^2 \;-\; 28\,\tfrac{m^2}{s^2} \\
0 &= (v'_A)^2 \;-\; (3\,\tfrac{m}{s})\,v'_A \;-\; 28\,\tfrac{m^2}{s^2}
\end{aligned}$$

Now we multiply the same first equation by v'_B

$$\begin{aligned}
(3\,\tfrac{m}{s})\,v'_B &= v'_B\,v'_A \;+\; (v'_B)^2 \\
(3\,\tfrac{m}{s})\,v'_B &= (v'_B)^2 \;-\; 28\,\tfrac{m^2}{s^2} \\
0 &= (v'_B)^2 \;-\; (3\,\tfrac{m}{s})\,v'_B \;-\; 28\,\tfrac{m^2}{s^2}
\end{aligned}$$

Thus v'_A and v'_B satisfy the same quadratic equation. We use the quadratic formula to find the solutions of this equation.

$$\begin{aligned}
v'_A = &\;\frac{-(-\,3\,\frac{m}{s}) \pm \sqrt{(-\,3\,\frac{m}{s})^2 \;-\; 4\,(-\,28\,\frac{m^2}{s^2})}}{2} \\
= &\;\frac{3 \pm \sqrt{(9 \;+\; 112)}}{2}\,\frac{m}{s} \;=\; \frac{3 \pm 11}{2}\,\frac{m}{s} \\
= &\;7\,\frac{m}{s} \quad \text{or} \quad -\,4\,\frac{m}{s}
\end{aligned}$$

The first ball before the collision had a velocity of $-\,4\,\frac{m}{s}$ and the second ball before the collision had a velocity of $7\,\frac{m}{s}$ so after the collision we find

$$v'_A \;=\; 7\,\frac{m}{s} \quad \text{and} \quad v'_B \;=\; -\,4\,\frac{m}{s}$$

Both balls are travelling opposite to their original directions.