## Department of Physics University of Maryland

Physics 121 Fall 2002 Homework Assignment # 6

Problem Solutions

3) Since the work W is defined by the equation  $W = F s \cos\theta$  it follows that

(a.) 
$$W = (94 N) (35 m) \cos(25^{\circ}) = 2980 J$$
  
(b.)  $W = (94 N) (35) \cos(0^{\circ}) = 3290 J$ 

9) The work done by the husband and wife must be respectively given by the equations

$$W_H = F_H s_H \cos \theta_H$$
,  $W_W = F_W s_W \cos \theta_W$ 

where  $F_H s_H$  and  $\theta_H$  are the force, displacement and angle associated with the husband and  $F_W s_W$  and  $\theta_W$  are the force, displacement and angle associated with the wife. Since the problem says they do the same amount of work we have

$$F_W s_W \cos\theta_W = F_H s_H \cos\theta_H \rightarrow F_W = \frac{F_H s_H \cos\theta_H}{s_W \cos\theta_W}$$

The problem states that  $s_H = s_W$  and the numerical data are

$$F_H = 67 N$$
 ,  $\theta_H = 58^o$  ,  $\theta_W = 38^o$  ,

so that

$$F_W = \frac{67 N \cos(58^{\circ})}{\cos(38^{\circ})} = 45 N$$

10) The work done by force P and the frictional force  $F_f$  are respectively given by

$$W_P = P s \cos(30^\circ)$$
,  $W_f = \mu_k N_f s \cos(180^\circ) = -\mu_k N_f s$ 

We must now find the normal force which follows from looking at Newton's second law in the y-direction and realizing that  $a_y = 0$ .

$$0 = N_f + P \sin(30^\circ) - mg$$

and so the work done by friction is just

$$W_f = -\mu_k [mg - P \sin(30^o)] s$$

The sum of the  $W_P$  and  $W_f$  is zero so

$$0 = P s \cos(30^{\circ}) - \mu_{k} [mg - P \sin(30^{\circ})] s$$
  

$$0 = P \cos(30^{\circ}) - \mu_{k} [mg - P \sin(30^{\circ})]$$
  

$$0 = P [\cos(30^{\circ}) + \mu_{k} \sin(30^{\circ})] - \mu_{k} mg$$
  

$$\rightarrow P = \frac{\mu_{k} mg}{[\cos(30^{\circ}) + \mu_{k} \sin(30^{\circ})]}$$

Using the numerical data this becomes

$$\rightarrow P = \frac{(0.2)(100\,kg)(9.8\,\frac{m}{s^2})}{[0.866 + (0.2)(0.5)]} = \frac{(0.2)(980\,N)}{[0.966]} = 203\,N$$

21) First the Work-Energy Theorem tell us

$$\frac{1}{2} m (v_f)^2 - \frac{1}{2} m (v_0)^2 = W$$

Next we can draw a force diagram using the tilted coordinate system. When this is done Newton's second law takes the form

$$m a_x = F_g \sin(25^\circ) - F_f$$
  
=  $F_g \sin(25^\circ) - \mu_k N_f$   
=  $m g \sin(25^\circ) - \mu_k N_f$   
 $m a_y = N_f - F_g \cos(25^\circ)$   
=  $N_f - m g \cos(25^\circ)$ 

Since there is no acceleration in the y-direction we find

 $N_f = m g \cos(25^o)$ 

so that the acceleration in the x-direction is given by

$$a_x = g [sin(25^o) - \mu_k cos(25^o)]$$

One of our kinematic equations tell us

$$(v_f)^2 - (v_0)^2 = 2 a_x s (v_f)^2 - (v_0)^2 = 2 g [sin(25^o) - \mu_k cos(25^o)] s$$

Next we multiply this by  $\frac{1}{2}m$  and find

$$\frac{1}{2}m(v_f)^2 - \frac{1}{2}m(v_0)^2 = mgs[sin(25^o) - \mu_k cos(25^o)]$$

Comparing this to the Work-Energy Theorem we see

$$W = m g [sin(25^{\circ}) - \mu_k cos(25^{\circ})] s$$

The first term is the work done by gravity and the second the work done by friction. Since she starts from rest at the top  $v_0 = 0$  and we have

$$(v_f)^2 = 2 g s [sin(25^o) - \mu_k cos(25^o)] (v_f) = \sqrt{2 g s [sin(25^o) - \mu_k cos(25^o)] }$$

The numerical data can be substituted into this to find

$$v_{f} = \sqrt{2 (9.8 \frac{m}{s^{2}}) (10.4 m) [sin(25^{o}) - (0.2) cos(25^{o})]}$$

$$v_{f} = \sqrt{(203.84) [sin(25^{o}) - (0.2) cos(25^{o})]} \frac{m}{s}$$

$$v_{f} = \sqrt{(203.84) [(0.42) - (0.2) (0.90)]} \frac{m}{s}$$

$$v_{f} = \sqrt{(203.84) [(0.42) - (0.18)]} \frac{m}{s}$$

$$v_{f} = \sqrt{(203.84) [(0.24)]} \frac{m}{s} = 7.01 \frac{m}{s}$$

For the second part of this problem the speed we just found. The Work-Energy Theorem states (now while she is falling)

$$\frac{1}{2} m (v_f)^2 - \frac{1}{2} m (v_0)^2 = W_g$$

where now the work being done is just that done by gravity

$$W_g = m g H$$

where *H* is her height about the ground after leaving the incline (i.e.  $H = 3.5 \ m$ ) with a speed of 7.01  $\frac{m}{s}$ .

$$\frac{1}{2}m(v_f)^2 - \frac{1}{2}m(v_0)^2 = mgH$$

After dividing by m, multiplying by 2 we find

$$(v_f) = \sqrt{(v_0)^2 + 2gH}$$

which after using the numerical data becomes

$$(v_f) = \sqrt{(7.01\frac{m}{s})^2 + 2(9.8\frac{m}{s^2})(3.5m)}$$
  
$$(v_f) = \sqrt{(49.1)\frac{m^2}{s^2} + (98.62)\frac{m^2}{s^2}} = \sqrt{117.7\frac{m}{s}} = 10.8\frac{m}{s}$$

24) The work done on the airplane follows from the Work-Energy Theorem.

$$\frac{1}{2}m[(v_f)^2 - (v_0)^2] = W$$

The use of Newton's second law for the initial configuration and the final configuration yields

$$m \frac{(v_0)^2}{r_0} = T_f , \quad m \frac{(v_f)^2}{r_f} = T_f$$

and the problems states that  $T_f = 4 T_0$  so that

$$4 m \frac{(v_0)^2}{r_0} = m \frac{(v_f)^2}{r_f} \longrightarrow (v_f)^2 = \frac{4 (v_0)^2}{r_0} r_f$$

This expression for the final velocity can be used in the Work-Energy Theorem

$$W = \frac{1}{2} m \left[ \frac{4 (v_0)^2}{r_0} r_f - (v_0)^2 \right] = \frac{1}{2} m (v_0)^2 \left[ \frac{4 r_f}{r_0} - 1 \right]$$

Now the numerical values can be used

$$W = \frac{1}{2} (0.9 \, kg) \, (22 \, \frac{m}{s})^2 \left[ \frac{4 \, (14)}{16} - 1 \right]$$
  
= 5.4 × 10<sup>2</sup> J

32) Even though there is a non-conservative force acting in this problem, it does no work because it acts at right angles to the direction of motion of the gymnast. So energy conservation can be used as usual. We choose our origin at the location of the bar. With this choice we find

$$\frac{1}{2}m(v_f)^2 + mg(-r_0) = \frac{1}{2}m(v_0)^2 + mg(r_0)$$
  
$$\frac{1}{2}m(v_f)^2 + mg(-r_0) = mg(r_0)$$

where we have used the fact that the initial velocity is zero, the initial y-coordinate is  $r_0$  and the final y-coordinate is  $-r_0$ . So this equation tells us

$$v_f = \sqrt{2 g (2 r_0)} = \sqrt{4 g r_0} = \sqrt{4 (9.8 \frac{m}{s^2}) (1.1 m)} = 6.6 \frac{m}{s}$$

46) The work done by air resistance can be deduced from the fact that the missing mechanical energy must correspond to the work done by the friction

$$W_f = \left[\frac{1}{2}m(v_f)^2 + mg h_f\right] - \left[\frac{1}{2}m(v_0)^2 + mg h_0\right]$$

Now we use all of the numerical data

$$W_{f} = \left[\frac{1}{2}\left(0.6\,kg\right)\left(4.2\,\frac{m}{s}\right)^{2} + \left(0.6\,kg\right)\left(9.8\,\frac{m}{s^{2}}\right)\left(3.1\,m\right)\right] \\ - \left[\frac{1}{2}\left(0.6\,kg\right)\left(7.2\,\frac{m}{s}\right)^{2} + \left(0.6\,kg\right)\left(9.8\,\frac{m}{s^{2}}\right)\left(2\,m\right)\right] \\ = \left[5.29\,J + 18.23\,J\right] - \left[15.55\,J + 11.76\,J\right] \\ = -3.8\,J$$

47) The work done by the non-conservative force can related to the change in the kinetic plus the change in the potential energy via

$$W_{non-c} = \Delta KE + \Delta PE$$
  
$$\Delta KE = \frac{1}{2}m(v_f)^2 - \frac{1}{2}m(v_0)^2$$
  
$$\Delta PE = mgh_f - mgh_0$$

We can choose our reference frame such that  $h_0 = 0$  and we have the numerical data  $v_0 = 1.8 \frac{m}{s}$ ,  $v_0 = 6 \frac{m}{s}$  and m = 55 kg so that

$$\begin{split} \Delta \, KE \; = \; \frac{1}{2} \, (55 \, kg) \, (6 \, \frac{m}{s})^2 \; - \; \frac{1}{2} \, (55 \, kg) \, (1.8 \, \frac{m}{s})^2 \\ & = \; \frac{1}{2} \, (55 \, kg) \, [ \; 36 \; - \; 3.24 \; ] \, \frac{m^2}{s^2} \; = \; 900.9 \, J \\ \Delta \, PE \; = \; m \, g \, h_f \end{split}$$

The two non-conservative forces both contributed to the  $W_{non-c}$  so that

 $W_{non-c} = 80 J - 265 J = `-185 J$ 

Now we can use the first equation in this solution to write

$$-185 J = 900.9 J + \Delta PE \rightarrow$$
$$\Delta PE = -1085.9 J$$

This result can next be used to find the change in the height

$$-1085.9 J = m g h_f \rightarrow h_f = \frac{-1085.9 J}{(55 kg)(9.8 \frac{m}{s^2})}$$
$$h_f = -2.01 m$$

The minus sign means the skater is below the origin of the reference frame.

54) This problem is very similar to the last one. We use the same equations

$$W_{non-c} = \Delta KE + \Delta PE$$
  
$$\Delta KE = \frac{1}{2}m(v_f)^2 - \frac{1}{2}m(v_0)^2$$
  
$$\Delta PE = mgh_f - mgh_0$$

The reference frame can again be chosen so that  $h_0 = 0$  and at the highest point the rocket has  $v_f = 0$ . We are given the numerical data  $h_f = 100 m$ , m = 3 kg and  $W_{non-c} = -800 J$ . So that

$$-800 J = \Delta KE + (3 kg) (9.8 \frac{m}{s^2}) (100 m)$$
  
-800 J = 0 - KE<sub>0</sub> + 2940 J  
$$\rightarrow KE_0 = 3740 J$$

If there were no non-conservative forces, all of this energy would be equal to potential energy at the highest point. This implies

$$(3 kg) (9.8 \frac{m}{s^2}) H_f = 3740 J$$
  

$$\rightarrow H_f = \frac{3740 J}{29.4 \frac{kg m}{s^2}} = 127.2 m$$

56) The work done turning the crank is just

$$W = F(2\pi R) = (22N)(2(3.14)(0.28m)) = 39J$$

and the power is just

$$P = \frac{W}{t} = \frac{39 J}{(1.3 s)} = 30 W$$