Department of Physics
University of Maryland

Physics 121 Fall 2002
Homework Assignment # 5

Problem Solutions

3) This problem can be solved by using the formula that relates linear speed V to angular speed (ω)

$$V = R\omega \quad \rightarrow \omega = \frac{V}{R}$$

where $v = 21\,\frac{m}{s}$ and $R = 0.053\ m$. Here ω is the angular speed measure in radians per second. The time it takes to complete one revolution is the period T and to complete one revolution the total total angle must be 2 π. So

$$2\,\pi = \omega\,T \quad \rightarrow \quad T = \frac{2\,\pi\,R}{V} \quad \rightarrow \quad T = \frac{2\,\pi\,(0.053)\,m}{21\,\frac{m}{s}}$$

$$T = 1.6 \times 10^{-2}\,s$$

6) Two revolutions per second implies an angular speed of $\omega = \frac{4\pi}{s}$ and the problems states the radius $R = 0.12\ m$. We know that

$$a_C = \frac{V^2}{R} \;,\; V = R\,\omega \quad \rightarrow$$

$$a_C = R\,\omega^2 = `(0.12\,m)\,(\tfrac{4\,\pi}{s})^2 = 18.95\,\frac{m}{s^2}$$

14) We recall that the centripetal force is given by

$$F_C = \frac{m\,V^2}{R} \quad \rightarrow \quad R = \frac{m\,V^2}{F_C}$$

Using the numerical values we see

$$R = \frac{(83\,kg)\,(3.2\,\frac{m}{s})^2}{560\,N} = 1.5\,m$$

20) Putting numerical values into equation (5.4) we see

$$tan\theta = \frac{V^2}{R\,g} = \frac{(25\,\frac{m}{s})^2}{(150\,m)\,9.8\,\frac{m}{s^2}} = 0.43 \quad \rightarrow \quad \theta = tan^{-1}(0.43)$$

Or for the angle $\theta = 23^o$.

23) The formula we used in problem (20) can also be written as

$$V \;=\; \sqrt{g\,R\,tan\theta}$$

From the drawing we see that the height in the y-direction is $H = 18\ m$. To figure out the length of the base we have to subtract the length from the center to the bottom of the incline from the length from the center to the of top the incline $L = 165\ m$ - $112\ m = 53\ m$. So the angle of the incline is given by $tan\theta \;=\; \frac{H}{L}$ and thus

$$V \;=\; \sqrt{\frac{g\,R\,H}{L}}$$

The mimimum velocity occurs when the radius is at a minimum (i.e. $R = 112\ m$) at the bottom of the incline. The maximum velocity occurs when the radius is at a maximum (i.e. $R = 165\ m$) at the bottom of the incline.

$$\begin{aligned} V_{min} &= \sqrt{\frac{g\,R_{min}\,H}{L}} \;=\; \sqrt{\frac{(9.8\,\frac{m}{s^2})\,(112\,m)\,(18\,m)}{53\,m}} \\ &= 19.31\,\frac{m}{s} \\ V_{max} &= \sqrt{\frac{g\,R_{max}\,H}{L}} \;=\; \sqrt{\frac{(9.8\,\frac{m}{s^2})\,(165\,m)\,(18\,m)}{53\,m}} \\ &= 23.43\,\frac{m}{s} \end{aligned}$$

29) For an object with speed V in orbit around Jupiter at radius R we must have

$$V \;=\; \sqrt{\frac{G\,M_J}{R}}$$

The radius of the orbit must be equal to the radius of the planet $7.14\,\times\,10^7\,m$ plus the height above the surface of Jupiter $6\,\times\,10^5\,m$ or

$$\begin{aligned} R &= 7.14\,\times\,10^7\,m \;+\; 6\,\times\,10^5\,m \\ &= 714\,\times\,10^5\,m \;+\; 6\,\times\,10^5\,m \;=\; 720\,\times\,10^5\,m \end{aligned}$$

So the speed is given by

$$V \;=\; \sqrt{\frac{(6.67\,\times\,10^{-11}\frac{N\,m^2}{kg^2})\,(1.9\,\times\,10^{27}\,kg)}{720\,\times\,10^5\,m}}$$

$$V \;=\; 4.2 \times 10^4\,\frac{m}{s}$$

33) Many times previously we have seen that the acceleration of gravity at the surface of a planet with mass M_P and radius R_P is given by

$$g_P \;=\; \frac{G\,M_P}{(R_P)^2}$$

On the otherhand, if we look above equation (5.6) in the textbook we see

$$M_P \;=\; \frac{4\,\pi^2\,r^3}{G\,T^2} \;=\; \frac{4\,\pi^2\,(R_P \,+\, h)^3}{G\,T^2}$$

where h is the height of the orbit above the surface of the planet. So combining these two equations yields

$$\begin{aligned} g_P = \;& \frac{4\,\pi^2\,(R_P \,+\, h)^3}{(R_P\,T)^2} \;=\; \frac{4\,\pi^2\,R_P}{T^2}\left[1 \,+\, (\tfrac{h}{R_P})\right]^3 \\ =\;& \frac{4\,\pi^2\,(R_P \,+\, h)^3}{(R_P\,T)^2} \;=\; \frac{4\,\pi^2\,R_P}{T^2}\left[1 \,+\, (\tfrac{h}{R_P})\right]^3 \end{aligned}$$

Using the numerical values gives

$$\begin{aligned} g_P \;=\;& \frac{4\,\pi^2\,(4.15 \times\, 10^6)}{(7.2\;\times\;10^3)^2}\left[1 \,+\, (\tfrac{4.1\times 10^5}{41.5\times 10^5})\right]^3 \tfrac{m}{s^2} \\ =\;& \frac{4\,\pi^2\,(4.15 \times\, 10^6)}{(7.2\;\times\;10^3)^2}\left[1 \,+\, (\tfrac{4.1}{41.5})\right]^3 \tfrac{m}{s^2} \\ =\;& \frac{4\,\pi^2\,(4.15)}{(7.2)^2}\,(\,1.1)^3\,\tfrac{m}{s^2} \\ =\;& \frac{4\,(9.9)\,(4.15)}{(51.84)}\,(\,1.33)\,\tfrac{m}{s^2} \;=\; 4.2\,\tfrac{m}{s^2} \end{aligned}$$

So for the weight we find

$$W \;=\; m\,g_P \;=\; (5850\,kg)\,4.2\,\tfrac{m}{s^2} \;=\; 2.45 \times\, 10^4\,N$$

36) At the bottom of the loop the normal force points upward and using Newton's second law we see

$$N_F \;-\; m\,g \;=\; \frac{m\,V^2}{R}$$

The problem states that the normal force is three times the pilots weight so $N_F = 3\;m\,g$ which can be substituted into the above equation.

$$\begin{aligned} 3\,m\,g \;-\; m\,g \;=\;& \frac{m\,V^2}{R} \;\;\rightarrow\;\; 2\,m\,g \;=\; \frac{m\,V^2}{R} \\ \rightarrow\;\; R \;=\;& \frac{V^2}{2\,g} \;=\; \frac{(230\,\frac{m}{s})^2}{2\,(9.8\frac{m}{s^2})} \;=\; \frac{52,900\,\frac{m^2}{s^2}}{19.6\,\frac{m}{s^2}} \\ \rightarrow\;\; R \;=\;& 2.7\;\times\;10^3\,m \end{aligned}$$