Department of Physics
University of Maryland

Physics 121 Fall 2002
Homework Assignment # 1

Problem Solutions

4) The solution to this can be written as

$$
\begin{aligned}
515.2 \frac{km}{hr} &= 515.2\,(3281)\,\frac{ft}{hr} \quad , \\
515.2 \frac{km}{hr} &= 515.2\,\Big(\frac{3281}{5280}\Big)\,\frac{mi}{hr} \quad , \\
515.2 \frac{km}{hr} &= 320.13\,\frac{mi}{hr} \quad .
\end{aligned}
$$

because 1 $km = 3281\ ft$ and 1 $mi = 5280\ ft$.

5) Here we are told that $m = 5 \times 10^{-6}\,kg$.
(a.) Since 1 $kg = 1000\ g = 10^3\,g$, we have $m = 5 \times 10^{-3}\,g$.
(b.) Since 1 $mg = 10^{-3}\,g$, we have $m = 5\ mg$.
(c.) Since 1 $\mu g = 10^{-6}\,g$, we have $m = 5 \times 10^3\,\mu g$.

11) This problem can be solved by using the Pythagorean Theorem and the definition of the tangent. The shortest distance is the hypotenuse of a right triangle.

$$
\begin{aligned}
d &= \sqrt{a^2 + b^2} \quad , \quad a = -72\,km \quad , \quad b = -35\,km \\
d &= \sqrt{(-72\,km)^2 + (-35\,km)^2} = \sqrt{5184\,km^2 + 1225\,km^2} \\
d &= \sqrt{5184 + 1225}\,km = \sqrt{6409}\,km = 80.06\,km \quad .
\end{aligned}
$$

The angle ϕ is below the east-west axis in the third quadrant and determined by

$$
\begin{aligned}
tan\phi &= \frac{-35}{-72} = \frac{35}{72} = 0.49 \quad . \\
\rightarrow \quad \phi &= tan^{-1}(0.49) = 25.92^o \quad .
\end{aligned}
$$

12) This problem is also about right triangles. We have

$$
\begin{aligned}
a &= 100\,m \quad , \quad b = 12\,m \quad , \\
tan\phi &= \frac{12}{100} = 0.12 \quad . \\
\rightarrow \quad \phi &= tan^{-1}(0.12) = 6.84^o \quad .
\end{aligned}
$$

19) The law of cosines is given by (page A-6 in the textbook)

$$c^2 = a^2 + b^2 - 2\,a\,b\,cos\gamma \quad \rightarrow$$

$$cos\gamma = -\left[\frac{c^2 - a^2 - b^2}{2\,a\,b}\right]$$

and we just have to use this three times but with

(a.) $a = 95$, $b = 150$ and $c = 190$

(b.) $a = 150$, $b = 190$ and $c = 95$

(c.) $a = 190$, $b = 95$ and $c = 150$

to find the three angles. So using these values

$$cos\gamma_1 = -\left[\frac{(190)^2 - (95)^2 - (150)^2}{2\,(95)\,(150)}\right] = -\frac{4575}{28500} = -\,0.16 \quad ,$$

$$cos\gamma_2 = -\left[\frac{(95)^2 - (150)^2 - (190)^2}{2\,(150)\,(190)}\right] = \frac{49575}{57000} = 0.87 \quad ,$$

$$cos\gamma_3 = -\left[\frac{(150)^2 - (95)^2 - (190)^2}{2\,(95)\,(190)}\right] = \frac{22625}{36100} = 0.63 \quad ,$$

or

$$\gamma_1 = 99.21^o \quad , \quad \gamma_2 = 29.54^o \quad , \quad \gamma_3 = 50.95^o \quad .$$

It should be noted that $\gamma_1 + \gamma_2 + \gamma_3 = 179.7^o$ which is due to the round-off error since the sum of angles is suppose to equal 180^o.

24) For each of the vectors, the components can be found

Vector	x-component	y-component
$\vec{A}$	$8\,cm$	$8\,cm$
$\vec{B}$	$8\,cm$	$8\,cm$
$\vec{C}$	$-\,8\,cm$	$8\,cm$
$\vec{A} + \vec{B} + \vec{C}$	$8\,cm$	$24\,cm$

$$\left|\vec{A} + \vec{B} + \vec{C}\right| = \sqrt{(8\,cm)^2 + (24\,cm)^2} = 8\sqrt{10}\,cm = 25.3\,cm \ .$$

24) We can call the vector above the dashed line $\vec{A}$ and the one below $\vec{B}$. The components of these vectors are given by

Vector	x-component	y-component
$\vec{A}$	$2900\,cos(15^o)\,N$	$2900\,sin(15^o)\,N$
$\vec{B}$	$2900\,cos(15^o)\,N$	$-\,2900\,sin(15^o)\,N$
$\vec{A} + \vec{B}$	$5800\,cos(15^o)\,N$	0

(a.) A single rope has a force of $5800\,cos(15^o)\,N$, or $5602.37N$.

(b.) The single rope must be directed along the dashed line.

31) For the vector $\vec{B}$, we use the usual trigonometric relationships we see

$$B_x = 200cos(60^o)units \quad , \quad B_y = 200sin(60^o)units \quad ,$$
$$B_x = 100\,units \quad , \quad B_y = 173.21units \quad .$$

Next we write the components for each vector

Vector	x-component	y-component
$\vec{A}$	0	$100\,units$
$\vec{B}$	$100\,units$	$173.21\,units$
$\vec{C}$	$150\,units$	0

(a.) The vector $\vec{C}$ has the longest x-component.
(b.) The vector $\vec{B}$ has the longest y-component.

43) For the vector $\vec{C}$ in the diagram we have

$$C_x = 18\,cos(35^o)\,m \quad , \quad C_y = -\,18\,sin(35^o)\,m \quad ,$$
$$= 14.74\,m \quad , \quad = -\,10.32\,m \quad .$$

Next we add the components for each vector

Vector	x-component	y-component
$\vec{A}$	0	$5\,m$
$\vec{B}$	$15\,m$	0
$\vec{C}$	$14.74\,m$	$-\,10.32\,m$
$\vec{A} + \vec{B} + \vec{C}$	29.74	$-\,5.32\,m$

and from the Pythagorean Theorem

$$\left|\vec{A} + \vec{B} + \vec{C}\right| = \sqrt{(29.74\,m)^2 + (-\,5.32\,m)^2}$$
$$= \sqrt{(29.74\,)^2\,m^2 + (5.32\,)^2\,m^2}$$
$$= \sqrt{(884.47) + (28.30)}\;m$$
$$= \sqrt{909.77}\,m = 30.16\,m \quad .$$

To find the angle below the x-axis requires use of the tangent.

$$tan\,\theta = \left(\frac{5.32}{29.74}\right) \rightarrow \theta = 10.14^o$$

49) Once more everything begins by find the components of all the vectors.

Vector	x-component	y-component
$\vec{A}$	$3.2\, cos(40^o)\, km$	$3.2\, sin(40^o)\, km$
$\vec{B}$	$-\, 5.1\, cos(35^o)\, km$	$5.1\, sin(35^o)\, km$
$\vec{C}$	$-\, 4.8\, cos(23^o)\, km$	$-\, 4.8\, sin(23^o)\, km$
$\vec{D}$	$D\, cos(\theta^o)\, km$	$-\, D\, sin(\theta^o)\, km$

Since the starting point and ending point are at the same location, when all of these vectors are added together the sum must equal zero. First considering only the x-components leads to

$$\begin{aligned} 0 =&\; 3.2\, cos(40^o) \;-\; 5.1\, cos(35^o) \;-\; 4.8\, cos(23^o) \;+\; D\, cos(\theta^o) \\ =&\; 2.45 \;-\; 4.18 \;-\; 4.42 \;+\; D\, cos(\theta^o) \\ =&\; -\, 6.15 \;+\; D\, cos(\theta^o) \qquad \rightarrow \quad D\, cos(\theta^o) \;=\; 6.15 \quad , \end{aligned}$$

and for the y-components one finds

$$\begin{aligned} 0 =&\; 3.2\, sin(40^o) \;+\; 5.1\, sin(35^o) \;-\; 4.8\, sin(23^o) \;-\; D\, sin(\theta^o) \\ =&\; 2.06 \;+\; 2.93 \;-\; 1.88 \;-\; D\, sin(\theta^o) \\ =&\; 3.11 \;-\; D\, sin(\theta^o) \qquad \rightarrow \quad D\, sin(\theta^o) \;=\; 3.11 \quad . \end{aligned}$$

So it follows that the angle satifies

$$\begin{aligned} &\frac{D\, sin(\theta^o)}{D\, cos(\theta^o)} \;=\; \frac{3.11}{6.15} \quad \rightarrow \; tan(\theta^o) \;=\; \frac{3.11}{6.15} \;=\; 0.51 \\ &\theta^o \;=\; tan^{-1}(0.51) \;=\; 26.83^o \end{aligned}$$

and for the magnitude

$$\begin{aligned} &(D\, cos(\theta^o))^2 \;+\; (D\, sin(\theta^o))^2 \;=\; (3.11)^2 \;+\; (6.15)^2 \\ &D^2\, (\, cos(\theta^o)^2 \;+\; sin(\theta^o))^2 \;=\; (3.11)^2 \;+\; (6.15)^2 \\ &D^2 \;=\; (3.11)^2 \;+\; (6.15)^2 \\ &D \;=\; \sqrt{9.67 \;+\; 37.82} \\ &D \;=\; \sqrt{47.49} \;=\; 6.89 \end{aligned}$$