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PHYSICS 121 Prof. S. J. Gates

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This is a closed book examination. Read the entire examination before you begin to work. Be sure to read each problem carefully. Any questions should be directed to the proctors. *ALL* work should be written within the examination booklet provided. There is a two hour time limit. Show all of your work for analytical questions. Use the backs of pages if necessary or request an extra booklet. Be sure to complete the front page of the examination booklet including your name and recitation section. Show all calculations needed to support your answers, where necessary.

Section I. Useful Constants & Formulae

Constants: $g = 9.8\,\frac{m}{s^2}$, $1\ mi = 1700\ m$, $1\ J\ =\ (kg \cdot m^2)/s^2$, $1\ Pa = 1\ N/m^2$,
$G = 6.673\ \times 10^{-11}\,(N\ \cdot\ m^2)/kg^2$, $P_0 = 1.01 \times 10^5\,Pa$,

Trigonometry and Vector Components

$$sin\theta\ =\ \frac{side\ opposite\ \theta}{hypotenuse}\ =\ \frac{A_y}{A}\quad,\quad cos\theta\ =\ \frac{side\ adjacent\ to\ \theta}{hypotenuse}\ =\ \frac{A_x}{A}\quad,$$

$$tan\theta\ =\ \frac{side\ opposite\ \theta}{side\ adjacent\ to\ \theta}\ =\ \frac{A_y}{A_x}\quad,$$

$$A_x\ =\ A\,cos\,\theta\ \ ,\ \ A_y\ =\ A\,sin\,\theta\ \ ,\ \ A^2\ =\ (A_x)^2\ +\ (A_y)^2\ \ ,\ \ tan\,\theta\ =\ \frac{A_y}{A_x}\ \ ,$$

Adding Vector Components

$$C_x\ =\ A_x\ +\ B_x\quad,\quad C_y\ =\ A_y\ +\ B_y\quad,$$

Displacement and Average Velocity & Average Acceleration

$$\Delta r\ =\ r_f\ -\ r_0\quad,\quad \Delta t\ =\ t_f\ -\ t_0\quad,\quad \Delta v\ =\ v_f\ -\ v_0\quad,$$

$$\bar{v}\ =\ \frac{\Delta r}{\Delta t}\quad,\quad \bar{a}\ =\ \frac{\Delta v}{\Delta t}\quad,$$

Kinematic Equations

$$x\ =\ x_0\ +\ v_{0,x}\,t\ +\ \tfrac{1}{2}\,a_{0,x}\,t^2\quad,\quad y\ =\ y_0\ +\ v_{0,y}\,t\ +\ \tfrac{1}{2}\,a_{0,y}\,t^2\quad,$$

$$x\ =\ x_0\ +\ \tfrac{1}{2}[\,v_{0,x}\ +\ v_x\,]\,t\quad,\quad y\ =\ y_0\ +\ \tfrac{1}{2}[\,v_{0,y}\ +\ v_y\,]\,t\quad,$$

$$v_x\ =\ v_{0,x}\ +\ a_{0,x}\,t\quad,\quad v_y\ =\ v_{0,y}\ +\ a_{0,y}\,t\quad,$$

$$v^2 = (v_0)^2 + 2\,a\,[\,r - r_0\,] \;\; ,$$

Projectile Motion Equations

$$V_{x,0} = V_0\, cos\,\theta \;\; , \;\; V_{y,0} = V_0\, sin\,\theta \;\; ,$$

$$x = x_0 + V_0\, cos\,\theta\, t \;\; , \;\; y = y_0 + V_0\, sin\,\theta\, t - \tfrac{1}{2} g\, t^2 \;\; ,$$

Quadratic Formula

$$a\,x^2 + b\,x + c = 0 \;\; \rightarrow \;\; x = \frac{-\,b \pm \sqrt{b^2 - 4\,a\,c}}{2a}$$

Newton's Law's

(I.) An object at rest remains at rest, and an object in motion continues in motion with constant velocity, unless it experiences a net external force.

(II.) The acceleration of an object is directly proportional to the net force on it and inversely proportional to its mass, $m\,\mathbf{a} = \sum \mathbf{F}$.

(III.) If two object interact, the force exerted on object 1 by object 2 is equal in magnitude but opposite in direction to the force exerted on object 2 by object 1.

Force of gravity $\mathbf{F}_g = m\mathbf{g}$ = weight, (near the earth's surface)

Work, Kinetic Energy, Work-Energy Theorem

$$W = (\,F\, cos\theta\,)\, s \;\; , \;\; KE = \tfrac{1}{2}\, m\, v^2 \;\; , \;\; KE_f - KE_i = W_{net} \;\; ,$$

Power, Friction, Frictional Work

$$\bar{P} = \frac{W}{\Delta t} = \frac{F \Delta x}{\Delta t} = F\,\bar{v} \;\; , \;\; f_s \leq \mu_s\, n \;\; , \;\; f = \mu_k\, n \;\; , \;\; W_f = -\,\mu\, n\, s \;\; ,$$

Force and Potential Energy (gravity)

$$W_g = -\,[\,mgy_f - mgy_i\,] \;\; , \;\; PE_g = mgy \;\; , \;\; F_g = m\,g \;\; ,$$

Force and Potential Energy (spring)

$$W_s = -\,\tfrac{1}{2}\,[\,k(x_f)^2 - k(x_i)^2\,] \;\; , \;\; PE_s = \tfrac{1}{2} k\, x^2 \;\; , \;\; F_s = -k\, x \;\; ,$$

Conservation of Energy

$$KE_f + PE_f = KE_i + PE_i$$

Conservation of Energy with Friction

$$KE_f + PE_f = KE_i + PE_i + W_{Friction}$$

Impulse, Momentum, Impulse-Momentum Theorem

$$Impulse = \mathbf{F}\,\Delta t \;\; , \;\; \mathbf{p} = m\,\mathbf{v} \;\; , \;\; \mathbf{F}\,\Delta t = m\,\mathbf{v}_f - m\,\mathbf{v}_i \;\; ,$$

Conservation of Momentum

$$\mathbf{P}_{Tot} \;=\; \sum m\,\mathbf{v} \quad , \quad \mathbf{P}_{Tot,\,f} \;=\; \mathbf{P}_{Tot,\,0} \quad ,$$

Circular Motion

$$\Delta\theta \;=\; \theta_f \;-\; \theta_0 \quad , \quad \Delta t \;=\; t_f \;-\; t_0 \quad , \quad \Delta\omega \;=\; \omega_f \;-\; \omega_0 \quad ,$$
$$\bar{\omega} \;=\; \frac{\Delta\theta}{\Delta t} \quad , \quad \bar{\alpha} \;=\; \frac{\Delta\omega}{\Delta t} \quad ,$$

Rotational Kinematic Equations and Centripetal Force

$$\theta \;=\; \theta_0 \;+\; \omega\,t \;+\; \tfrac{1}{2}\,\alpha\,t^2 \quad , \quad v_t \;=\; r\,\omega \quad , \quad s \;=\; R\,\theta \quad ,$$
$$\theta \;=\; \theta_0 \;+\; \tfrac{1}{2}[\,\omega_0 \;+\; \omega_f\,]\,t \quad , \quad a_t \;=\; r\,\alpha \quad ,$$
$$\omega \;=\; \omega_0 \;+\; \alpha\,t \quad , \quad a_C \;=\; r\,\omega^2 \;=\; \tfrac{(v_t)^2}{r} \quad ,$$
$$\omega^2 \;=\; (\omega_0)^2 \;+\; 2\,\alpha\,[\,\theta \;-\; \theta_0\,] \quad , \quad F_C \;=\; ma_C \;=\; m\,\tfrac{(v_t)^2}{r} \quad ,$$

Newton's Gravitational Law and Kepler's Third Law

$$|\,\mathbf{F}_g| \;=\; (\frac{G\,M_1\,M_2}{r^2}) \quad , \quad T^2 \;=\; (\frac{4\pi^2}{GM})\,r^3 \quad ,$$

Rotational Dynamics

$$\tau \;=\; F\,d\,sin\theta \quad , \quad \mathbf{L} \;=\; I\,\omega \quad , \quad KE_r \;=\; \tfrac{1}{2}\,I\,\omega^2 \quad ,$$

$$I \;=\; \sum m\,r^2 \quad , \quad I\,\alpha \;=\; \sum \tau \quad ,$$

Conservation of Angular Momentum

$$\mathbf{L}_{Tot} \;=\; \sum I\,\omega \quad , \quad \mathbf{L}_{Tot,\,f} \;=\; \mathbf{L}_{Tot,\,0} \quad ,$$

Properties of Matter

Moduli:

$(F/A) \;=\; Y\,(\Delta L/L_0)$, $(\Delta F/A) \;=\; -\,B\,(\Delta V/V_0)$,

Density and Pressure: $\rho \;\equiv\; (M/V)$, $P \;\equiv\; (F/A)$

Pressure in Fluid: $P \;=\; P_0 \;+\; \rho\,g\,h$,

Equation of Continuity: $A_1\,v_1 \;=\; A_2\,v_2$,

Bernoulli's Equation: $\frac{1}{2}\rho\,v^2 \;+\; P \;+\; \rho\,g\,h \;=\; \text{Constant}$,

Archimedes' Principle: When an object is partially or fully submerged, the fluid exerts a bouyant force upward. The magnitude of this force is equal to the weight of the fluid displaced by the object.

Thermodynamics

The zeroth law of thermodynamics: If two objects A and B are separately in in thermal equilibrium with a third object, then A and B are in thermal equili- with each other.

Temperature Scales:

$$T_C = T - 273.15 \quad , \quad T_F = \frac{9}{5} T_C + 32$$

T: Absolute or Kelvin temp., T_C: Celsius temp. and T_F: Fahrenheit temp.

Thermal Expansion

Coefficients:

$$\Delta L = \alpha L_0 \Delta T \quad , \quad \Delta V = \beta V_0 \Delta T \quad ,$$

Heat

The mechanical equivalent of heat: 1 $cal = 4.186\ J$

Specific heat: $Q = c\, m \Delta T$

Latent heat: $Q = m\, L$

Waves & Sound

Wavelength: $\lambda =$ crest-to-crest distance

Frequency: $f = \frac{1}{T}$

Speed of travelling wave $v = \lambda \frac{1}{T} = \lambda f$

Speed of wave on string $v = \sqrt{\frac{F}{(m/l)}}$

Speed of Sound: (a.) In gas $v = \sqrt{\frac{\gamma k_B T}{m}}$ (b.) In liquid $v = \sqrt{\frac{B_{ad}}{\rho}}$

(c.) In solid $v = \sqrt{\frac{B}{\rho}}$

Sound Intensity: $I = \frac{\mathcal{P}}{Area}$ where $\mathcal{P}$ is the sound power.

Decibels $\beta = (10dB)\, log(\frac{I}{I_0})$

Mathematical Formula for Waves

Right-moving Wave: $y = A\, sin(2\pi\, f\, t - \frac{2\pi\, x}{\lambda})$

Left-moving Wave: $y = A\, sin(2\pi\, f\, t + \frac{2\pi\, x}{\lambda})$

Standing Wave: $y = A\, cos(2\pi\, f\, t)\, sin(\frac{2\pi\, x}{\lambda})$

Doppler Effect

$$f_o = f_s \left(\frac{1 \pm \frac{v_o}{v}}{1 \mp \frac{v_s}{v}} \right)$$

Circumference and Area of a Circle: $C = 2\pi\, r \quad , \quad A = \pi\, r^2$

Area and Volume of a Sphere: $A = 4\pi\, r^2 \quad , \quad V = \frac{4}{3} \pi\, r^3$

Area and Volume of a Cylinder: $A = 2\pi\, r^2 + 2\pi r\, h \quad , \quad V = h\, \pi\, r^2$

Section II. Multiple Choice Questions

Each question in this section is worth eight (8) points. You should **NOT** take more than two minutes per question. If you do, it is advisable to continue on to the next question!

(1.) The speed of sound in air at 20 ^{o}C is 343 $\frac{m}{s}$. At 80 ^{o}C it must be; (a.) 686 $\frac{m}{s}$. (b.) 1372 $\frac{m}{s}$. (c.) 137 $\frac{m}{s}$ (d.) none of the above.

(2.) A ball accelerates uniformly from rest and covers a distance of 90 m in 3 s. Its velocity at 1 s must be; (a.) 30 $\frac{m}{s}$. (b.) 20 $\frac{m}{s}$. 10 $\frac{m}{s}$. (d.) none of the above.

(3.) A substance with a specific heat of 3600 $J/(kg \times C^o)$ changes its temperature by 2 ^{o}C when 14,400 J of heat is added to it. The mass of the substance must be: (a.) 8 kg. (b.) 4 kg. (c.) 2 kg. (d.) none of the above.

(4.) The displacement of an object is: (a.) dependent of direction. (b.) independent of the starting point (c.) independent of the ending point (d.) none of the above.

(5.) One wave has $\lambda = 4$ cm and a second wave has $\lambda = 8$ cm. If the frequency of the first wave is twice that of the second, then the first wave moves; (a.) faster than the second wave. (b.) at the same speed as the second wave. (c.) slower than the second wave. (d.) none of the above.

(6.) If one vector points above the x-axis and another points below, then the x-component of the first can: (a.) never equal the x-component of the second. (b.) must be the negative of x-component of the second. (c.) must be equal to x-component of the second. (d.) none of the above.

(7.) For an object tossed into the air in any manner and at the instant that it is at the top of its arc, it possesses; (a.) zero kinetic energy. (b.) maximum kinetic energy. (c.) the same kinetic energy as when it was first tossed. (d.) none of the above.

(8.) A 20 kg mass moving at 16 $\frac{cm}{s}$ collides head-on with an 80 kg mass moving at 4 $\frac{cm}{s}$. The collision last 0.2 s. The magnitude of the force exerted on the first mass is: (a.) greater than the magnitude of the force exerted on the second mass. (b.) equal to the magnitude of the force exerted on the second mass. (c.) less than the magnitude of the force exerted on the second mass. (d.) none of the above.

Section III. Analytical Questions

Problem (1.)

Two masses are moving in circular paths so that the centripetal acceleration of the first is 24 times that of the second. The following data is measured for each one

$$m_1 = 3\,kg \quad , \quad R_1 = 20\,cm \quad , \quad m_2 = 6\,kg \quad , \quad R_2 = 240\,cm$$

(a.) If the first mass has 65 J of kinetic energy, what is the kinetic energy of the second?

Problem (2.)

Before a collision, one ball of mass 10 kg is rolling in the positive x-direction with a speed of 7 $\frac{m}{s}$. A second ball with a mass of 5 kg is rolling in the positive y-direction with a speed of 20 $\frac{m}{s}$. After the collision, the two stick together.

(a.) What is the magnitude and direction of motion for the two joined balls after the collision?

Problem (3.)

Two friends decide to go to the beach and get into the surf. The first stands 12 m from the shoreline and the other 16 m from the shoreline. They notice that it takes 3 seconds a wave to travel from the location of the second to the first. They also note that when a crest reaches the first, the next crest reaches the second. Finally the distance from the crest of a wave to its trough is 0.3 m.

(a.) Write a mathematical expression to describe the travelling wave of water at the beach.

Problem (4.)

A perfect cube of length 0.6 m undergoes thermal expansion. Its coefficient of linear expansion is three times its coefficient of volume expansion.

(a.) Find the ratio of its change in volume to its change in length.

Problem (5.) ______________________

A ball starts at the top of an inclined plane that makes an angle of 33^o with regard to the horizontal. It is 2.3 meters above a level plane. There is a coefficient of friction between the $3\ kg$ ball and the incline of 0.3.

(a.) How fast is it travelling when it reaches the level plane? (5 points)

(b.) After rolling along the plane it bumps into a spring with constant given by $2\ \frac{N}{m}$, what is the maximum compression of the spring? (5 points)

Problem (6.) ______________________

A $5\ kg$ solid metal mass has a specific heat of $387\ J/(kg \cdot {}^oC)$, a melting point of 1083^oC, a Latent of Fusion of 207,000 (J/kg) and a temperature of 1000^oC.

(a.) How much heat must be added to the mass to completely melt it and then raise the temperature of the molten metal to 2000^oC.

Problem (7.) ______________________

A star that rotates at a rate of one revolution every 12 days suddenly explodes. This process reduces its radius by a factor of ten and its mass by a factor of two.

(a.) If the moment of inertia of a sphere is given by $I = \frac{2}{5} M\, R^2$, what is the angular velocity of the star after the explosion?

Problem (8.) ______________________

A diver on an alien planet steps off a cliff and accelerates at half the normal rate on Earth into a strange liquid. She has on her wrist a device for measuring pressure and notices at a depth of $5.5\ m$ that the device measures a pressure of $1.55 \times 10^5\ Pa$. At a depth of $10\ m$ the device measures a pressure of $3 \times 10^6\ Pa$.

(a.) What is the density of the fluid into which she dives.

Problem (9.)

At what temperature does the numerical value on the Fahrenheit scale equal to four times that on the centigrade scale?