Chapter 2

Greek cosmology

The first "cosmologies" were based on creation myths in which one or more deities made the universe out of sheer will, or out of their bodily fluids, or of the carcass of some god they defeated, etc. A few examples of such "theories" of the universe are provided in this chapter. These are hardly scientific theories in the sense that they have almost no support form observation and in that they predict very few things outside of the fact that there *is* a world (if everything is due to the whims of the Gods then there is very little one can predict). It is an interesting comment on the workings of the human mind that quite different cultures produced similar creation myths.

The first scientific cosmology was created by the Greeks more than 2000 years ago, and this chapter also describes these ideas and their origin. The Greeks used some of the knowledge accumulated by earlier civilizations, thus this chapter begins with a brief description of the achievements of the Egyptians and Babylonians. We then consider the highlights of Greek cosmology culminating with Ptolemy's system of the world.

2.1 Egypt and Babylon

2.1.1 Babylon

The Babylonians lived in Mesopotamia, a fertile plain between the Tigris and Euphrates rivers (see Fig. 2.1). They developed an abstract form of writing based on cuneiform (wedge-shaped) symbols. Their symbols were written on wet clay tablets which were baked in the sun; many thousands of these tablets have survived to this day; an example is shown in Fig. 2.1.

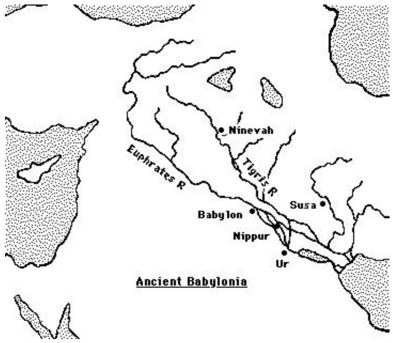


Figure 2.1: Left: Region dominated by the Babylonian civilization. Right: example of a cuneiform tablet containing Pythagorean triples.

The Babylonian apparently believed the Earth to be a big circular plane surrounded by a river beyond which lies an impassable mountain barrier, with the whole thing resting on a cosmic sea. No human may cross the river surrounding the Earth. The mountains support the vault of heaven, which is made of a very strong metal. There is a tunnel in the northern mountains that opens to the outer space and which also connects two doors, one in the East and one in the West. The sun comes out through the eastern door, travels below the metallic heavens and then exits through the western door; he spends the nights in the tunnel.

The creation myth is more lively than the Egyptian version. It imagines that the cosmic ocean *Apsu* mixed with chaos *Tiamat* and eventually generated life. For a while life was good for the gods but there came a time when Tiamat felt her domain was too small and made war against the other gods. All but *Marduk* were afraid of her, so Marduk, after getting all the powers from the frightened gods, fought Tiamat. When Tiamat opened her mouth to swallow him he thrust a bag full with hurricane winds into her so that she swelled and, taking advantage of her indisposition, Marduk pierced her with his lance and killed her. Then he split Tiamat's carcass making the lower half the earth and the upper the heavens. Finally Marduk mixed his bloodown blood with the earth to make men for the service of the gods.

Babylonians and Chaldeans observed the motion of the stars and planets from the earliest antiquity (since the middle of the 23rd century B.C.). They cataloged the motion of the stars and planets as well as the occurrence of eclipses and attempted to fit their behavior to some numerical theories. Many of these observations were used for astrological prophesying and, in fact, they were the originators of astrology. They believed that the motions and changes in the stars and planets determine (or so they believed) what occurs on this planet.

The Babylonians excelled in computational mathematics, they were able to solve algebraic equations of the first degree, understood the concept of function and realized the truth of Pythagoras' theorem (without furnishing an abstract proof). One of the clay tablets dated from between 1900 and 1600 B.C. contains answers to a problem containing Pythagorean triples, i.e. numbers a, b, c with $a^2 + b^2 = c^2$. It is said to be the oldest number theory document in existence. The Babylonians had an advanced number system with base 60 rather than the base 10 of common today. The Babylonians divided the day into 24 hours, each hour into 60 minutes, each minute into 60 seconds. This form of counting has survived for 40 centuries.

2.1.2 Egypt

The anciebloodnt Egyptians conceived the sky as a roof placed over the world supported by columns placed at the four cardinal points. The Earth was a flat rectangle, longer from north to south, whose surface bulges slightly and having (of course) the Nile as its center. On the south there was a river in the sky supported by mountains and on this river the sun god made his daily trip (this river was wide enough to allow the sun to vary its path as it is seen to do). The stars were suspended from the heavens by strong cables, but no apparent explanation was given for their movements.

There is no unique Egyptian creation myth, yet one of the most colorful versions states that at the beginning of the world,*Nuit*, the goddess of the night, was in a tight embrace with her husband *Sibû*, the earth god. Then one day, without an obvious reason, the god *Shû* grabed her and elevated her to the sky (to *become* the sky) despite the protests and painful squirmings of Sibû. But Shû has no sympathy for him and freezes Sibû even as he is thrashing about. And so he remains to this day, his twisted pose generating the irregularities we see on the Earth's surface (see Fig. 2.2). Nuit is supported by her arms and legs which become the columns holding the sky. The newly created world was divided into four regions or houses, each dominated by a god. Since the day of creation Sibû has been clothed in verdure and generations of animals prospered on his back, but his pain persists.

An extended version of this myth imagines that in the beginning the god *Tumu* suddenly cried "Come to me!" across the cosmic ocean, whence a giant lotus flower appeared which had the god *Ra* inside, then Ra separates Nuit and Sibû, and the story proceeds as above. It is noteworthy that creation did not come through muscular effort, but through Tumu's voiced

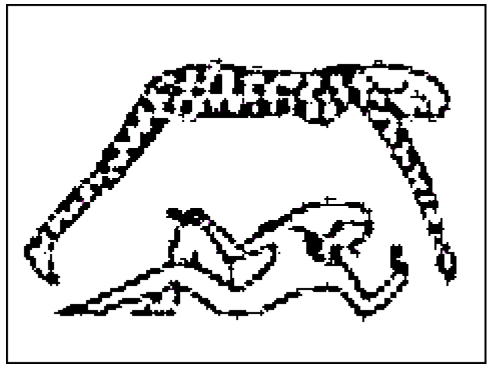


Figure 2.2: Nuit the sky above Sibû the Earth after being separated by Shû in a version of the Egyptian creation myth.

command. This later evolved into the belief that the creator made the world with a single word, then with a single sound (yet the creation through pure thought was not considered).

After creation the gods, especially *Thot* (Fig. 2.3), teach the arts and sciences to the Egyptians. In particular Thot taught the Egyptians how to observe the heavens and the manner in which the planets and the sun move, as well as the names of the (36) constellations (though he apparently neglected to tell them about eclipses which are never referred to).

Figure 2.3: The Egyptian god Thot.

The study of the heavens was not made for altruistic purposes but with very practical aims: a good calendar was necessary in order to prepare for the regular flooding of the Nile as well as for religious purposes. The Egyptian calendar had a year of precisely 365 days and was used for many centuries; curiously they never corrected for the fact that the year is 365

1/4 days in length (this is why every four years we have a leap year and add a day to February) and so their time reckoning was off one day every four years. After 730 years this deficiency adds up to 6 months so that the calendar announced the arrival of summer at the beginning of winter. After 1460 years the Egyptian calendar came back on track and big celebrations ensued.

Egyptians knew and used the water clock whose origin is lost in the mists of time. the oldest clock in existence dates from the reign of the pharo Thutmose III (about 1450 B.C.) and is now in th Berlin Museum.

Most of Egyptian mathematics was aimed at practical calculations such as measuring the Earth (important as the periodic Nile floods erased property boundary marks) and business mathematics. Their number system was clumsy (addition was not too bad but multiplication is very cumbersome). To overcome this deficiency the Egyptians devised cunning ways to multiply numbers, the method, however, was very tedious: to obtain $41 \times 59 = 2419$, nine operations had to be performed (all additions and subtractions); yet they were able to calculate areas and estimate the number π. Examples of calculations have survived in several papyri (Fig. 2.4).

Unlike the Greeks who thought abstractly about mathematical ideas, the Egyptians were only concerned with practical arithmetic. In fact the Egyptians probably did not think of numbers as abstract quantities but always thought of a specific collection of objects when a number was mentioned.

2.2 Other nations

None of the early civilizations lacked a cosmology or creation myths. In this section a brief summary of some of these myths is presented.

2.2.1 India

The traditional Indian cosmology states that the universe undergoes cyclic periods of birth, development and decay, lasting 4.32×10^9 years, each of these periods is called a *Kalpa* or "day of Brahma". During each Kalpa the universe develops by natural means and processes, and by natural means and processes it decays; the destruction of the universe is as certain as the death of a mouse (and equally important). Each Kalpa is divided into 1000 "great ages", and each great age into 4 ages; during each age humankind deteriorates gradually (the present age will terminate in 426,902 years). These is no final purpose towards which the universe moves, there is no progress, only endless repetition. We do not know how the universe began, perhaps

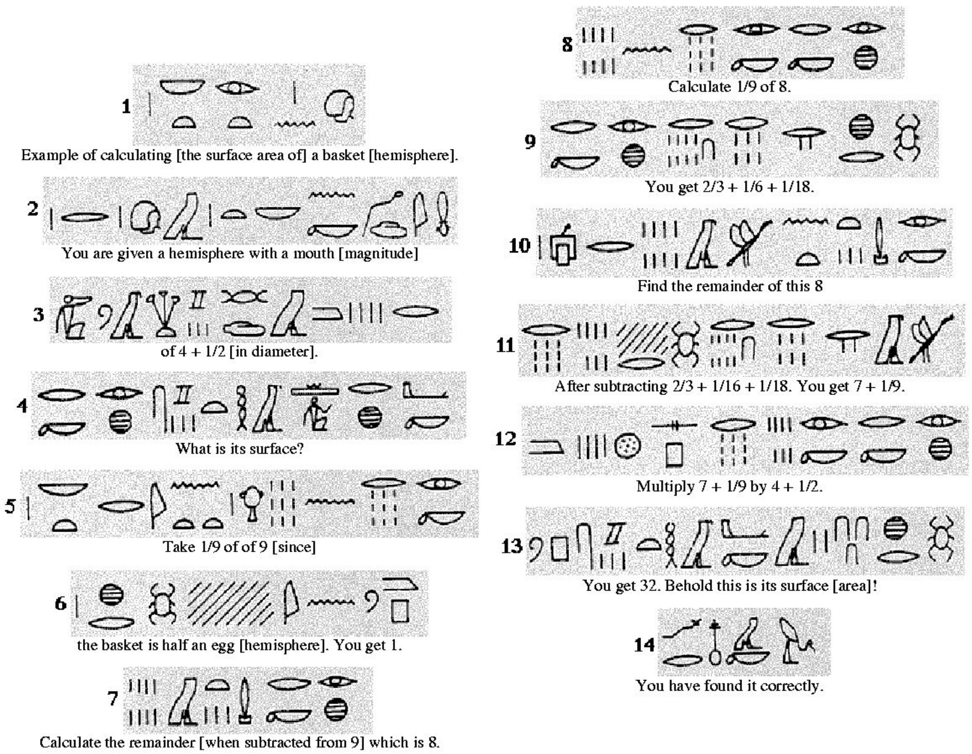


Figure 2.4: An example of Egyptian papyri, the Moscow papyrus and its translation; the text contains the estimate $\pi \simeq 256/81 = 3.1605$.

Brahma laid it as an egg and hatched it; perhaps it is but an error or a joke of the Maker.

This description of the universe is remarkable for the enormous numbers it uses. The currently accepted age of the universe is about 10^{18} seconds and this corresponds to about 7 Kalpas+335 great ages. A unique feature of Indian cosmology is that no other ancient cosmology manipulates such time periods.

In the Surya Siddanta it is stated that the stars revolved around the cosmic mountain Meru at whose summit dwell the gods. The Earth is a *sphere* divided into four continents. the planets move by the action of a cosmic wind and, in fact, the Vedic conception of nature attributes *all* motion to such a wind. It was noted that the planets do not move in perfect circles and this was attributed to "weather forms" whose hands were tied to the planets by "cords of wind"

2.2.2 China

The Chinese have a very long history of astronomical observations reaching back to the 13th century B.C. They noted solar eclipses as well as supernova events (exploding stars). The most impressive of these events was the observation on 1054 A.D. of such a supernova event which lasted for 2 years, after that the star dimmed and disappeared from view. The astronomical observations were sufficiently precise for later astronomers to determine that the location of that exploding star is now occupied by the crab nebula (Fig. 2.5); it was then shown that this nebula is expanding and, extrapolating backwards, that this expansion started in 1054 A.D.

Figure 2.5: The Crab nebula, the remnant of a supernova.

The first Chinese cosmography imagines a round sky over a square Earth with the sun and heavens revolving around the Earth. Later this was replaced by a round Earth around which all heavenly bodies rotate. These theories propagated throughout Eastern Asia.

2.3 Early Greeks

The Greeks were apparently the first people to look upon the heavens as a set of phenomena amenable to human comprehension and separated from the sometimes fickle whims of the gods. They were able to extract an great amount of information using nothing but basic reasoning and very elementary observations. This makes their results all the more amazing.

In the earliest times their view of the world and its origin was firmly based on creation myths consolidated by Homer in the Iliad and Odyssey, as the culture evolved this view of the universe evolved and distanced itself from the purely religious outlook.

2.3.1 Mythology

A simplified version of the Greek creation myth follows.

In the beginning there was only chaos. Then out of the void appeared Night and Erebus, the unknowable place where death dwells. All else was empty, silent, endless, darkness. Then somehow Love (Eros) was born bringing a start of order. From Love came Light and Day. Once there was Light and Day, Gaea, the earth appeared.

Then Erebus slept with Night, who gave birth to Aether, the heavenly light, and to Day, the earthly light. Then Night alone produced Doom, Fate, Death, Sleep, Dreams, Nemesis, and others that come to man out of darkness.

Meanwhile Gaea alone gave birth to Uranus, the heavens. Uranus became Gaea's mate covering her on all sides. Together they produced the three Cyclops, the three Hecatoncheires, and twelve Titans.

However, Uranus was a bad father and husband. He hated the Hecatoncheires and imprisoned them by pushing them into the hidden places of the earth, Gaea's womb. This angered Gaea and she plotted against Uranus. She made a flint sickle and tried to get her children to attack Uranus. All were too afraid except, the youngest Titan, Cronus.

Gaea and Cronus set up an ambush of Uranus as he lay with Gaea at night. Cronus grabbed his father and castrated him, with the stone sickle, throwing the severed genitals into the ocean. The fate of Uranus is not clear. He either died, withdrew from the earth, or exiled himself to Italy. As he departed he promised that Cronus and the Titans would be punished. From his spilt blood came the Giants, the Ash Tree Nymphs, and the Erinyes. From the sea foam where his genitals fell came Aphrodite.

Cronus became the next ruler. He imprisoned the Cyclops and the Hecatoncheires in Tartarus. He married his sister Rhea, under his rule he and the other Titans had many offspring. He ruled for many ages. However, Gaea and Uranus both had prophesied that he would be overthrown by a son. To avoid this Cronus swallowed each of his children as they were born. Rhea was angry at the treatment of the children and plotted against Cronus. When it came time to give birth to her sixth child, Rhea hid herself, and after the birth she secretly left the child to be raised by nymphs. To conceal her act she wrapped a stone in swaddling clothes and passed it off as the baby to Cronus, who swallowed it.

This child was Zeus. He grew into a handsome youth on Crete. He consulted Metis on how to defeat Cronus. She prepared a drink for Cronus designed to make him vomit up the other children. Rhea convinced Cronus to accept his son and Zeus was allowed to return to Mount Olympus as Cronus's cup-bearer. This gave Zeus the opportunity to slip Cronus the specially prepared drink. This worked as planned and the other five children were vomited up. Being gods they were unharmed. They were thankful to Zeus and made him their leader.

Cronus was yet to be defeated. He and the Titans, except Prometheus, Epimetheus, and Oceanus, fought to retain their power. Atlas became their leader in battle and it looked for some time as though they would win and put the young gods down. However, Zeus was cunning. He went down to Tartarus and freed the Cyclops and the Hecatoncheires. Prometheus joined Zeus as well who returned to battle with his new allies. The Cyclops provided Zeus with lighting bolts for weapons. The Hecatoncheires he set in ambush armed with boulders. With the time right, Zeus retreated drawing the Titans into the Hecatoncheires's ambush. The Hecatoncheires rained down hundreds of boulders with such a fury the Titans thought the mountains were falling on them. They broke and ran giving Zeus victory.

Zeus exiled the Titans who had fought against him into Tartarus. Except for Atlas, who was singled out for the special punishment of holding the world on his shoulders.

However, even after this victory Zeus was not safe. Gaea angry that her children had been imprisoned gave birth to a last offspring, Typhoeus. Typhoeus was so fearsome that most of the gods fled. However, Zeus faced the monster and flinging his lighting bolts was able to kill it. Typhoeus was buried under Mount Etna in Sicily.

Much later a final challenge to Zeus rule was made by the Giants. They went so far as to attempt to invade Mount Olympus, piling mountain upon mountain in an effort to reach the top. But, the gods had grown strong and with the help of Heracles the Giants were subdued or killed.

One of the most significant features of the Greek mythology is the presence of the Fates: these were three goddesses who spend the time weaving a rug where all the affairs of men and gods appear. There is nothing that can be done to alter this rug, even the gods are powerless to do so, and it is this that is interesting. For the first time the idea appears of a force which rules *everything*, even the gods.

2.3.2 Early cosmology

In their many travels the early Greeks came into contact with older civilizations and learned their mathematics and cosmologies. Early sailors relied heavily on the celestial bodies for guidance and the observation that the heavens presented very clear regularities gave birth to the concept that these regularities resulted, not from the whims of the gods, but from physical laws. Similar conclusions must have been drawn from the regular change of the seasons. This realization was not sudden, but required a lapse of many centuries, yet its importance cannot be underestimated for it is the birth of modern science.

The earliest of the Greek cosmologies were intimately related to mythology: earth was surrounded by air above, water around and Hades below; ether surrounded the earth-water-Hades set (Fig. 2.6),

Figure 2.6: The universe according to Greek mythology.

This system was soon replaced by more sophisticated views on the nature of the cosmos. Two interesting examples were first the claim of Anaxagoras of Clazomenae that the Moon shines only through the light it reflects from the sun, and that that lunar eclipses are a result of the earth blocking the sunlight in its path to the moon; he also believed the Sun to be a ball of molten iron larger than the Peloponesus.

Another remarkable feat was the prediction of a solar eclipse by Thales in 585 B.C. (for which he used the data obtained by Babylonian astronomers). During this period other ideas were suggested, such as the possibility of an infinite, eternal universe (Democritus) and a spherical immovable Earth (Parmenides).

Thales of Miletus (624 B.C. - 546 B.C.). Born and died in Miletus, Turkey. Thales of Miletus was the first known Greek philosopher, scientist and mathematician. None of his writing survives so it is difficult to determine his views and to be certain about his mathematical discoveries. He is credited with five theorems of elementary geometry: *(i)* A circle is bisected by any diameter. *(ii)* The base angles of an isosceles triangle are equal. *(iii)* The angles between two intersecting straight lines are equal. *(iv)* Two triangles are congruent if they have two angles and one side equal. *(v)* An angle in a semi-circle is a right angle. Thales is believed to have been the teacher of Anaximander and he is the first natural philosopher in the Milesian School. [1].

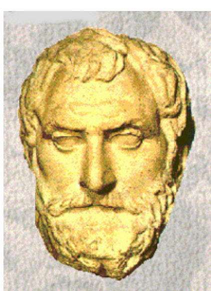
Despite these strikingly "modern" views about the sun and moon, the accepted cosmologies of the time were not so advanced. For example, Thales believed that the Earth floats on water (and earthquakes were the result of waves in this cosmic ocean), and all things come to be from this cosmic ocean. In particular the stars float in the upper waters which feed these celestial fires with their "exhalations". The motion of the stars were assumed to be governed by (then unknown) laws which are responsible for the observed regularities.

A good example of the manner in which the Greeks drew logical conclusions from existing data is provided by the argument of Anaxagoras who pointed out that meteors, which are seen to fall from the heavens, are made of the same materials as found on Earth, and then hypothesized that the heavenly bodies were originally part of the Earth and were thrown out by the rapid rotation of the Earth; as the rapid rotation of these bodies decreases they are pulled back and fall as meteors. This conclusion is, of course, wrong, but the hypothesis proposed does demonstrate imagination as well as close adherence to the observed facts.

The early Greek cosmological theories *did* explain all the data available at the time (though they made no predictions). And, even with these deficiencies, this period is notable for the efforts made to understand the workings of Nature using a rational basis. This idea was later adopted by Plato and is the basis of all modern science.

There are many other early cosmologies, for example, Anaximander believed the Earth to be surrounded by a series of spheres made of mist and surrounded by a big fire; the Sun, Moon and stars are glimpses of this fire through the mist. In a different version of his cosmology he imagined the Earth to be a cylinder floating in space. In a more poetical vein, Empedocles believed the cosmos to be egg-shaped and governed by alternating reigns of love and hate.

Parmenides of Elea (515 B.C. - 445 B.C.). Born in Elea, a Greek city in southern Italy (today called Velia); almost certainly studied in Athens and there is ample evidence that he was a student of Anaximander and deeply influenced by the teachings of the Pythagoreans, whose religious and philosophical brotherhood he joined at their school in Crotona. All we have left of his writings are about 160 lines of a poem called Nature, written for his illustrious student Zeno and preserved in the writings of later philosophers such as Sextus Empiricus. His style influenced by Pythagorean mysticism.



Anaxagoras of Clazomenae (499 B.C. - 428 B.C.). Greek, born in Ionia, lived in Athens He was imprisoned for claiming that the sun was not a god and that the moon reflected the sun's light. While in prison he tried to solve the problem of squaring the circle, that is constructing with ruler and compasses a square with area equal to that of a given circle (this is the first record of this problem being studied). He was saved from prison by Pericles but had to leave Athens.

The early Greeks also considered the composition of things. It was during these times that it was first proposed (by Anaximines of Miletus, c. 525 B.C.) that everything was supposed to be made out of four "elements":

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earth, water, air and fire. This idea prevailed for many centuries. It was believed that earth was some sort of condensation of air, while fire was some sort of emission form air. When earth condenses out of air, fire is created in the process. Thus we have the first table of the elements (see Fig. 2.7)

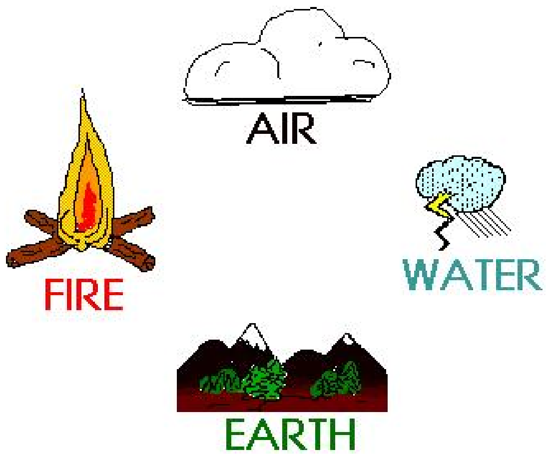
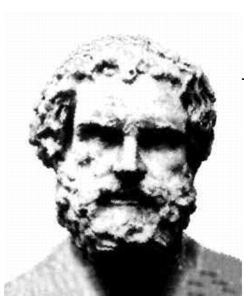


Figure 2.7: The earliest table of the elements.

This, however was not universally accepted. The most notable detractor was Democritus who postulated the existence of indestructible atoms (from the Greek *a-tome:* that which cannot be cut) of an infinite variety of shapes and sizes. He imagined an infinite universe containing an infinite number of such atoms, in between the atoms there is an absolute void.



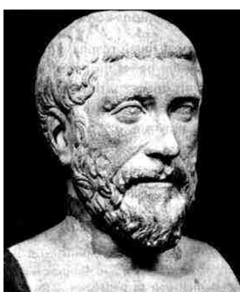
Democritus of Abdera (460 B.C. - 370 B.C.). Democritus is best known for his atomic theory but he was also an excellent geometer. Very little is known of his life but we know that Leucippus was his teacher. He's believed to have traveled widely, perhaps spent a considerable time in Egypt, and he certainly visited Persia. Democritus wrote many mathematical works but none survive. He claimed that the universe was a purely mechanical system obeying fixed laws. He explained the origin of the universe through atoms moving randomly and colliding to form larger bodies and worlds. He also believed that space is infinite and eternal, and that the number of atoms are infinite. Democritus's philosophy contains an early form of the conservation of energy.

2.3.3 The Pythagoreans

About five centuries B.C. the school founded by the Greek philosopher, mathematician and astronomer Pythagoras flourished in Samos, Greece. The Pythagoreans believed (but failed to prove) that the universe could be understood in terms of whole numbers. This belief stemmed from observations in music, mathematics and astronomy. For example, they noticed that vibrating strings produce harmonious tones when the ratios of the their lengths are whole numbers. From this first attempt to express the universe in terms of numbers the idea that the world could be understood through mathematics was born, a central concept in the development of mathematics and science.

The Pythagoreans originated the idea that the world could be understood through mathematics was born

The importance of pure numbers is central to the Pythagorean view of the world. A point was associated with 1, a line with 2 a surface with 3 and a solid with 4. Their sum, 10, was sacred and omnipotent [2].



Pythagoras of Samos (580–500 B.C.). Born Samos, Greece, died in Italy. Pythagoras was a Greek philosopher responsible for important developments in mathematics, astronomy, and the theory of music. He founded a philosophical and religious school in Croton that had many followers. Of his actual work nothing is directly known. His school practiced secrecy and communalism making it hard to distinguish between the work of Pythagoras and that of his followers.

Pythagoras also developed a rather sophisticated cosmology. He and his followers believed the earth to be perfectly spherical and that heavenly bodies, likewise perfect spheres, moved as the Earth around a central fire invisible to human eyes (this was *not* the sun for it *also* circled this central fire) as shown in Fig. 2.8. There were 10 objects circling the central fire which included a counter-earth assumed to be there to account from some eclipses but also because they believed the number 10 to be particularly

[2] Some relate this to the origin of the decimal system, but it seems to me more reasonable to associate the decimal system to our having ten fingers.

sacred. This is the first coherent system in which celestial bodies move in circles, an idea that was to survive for two thousand years.

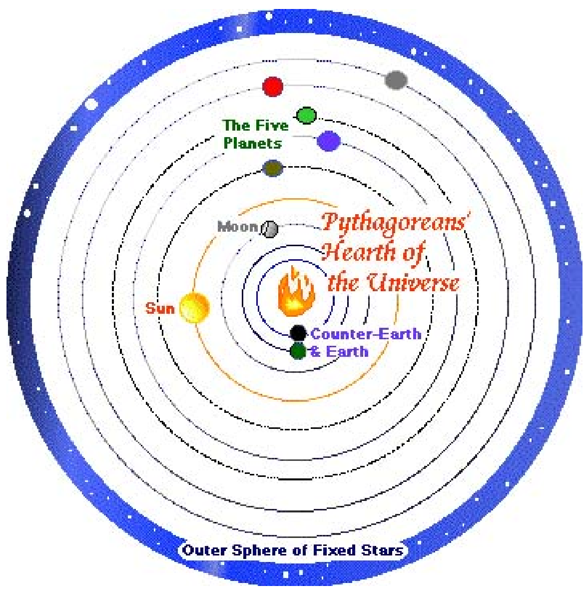


Figure 2.8: The universe according to the Pythagoreans.

It was also stated that heavenly bodies give forth musical sounds "the harmony of the spheres" as they move in the cosmos, a music which we cannot discern, being used to it from childhood (a sort of background noise); though we would certainly notice if anything went wrong! The Pythagoreans did not believe that music, numbers and cosmos were just related, they believed that music *was* number and that the cosmos *was music*

Pythagoras is best known for the mathematical result (Pythagoras' theorem) that states that the sum of the squares of the sides of a right triangle equals the square of the diagonal; see Fig. 2.9. This result, although known to the Babylonians 1000 years earlier, was first proved by Pythagoras (allegedly: no manuscript remains). Pythagoras' theorem will be particularly important when we study relativity for, as it turns out, it is *not* valid in the vicinity of very massive bodies! Similar statements hold for Euclid's postulate that parallel lines never meet, see Sect. **??**.

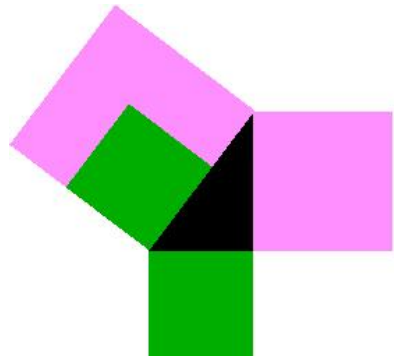
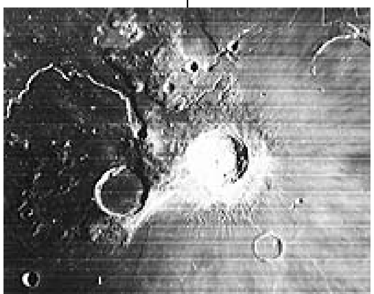


Figure 2.9: Pythagoras' theorem (the areas of the squares attached to the smaller sides of the triangle equal the area of the largest square).

2.4 Early heliocentric systems

By the IV century B.C. observations had shown that there are two types of stars: fixed stars whose relative position remained constant, and "wandering stars", or planets, whose position relative to the fixed stars changed regularly. Fixed stars moved as if fixed to a sphere that turned with the earth at its center, the planets moved about these fixed stars driven by an unknown agency. In fact, Plato regarded the investigation of the rules that determined the motion of the planets as a very pressing research problem.

A remarkable answer was provided by the heliocentric (!!) system of Aristarchus of Samos. Using a clever geometric argument Aristarchus estimated the size of the Sun and concluded it must be enormously larger than the Earth; he then argued that it was inconceivable that such a behemoth would slavishly circle a puny object like the Earth. Once he concluded this, he concluded that the Earth must rotate on its axis in order to explain the (apparent) motion of the stars. Thus Aristarchus conceived the main ingredients of the Copernican system 17 centuries before the birth of Copernicus! Unfortunately these views were soundly rejected by Aristotle: if the Earth is rotating, how is it that an object thrown upwards falls on the same place? How come this rotation does not generate a very strong wind? Due to arguments such as this the heliocentric theory was almost universally rejected until Copernicus' answered these criticisms.



Aristarchus of Samos (310 B.C. - 230 B.C.). Born and died in Greece. Aristarchus was a mathematician and astronomer who is celebrated as the exponent of a Sun-centered universe and for his pioneering attempt to determine the sizes and distances of the Sun and Moon. Aristarchus was a student of Strato of Lampsacus, head of Aristotle's Lyceum, coming between Euclid and Archimedes. Little evidence exists concerning the origin of his belief in a heliocentric system; the theory was not accepted by the Greeks and is known only because of a summary statement in Archimedes' *The Sand-Reckoner* and a reference by Plutarch. The only surviving work of Aristarchus, *On the Sizes and Distances of the Sun and Moon*, provides the details of his remarkable geometric argument, based on observation, whereby he determined that the Sun was about 20 times as distant from the Earth as the Moon, and 20 times the Moon's size. Both these estimates were an order of magnitude too small, but the fault was in Aristarchus' lack of accurate instruments rather than in his correct method of reasoning. Aristarchus also found an improved value for the length of the length of the solar year.

Astronomy also progressed, with the most striking result, due to Eratosthenes, was accurate measurement of the Earth's circumference [3] (the fact that the Earth is round was common knowledge) He noted that the distance from Alexandria to Aswan is 5,000 stadia and that when the sun casts no shadow in Alexandria it casts a shadow corresponding to an angle of 7.2^o (see Fig. 2.10). From this he determined the circumference of the Earth less than 2% accuracy!

The fact that the Earth is round was common knowledge

It is important to remember that the realization that the Earth was round was not lost to the following centuries, so that neither Columbus nor any of his (cultivated) contemporaries had any fear of falling off the edge of the world when traveling West trying to reach the Indies. The controversy surrounding Columbus' trip was due to a disagreement on the *size* of the Earth. Columbus had, in fact, seriously underestimated the radius of the Earth and so believed that the tiny ships he would command had a fair chance of getting to their destination. He was, of course, unaware of the interloping piece of land we now call America, had this continent not existed, Columbus and his crew would have perished miserably in the middle of the ocean.

[3] Aristotle had previously estimated a value of 400,000 stadia (1 stadium=157.5m) which is about 1.6 times its actual size.

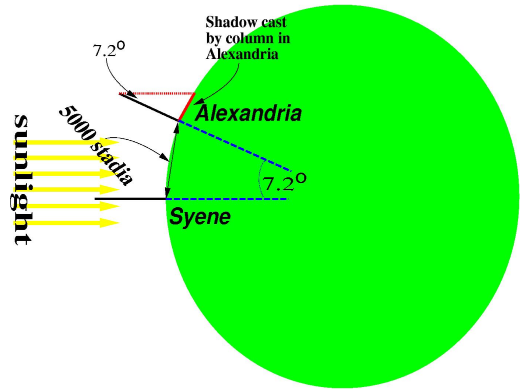


Figure 2.10: Description of Eratosthenes' procedure for measuring the Earth. He reasoned that the change in angle of the shadow was caused by moving about the Earth. By measuring the angle of the shadow at Seyene, and then in a city that was directly north of Seyene (Alexandria), he determined that the two cities were 7 degrees apart. That is to say, out of the 360 degrees needed to travel all the way around the world, the two cities were 360/7 of that distance. Since he knew that the two cities were about 500 miles apart, he concluded that the the Earth must be $(360/7) \times 500$ miles in circumference, or roughly $25,000$ miles.

Eratosthenes of Cyrene (276 B.C. - 197 B.C.). Greek, lived in Alexandria He was born in Cyrene which is now in Libya. He worked on geometry and prime numbers. He is best remembered for his prime number sieve which, in modified form, is still an important tool in number theory research. Eratosthenes measured the tilt of the earth's axis with great accuracy and compiled a star catalogue containing 675 stars; he suggested that a leap day be added every fourth year and tried to construct an accurately-dated history. He became blind in his old age and is said to have committed suicide by starvation.

2.5 Aristotle and Ptolemy

There are few instances of philosophers that have had such a deep influence as Aristotle, or of cosmologists whose theories have endured as long as Ptolemy's. Aristotle's influence is enormous ranging form the sciences to logic. Many of his ideas have endured the test of the centuries. His cosmology, based on a geocentric system, is not one of them. In the words of W. Durant

> His curious mind is interested, to begin with, in the process and techniques of reasoning; and so acutely does he analyze these that his *Organon*, or Instrument–the name given after his death to his logical treatises–became the textbook of logic for two thousand years. He longs to think clearly, though he seldom, in extant works, succeeds; he spends half his time defining his terms, and then he feels that he has solved the problem.

It must me noted, however, that he forcefully argued for the sphericity of the Earth *based on data:* he noted that only a spherical Earth can account for the shadow seen on the Moon during a lunar eclipse

Ptolemy enlarged Aristotle's ideas creating a very involved model of the solar system which endured until the Copernican revolution of the middle 16th century. When comparing the Ptolemaic system with the Copernican heliocentric system Occam's razor (Sect. **??**) instantly tells us to consider the latter first: it provide a much simpler explanation (and, as it turns out, a much better one) that the former.

2.5.1 Aristotelian Cosmology

Aristotle's cosmological work *On The Heavens* is the most influential treatise of its kind in the history of humanity. It was accepted for more that 18 centuries from its inception (around 350 B.C.) until the works of Copernicus in the early 1500s. In this work Aristotle discussed the general nature of the cosmos and certain properties of individual bodies.

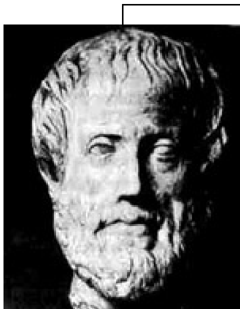
Aristotle believed that all bodies are made up of four elements: earth, water, air and fire (see Fig. 2.7). These elements naturally move up or down, fire being the lightest and earth the heaviest. A composite object will have the features of the element which dominates; most things are of this sort. But since the elements in, for example, a worm, are not where they belong (the fiery part is too low being bound by the earth part, which is

Aristotle believed that all bodies are made up of four elements: earth, water, air and fire

The elements naturally move up or down, fire being the lightest and earth the heaviest a bit too high), then the worm is imperfect. All things on earth are thus imperfect.

According to Aristotle all bodies, *by their very nature*, have a natural way of moving

The idea that all bodies, *by their very nature*, have a natural way of moving is central to Aristotelian cosmology. Movement is *not*, he states, the result of the influence of one body on another



Aristotle (384 B.C. - 322 B.C.). Born Stagirus, Greece, died Chalcis, Greece. In 367 Aristotle became a student at Plato's Academy in Athens. Soon he became a teacher at the Academy. After Plato's death in 347 B.C., Aristotle joined the court of Hermias of Atarneus. In 343 B.C. he became tutor to the young Alexander the Great at the court of Philip II of Macedonia. In 335 B.C. Aristotle founded his own school the Lyceum in Athens. The Academy had become narrow in its interests after Plato's death but the Lyceum under Aristotle pursued a broader range of subjects. Prominence was given to the detailed study of nature. After the death of Alexander the Great in 323 B.C., anti-Macedonian feeling in Athens made Aristotle retire to Chalcis where he died the following year. Aristotle was not primarily a mathematician but made important contributions by systematizing deductive logic. He wrote on physical subjects; some parts of his *Analytica Posteriora* show an unusual grasp of mathematics. He also had a strong interest in anatomy and the structure of living things in general which helped him to develop a remarkable talent for observation.

Some bodies naturally move in straight lines, others naturally stay put. But there is yet another natural movement: the circular motion. Since to each motion there must correspond a substance, there ought to be some things that naturally move in circles. Aristotle then states that such things are the heavenly bodies which are made of a more exalted and perfect substance than all earthly objects.

Since the stars and planets are made of this exalted substance and then move in circles, it is also natural, according to Aristotle, for these objects to be spheres also. The cosmos is then made of a central earth (which he accepted as spherical) surrounded by the moon, sun and stars all moving in circles around it. This conglomerate he called "the world". Note the strange idea that all celestial bodies are perfect, yet they must circle the imperfect Earth. The initial motion of these spheres was caused by the action of a "prime mover" which (who?) acts on the outermost sphere of the fixed stars; the motion then trickles down to the other spheres through a dragging force.

Aristotle also addresses the question whether this world is unique or not; he argues that it *is* unique. The argument goes as follows: earth (the substance) moves naturally to the center, if the world is not unique there ought to be at least two centers, but then, how can earth know to which of the two centers to go? But since "earthy" objects have no trouble deciding how to move he concludes that there can only be one center (the Earth) circled endlessly by all heavenly bodies. The clearest counterexample was found by Galileo when he saw Jupiter and its miniature satellite system (see Fig. 2.11), which looks like a copy of our "world". Aristotle was wrong not in the logic, but in the initial assumptions: things do *not* have a natural motion.

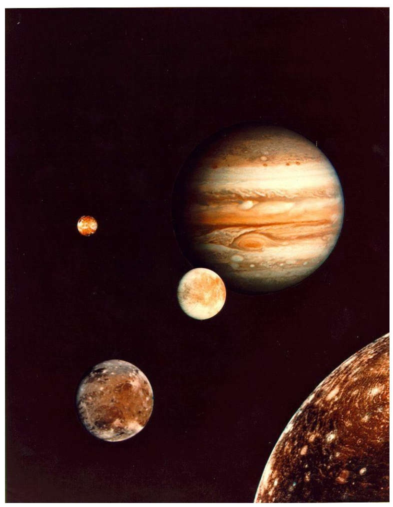


Figure 2.11: Montage of Jupiter and the Galilean satellites, Io, Europa, Ganymede, and Callisto.

It is interesting to note that Aristotle asserts that the world did not come into being at one point, but that it has existed, unchanged, for all eternity (it had to be that way since it was "perfect"); the universe is in a kind of "steady state scenario". Still, since he believed that the sphere was the most perfect of the geometrical shapes, the universe did have a center (the Earth) and its "material" part had an edge, which was "gradual" starting in the lunar and ending in the fixed star sphere. Beyond the sphere of the stars the universe continued into the spiritual realm where material things

Aristotle asserts that the world did not come into being at one point, but that it has existed, unchanged, for all eternity cannot be (Fig. 2.12). This is in direct conflict with the Biblical description of creation, and an enormous amount of effort was spent by the medieval philosophers in trying to reconcile these views.

Figure 2.12: A pictorial view of the Aristotelian model of the cosmos.

On the specific description of the heavens, Aristotle created a complex system containing 55 spheres(!) which, despite it complexity, had the virtue of explaining *and predicting* most of the observed motions of the stars and planets. Thus, despite all the bad publicity it has received, this model had all the characteristics of a scientific theory (see Sect. **??**): starting from the hypothesis that heavenly bodies move in spheres around the Earth, Aristotle painstakingly modified this idea, matching it to the observations, until all data could be accurately explained. He then used this theory to make predictions (such as where will Mars be a year from now) which were confirmed by subsequent observations.

2.5.2 The motion according to Aristotle

One of the fundamental propositions of Aristotelian philosophy is that there is no effect without a cause

One of the fundamental propositions of Aristotelian philosophy is that there is no effect without a cause. Applied to moving bodies, this proposition dictates that there is no motion without a force. Speed, then is proportional to force and inversely proportional to resistance

$$force = (resistance) \times (speed)$$

(though none of these quantities were unambiguously defined). This notion is not at all unreasonable if one takes as one's defining case of motion, say, an ox pulling a cart: the cart only moves if the ox pulls, and when the ox stops pulling the cart stops.

Aristotle's law of motion. (from *Physics*, book VII, chapter 5). Then, A the movement have moved B a distance Γ in a time Δ, then in the same time the same force A will move $\frac{1}{2}B$ twice the distance Γ, and in $\frac{1}{2}\Delta$ it will move $\frac{1}{2}B$ the whole distance for Γ: thus the rules of proportion will be observed.
The translation into modern concepts is $A \to F$ =force, $B \to m =$ mass, $\Gamma \to d$ =distance, and $\Delta \to t$ =time.
The statements then mean

- The distance is determined by the force F, the mass m and the time t
- Given a force F which moves a mass m a distance d in a time t, it will also move half the mass by twice the distance in the same time.
- Given a force F which moves a mass m a distance d in a time t, it will also move half the mass the same distance in half the time.

These three rules imply that the product of the mass and the average speed depends only on the force. For example, a body of constant mass under the action of a constant force will have a constant speed. This is wrong: the speed increases with time.

Qualitatively this implies that a body will traverse a thinner medium in a shorter time than a thicker medium (of the same length): things will go faster through air than through water. A natural (though erroneous) conclusion is that there could be no vacuum in Nature, for if the resistance became vanishingly small, a tiny force would produce a very large "motion"; in the limit where there is no resistance any force on any body would produce an infinite speed. This conclusion put him in direct contradiction with the ideas of the atomists such as Democritus (see Sect. 2.3.2). Aristotle (of course) concluded the atomists were wrong, stating that matter is in fact continuous and infinitely divisible.

Aristotle argued that there could be no vacuum in Nature

For falling bodies, the force is the weight pulling down a body and the resistance is that of the medium (air, water, etc.). Aristotle noted that a falling object gains speed, which he then attributed to a gain in weight. If weight determines the speed of fall, then when two different weights are dropped from a high place the heavier will fall faster and the lighter slower, in proportion to the two weights. A ten pound weight would reach the Earth by the time a one-pound weight had fallen one-tenth as far.

Aristotle asserted that when two different weights are dropped from a high place the heavier will fall faster and the lighter slower

2.5.3 Ptolemy

The Aristotelian system was modified by Hipparchus whose ideas were popularized and perfected by Ptolemy. In his treatise the *Almagest* ("The Great System") Ptolemy provided a mathematical theory of the motions of the Sun, Moon, and planets. Ptolemy vision (based on previous work by Hipparchus) was to envision the Earth surrounded by circles, on these circles he imagined other (smaller) circles moving, and the planets, Sun, etc. moving on these smaller circles. This model remained unchallenged for 14 centuries.

The system of circles upon circles was called a system of *epicycles* (see Fig. 2.14). It was extremely complicated (requiring several correction factors) but it did account for all the observations of the time, including the peculiar behavior of the planets as illustrated in Fig. 2.15. The Almagest was not superseded until a century after Copernicus presented his heliocentric theory in Copernicus' *De Revolutionibus* of 1543.

Ptolemy (100 - 170). Born in Ptolemais Hermii, Egypt, died Alexandria, Egypt. One of the most influential Greek astronomers and geographers of his time, Ptolemy propounded the geocentric theory that prevailed for 1400 years. Ptolemy made astronomical observations from Alexandria Egypt during the years A.D. 127-41. He probably spent most of his life in Alexandria. He used his observations to construct a geometric model of the universe which accurately predicted the positions of all significant planets and stars. This model employed combinations of circles known as epicycles, within the framework of the basic Earth-centered system supplied by Aristotle. His model is presented in his treatise *Almagest*. In a book entitled *Analemma* he discussed the projection of points on the celestial sphere. In *Planisphaerium* he is concerned with stereographic projection. He also devised a calendar that was followed for many centuries. There where problems with it, however, and this required corrections of about 1 month every 6 years. This generated a lot of problems in particular in agriculture and religion!

This model was devised in order to explain the motion of certain planets. Imagine that the stars are a fixed background in which the planets move, then you can imagine tracing a curve which joins the positions of a given planet everyday at midnight (a "join the dots" game); see, for example Fig 2.13. Most of the planets move in one direction, but Mars does not, its motion over several months is seen sometimes to backtrack (the same behavior would have been observed for other celestial objects had Ptolemy had the necessary precision instruments).

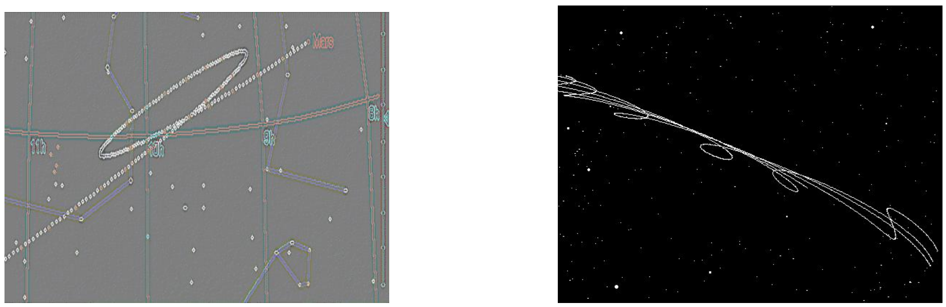


Figure 2.13: This computer simulations shows the retrograde motion of Mars (left) and the asteroid Vesta (right). Vesta's trajectory is followed over several years; it moves from right to left (west to east), and each loop occurs once per year. The shape of the retrograde loop depends on where Vesta is with respect to Earth.

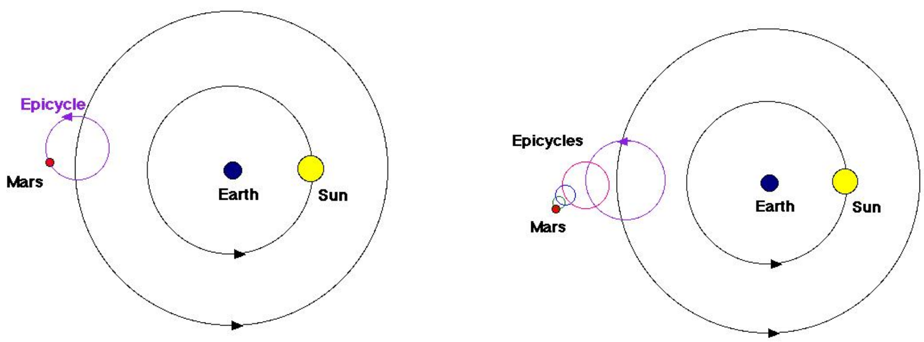


Figure 2.14: The simplest form of an epicycle (left) and the actual form required to explain the details of the motion of the planets (right).

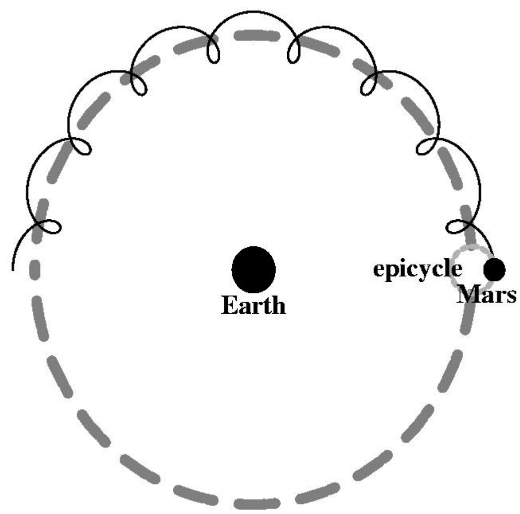


Figure 2.15: Example of how a system of epicycles can account for the backtracking in the motion of a planet. The solid line corresponds to the motion of Mars as it goes around the epicycle, while the epicycle itself goes around the Earth. As seen from Earth, Mars would move back and forth with respect to the background stars.