Physics 117

Homework Solutions to HW Set # 8 due April 6, '05 (WED)

Page 1 of 5.

By ____ On __/__/__

CH 10: CQ 23, 33, 34; Ex 7, 13, 17

CH 11: CQ 5, 9; Ex 3, 8

Ch 10 CQ 23. To an observer in the lab frame, the time interval measured by a clock (or the average lifetime of a decaying particle) moving through the lab at speed, v, is dilated (i.e., increased) by the adjustment factor, $\gamma = (1 - v^2/c^2)^{-1/2} \geq 1$, as compared with a clock at rest in the lab.

Then if the mu meson lifetime at rest (i.e. observed in its own rest frame) is τ_μ^0, the lab observer will measure this lifetime to be $\tau_\mu^v = \gamma\, \tau_\mu^0 \geq \tau_\mu^0$. Thus the lab lifetime of 8 μs (μs = micro-seconds, and this μ, - for "micro" = 10^{-6} has nothing to do with the name and symbol, μ, for the mu-meson, or muon.) is greater than that of the muon at rest by the factor $\gamma \geq 1$, and $\tau_\mu^0 \leq \tau_\mu^v$. I.e muon's lifetime at rest is less than its lifetime measured in a lab through which it moves.

By [signature]
On 3/15/04

Ch 10 CQ 33. If $T_\mu^0 = 2.2\ \mu s$ is the average lifetime of a muon at rest (i.e. as measured by an observer in its own rest frame), then at a speed c (and no speed can be greater!), it could travel $2.2\times10^{-6} s \cdot 3\times10^{8}\ m/s = 660\ m$, at most, before decaying.

But if the muon is moving in a lab frame on earth, then its lifetime is dilated by a factor of γ, and the maximum distance would by $c\cdot\gamma T_\mu^0 >> cT_\mu^0 = 660\ m$.

In this case the observer must be in the reference system through which the muon is moving (e.g. earth's lab frame).

An observer in the muon rest frame would report

(a) that muon is at rest, and earth is moving with velocity $-\vec{v}$, just opposite to what earth's observer says about muon, and

(b) That the distance from upper atmosphere to earth's surface, which earthlab measures to be D is actually "Contracted" to the smaller distance $D/\gamma = D'$ for the observer at rest on the muon. He says that the small earth-atmosphere distance D' travels past the muon at speed

$$\frac{D'}{T_\mu^0} = v_E = \frac{D}{\gamma T_\mu^0},$$ since $D' = \frac{D}{\gamma}$ is a contracted moving length.

The earth lab observer says that the muon is moving with speed $v_\mu = D/T_\mu^v = D/\gamma T_\mu^0$, given in his frame by the distance D (at rest & uncontracted) divided by the dilated lifetime of the moving meson $T_\mu^v = T_\mu^0\gamma$

The two observers agree only on the value of $|v|$.

Ch 10 CQ 34: If the muon's arrival at earth's surface after traversing $D >> 660$ m is to be explained in terms of length contraction, then the distance travelled must be moving in the frame of the observer. If D in the earth's rest frame is the height of the top of the earth's atmosphere, and is to be moving, then the observer is in the rest frame of the moving muon. He measures objects moving in in his own rest frame to be shortened (contracted) in the direction parallel to their velocity by a factor, $1/\gamma$. Then he measures the height D to be the much smaller value $D' = D/\gamma << 660$ m, and says ~~that it can~~ pass by the muon in the span of its lifetime in its rest frame, T_μ^0

To the earth lab observer D is at rest, so no question of contraction arises. On the other hand the muon is moving, so its internal clock (which defines its average lifetime) shows a time dilation. He explains the great distance travelled (D) by his observation that the muon's average lifetime is $\gamma T_\mu^0 = T_\mu^v >> T_\mu^0$.

Thus the two observers disagree both about clock times & distances ... they agree only on the relative speed $v = \frac{D'}{T_\mu^0} = \frac{D}{\gamma T_\mu^0} = \frac{D}{T_\mu^v}$.

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By [illegible]
On 3/15/04

Ch 10 Ex 7.
$$\tau_\pi^v = \gamma\,\tau_\pi^0 = (1 - v^2/c^2)^{-1/2}\,\tau_\pi^0$$
$$2.69\times10^{-9}\,s = \left[1-(0.99)^2\right]^{-1/2}\tau_\pi^0$$
$$\frac{2.69\times10^{-9}}{7.09} = \boxed{\tau_\pi^0 = 3.79\times10^{-10}\,\text{sec} = \text{Lifetime of pion at rest}}$$

Ex 13 Exactly the same answer since she is observing the train in the rest frame of the train. An observer in the station's rest frame, however, would measure the moving train's length to be contracted by a factor $1/\gamma\ (\leq 1)$. Then she again measures $\boxed{200\,m}$

Ex 17. Since $p^R = \gamma m v$ is the relativistic momentum (replacing $p^{NR} = mv$, non relativistically) we have $F_R\Delta t$ = impulse = change in momentum $= p_f^R - p_i^R$
$$F_R\Delta t = \gamma_f m v_f - 1\cdot m\cdot 0 = \gamma_f\cdot m v_f.$$
and the corresponding non-relativistic result is $F_{NR}\Delta t = 1\cdot m v_f.$

Since Δt is the same for both $F_R = \dfrac{\gamma_f\cdot m v_f}{\Delta t} = \gamma_f\cdot F_{NR}$

The relativistic force must be larger by the factor γ:
$$\gamma = \left[1-(1-10^{-4})^2\right]^{-1/2} = (1-1+2\cdot10^{-4}-10^{-8})^{-1/2} = \sqrt{1/2\times10^{-4}}$$
$$= \frac{10^2}{\sqrt{2}} = 70.7$$
Therefore, $\boxed{F_R = (9.5)(70.7) = 672\,N}$

{Note that calculator rounding error can lead to false results when the answer gets too small. Then writing $v/c = 1-\epsilon$ yields $(1-v^2/c^2)^{-1/2} =$
$$\left[1-(1-\epsilon)^2\right]^{-1/2} = \left[\cancel{1}-(\cancel{1}-2\epsilon+\epsilon^2)\right]^{-1/2} \approx \frac{1}{\sqrt{2\epsilon}}$$
for small ϵ. Since $\epsilon^2 \ll \epsilon$ when $\epsilon \ll 1$.}

PHYSICS 117 SPRING '05

Homework Solutions to HW Set # 8, due 4/6/05 (WED)

Page 5 of 5.

By [signature]
On 3/15/04

Ch 11: CQ 5, 9 / Ex 3, 8.

Q5) Additional examples in which experimental results agree with a model strengthen our belief in it However a model can never be proven true

11· Q9) Salt granite and water are not elements. Each is composed of more than one element

11· Ex3) Only 4 g of hydrogen is required to use up 32g of oxygen fully. Amount of water that can be formed $= 4(1g + 32g$ $\boxed{36g}$, and 4 g of H will be left over

11: Ex 8) Molecular mass of sulphur 2 amu

1 mole of sulfur weighs 32 g

There are Avogadro number of molecules one mole $N_A = 6 \times 10^{23}$

32g of sulfur contain N_A $6\ 02 \times 10^{23}$ molecules

of sulfur contains $\frac{6.02 \times 10^{23}}{(32)}$ molecules

$\boxed{= 1.88 \times 10^{22}}$ molecules of S

END of Solutions #8