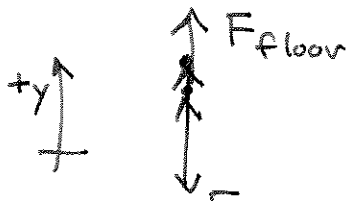
Homework 6 Soln

① sys = Simplicio



$F_{grav} = m_{simplicio} \cdot g$

$a_{simplicio} > 0$ (upward) $\Rightarrow$ $F_{net} > 0$ (upward)

$\Rightarrow$ $F_{floor} > F_{grav}$

Newton is right!
Simplicio feels "heavier" because the force on his feet due to the floor is larger.

② ~~to~~ (a) Total mass $= 6 \times 10^9 \text{ people} \times \dfrac{150\,\text{lb}}{\text{person}} \times \dfrac{\text{kg}}{2\,\text{lb}}$

$\simeq 4 \times 10^{11}$ kg

F_{grav} = force on people due to earth

$= (4 \times 11^{11}\text{ kg}) \cdot (10\text{ m/s}^2)$

$\simeq 4 \times 10^{12}$ N

Newton's 3rd law $\Rightarrow$ force on earth due to people = same magnitude

$\Rightarrow$ $\boxed{4 \times 10^{12}\text{ N}}$

(b) $a_{earth} = \dfrac{4 \times 10^{12}\text{ N}}{m_{earth}}$

m_{earth} = density × volume

$$= \frac{1\text{ gm}}{\text{cm}^3} \times \left(\frac{100\text{ cm}}{\text{m}}\right)^3 \times \left(\frac{\text{kg}}{1000\text{ gm}}\right) \times \frac{4}{3}\pi R_e^3$$

$$\simeq 10^3 \frac{\text{kg}}{\text{m}^3} \times \frac{4}{\cancel{3}} \cdot \cancel{3} \times \left(6000\,\cancel{\text{km}} \times \frac{1000\text{ m}}{\cancel{\text{km}}}\right)^3$$

$$\simeq 4\times 10^{3} \times \underbrace{6\cdot \underbrace{6\cdot 6}_{=36}}_{\simeq 200} \times 10^{18}$$

$$\boxed{m_{earth} \simeq 8\times 10^{23}\ \text{kg}}$$

$$\Rightarrow \quad a_{earth} = \frac{4\times 10^{12}}{8\times 10^{23}} \simeq 5\times 10^{-12}\ \text{m/s}^2 \quad \Rightarrow \text{tiny!}$$

This estimate of the mass of the earth is off by a factor of 8. It doesn't make any difference because the acceleration is so small.

③ (a) system = box + chain

$$\boxed{a = \frac{F_{net}}{m} = \frac{15\ \text{N}}{2.5\ \text{kg} + m_{chain}}}$$

$m_{chain} \to \infty \quad \Rightarrow \quad a \to 0$ (system too heavy to move)

$m_{chain} \to 0 \quad \Rightarrow \quad a \to \dfrac{15\ \text{N}}{2.5\ \text{kg}}$ = same as if force applied directly to box

(b) sys = box

F_{chain}

The force the chain exerts is the only force on the box

$$a = \frac{F_{chain}}{2.5\ \text{kg}} = \frac{15\ \text{N}}{2.5\ \text{kg} + m_{chain}}$$

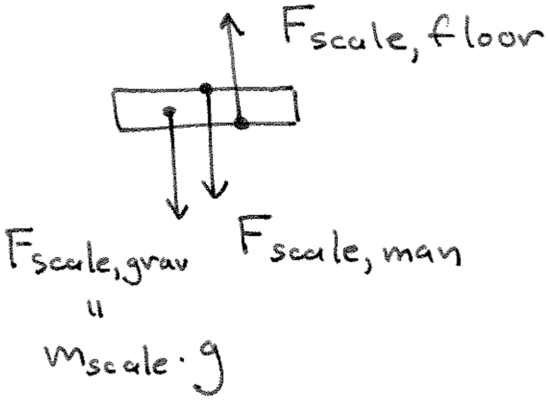
$\cancel{a = 2.5 + m_{chain}}$

$$\boxed{F_{chain} = 15\ \text{N}\ \frac{2.5\ \text{kg}}{2.5\ \text{kg} + m_{chain}}}$$

$m_{chain} \to 0 \Rightarrow F_{chain} \to 15\,N$ same as force applied to chain

$m_{chain} \to \infty \Rightarrow F_{chain} \to 0$

4 sys = scale



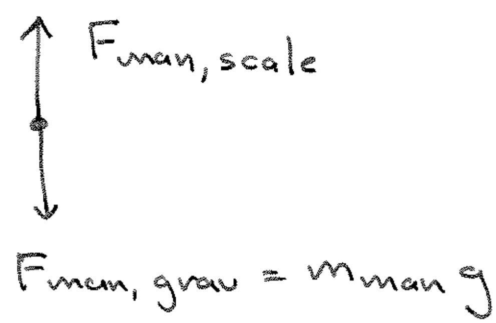
The reading of the scale corresponds to $F_{scale,man}$

$n \propto F_{scale,man}$

$\Rightarrow n = c\,F_{scale,man}$ $\qquad c =$ constant $= ?$

(a) At rest: ~~$F_{scale,man} =$~~

sys = man



N3: $F_{man,scale} = F_{scale,man}$ ~~= scale readi~~

$F_{man,scale} = m_{man}\,g$

$\Rightarrow$ ~~scale~~ $n = c\,F_{scale,man} =$

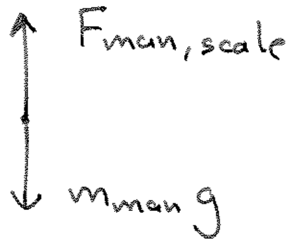
$= c\,F_{man,scale}$

$= c \cdot m_{man}\,g$

$$c = \frac{n}{m_{man} \cdot g} = \frac{5\cancel{0}}{5\cancel{0} \cdot 9.8\ m/s^2} = \frac{1}{9.8\ m/s^2}$$

(b) $F_{man,scale} = \frac{n}{c} = n \cdot 9.8\ m/s^2$

sys = man



net force upward $= F_{man,scale} - m_{man}\, g$

$= (9.8\ m/s^2)\, n - (50\ kg)\ 9.8\ m/s^2$

$= (n - 50\ kg)(9.8\ m/s^2)$

$\Rightarrow a_{man} = \frac{F_{net}}{m_{man}}$

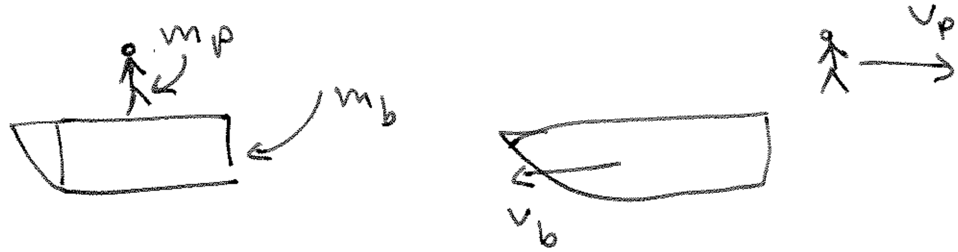
$$\boxed{a_{man} = \frac{n - 50\ kg}{50\ kg}\,(9.8\ m/s^2)}$$

(c) $n = 0 \Rightarrow a_{man} = -9.8\ m/s^2$

downward

This is what happens when the elevator falls freely!

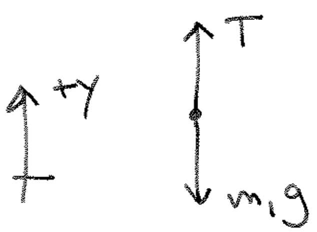
⑤ As in the problem with the spring loaded carts discussed in the lecture and the book, we have



$$m_p v_p = m_b v_b$$

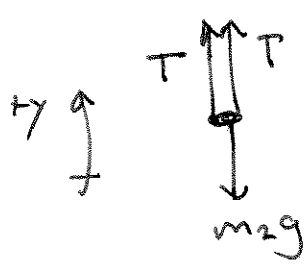
$$v_b = \frac{m_p}{m_b} v_p = \frac{50\ kg}{100\ kg} \cdot 4\ m/s = 2\ m/s$$

⑥ (a) sys = m_1



$$T - m_1 g = m_1 a_1$$

(b) sys = m_2 (including pulley)



$$2T - m_2 g = m_2 a_2$$

(c) If an amount of string Δl goes over the big pulley, from right to left, then the m_1 goes down by Δl. But the shortened length on the right side of the big pulley is shared by the two strings on the right. Therefore, m_2 only goes up by $\frac{1}{2}\Delta l$. Thus

$$\Delta y_1 = -2\Delta y_2$$

↑ accounts for different direction

$$v_{1,2} = \lim_{\Delta t \to 0} \frac{\Delta y_{1,2}}{\Delta t} \Rightarrow v_1 = -2v_2$$

$$\Rightarrow \Delta v_1 = -2\Delta v_2$$

$$\Rightarrow a_1 = -2a_2$$

(d) $\cancel{a_1 = -2a_2}$ $a_1 = -2a_2 \Rightarrow$

$$\cancel{T \, m_1} \begin{cases} T - m_1 g = m_1 a_1 \\ 2T - m_2 g = m_2 \left(-\frac{a_1}{2}\right) \end{cases}$$

Eliminate T:

$$T = m_1 (a_1 + g)$$

$$2 \cdot m_1 (a_1 + g) - m_2 g = -\frac{1}{2} m_2 a_1$$

$$\left(2m_1 + \frac{1}{2}m_2\right) a_1 + 2m_1 g - m_2 g = 0$$

$$a_1 = \frac{m_2 - 2m_1}{2m_1 + \frac{1}{2}m_2} g = \frac{1.5\text{ kg} - 2(1\text{ kg})}{2(1\text{ kg}) + \frac{1}{2}(1.5\text{ kg})} \cdot 9.8\text{ m/s}^2$$

$$= -1.8\text{ m/s}^2 \qquad \Rightarrow \underline{\text{downward}}$$