1. The scattering amplitude for neutrons of energy $E$ incident on a certain species of heavy nuclear target is given to a good approximation by $f(\theta) = A + B \cos \theta$.

   (a) For approximately what range of energies $E$ could this be true?

   (b) What is the $s$-wave phase shift?

   (c) If the incident beam has a number flux $I$, how many neutrons per unit time are back-scattered into a small solid angle $\Delta\Omega$ about the backward direction $\theta = \pi$?

2. Consider scattering of a particle of energy $E$ and mass $m$ from the potential $V(r) = V_0 \theta(a - r)$, where $V_0$ can have either sign.

   (a) Give an “observational” definition of the differential scattering cross section $d\sigma/d\Omega$.

   (b) Express the differential cross section in terms of the scattering amplitude.

   (c) State two different conditions on $(E, m, V_0, a)$ which are independently sufficient for validity of the Born approximation. One condition should depend on $E$ and the other should not.

   (d) State the conditions on $(E, m, V_0, a)$ for $s$-wave scattering to dominate the partial wave expansion. What is the angular dependence of the scattering amplitude in this case?

   (e) Find an exact transcendental equation for the $s$-wave phase shift $\delta_0$. Write the scattering amplitude in the $s$-wave approximation in terms of $\delta_0$.

   (f) Find the scattering amplitude using the Born approximation in the common domain of validity of the Born and $s$-wave approximations.

   (g) Show that in the common domain of validity of the Born and $s$-wave approximations, your results for parts 2f and 2e agree. (Hint: You may wish to use the expansion $\tan x = x + x^3/3 + \cdots$.)

3. You showed in HW#10 that the first Born approximation gives a finite, nonzero cross section for a delta-function potential, while the second Born approximation is infinite. (a) Now use the partial wave expansion to show that the exact cross section is zero (!) for the delta-function limit of the spherical square potential $V(r) = V_0 \theta(a - r)$ (i.e. the limit $a \to 0$, $V_0 \to \infty$, with $\int d^3x V(\vec{x})$ held fixed). (b) Show that if, instead of the delta-function limit, $V_0$ times the surface area of the potential is held fixed, then the limit is finite and nonzero.