A general solution is a linear combination

\[ N(r, t) = \sum_n A_n \exp(\mu_n t) \sin(k_n r)/r, \]

where \( A_n \) are arbitrary constants, and \( \mu_n \) and \( k_n \) are determined by the diffusion constant \( \kappa \), the production rate \( \lambda \), the radius of the sphere \( R \) and the integer \( n \). See the solution to problem 5 in hw5, where \( k_n \) is denoted \( \sqrt{|\nu|} \).

At time \( t = 0 \) we have

\[ N(r, t) = \sum_n A_n \sin(k_n r)/r = \begin{cases} \bar{N} & \text{if } r < a \\ 0 & \text{otherwise} \end{cases} \]

To pick off the coefficients \( A_n \), I suggest you multiply both sides of this equation by \( r \sin(k_m r) \) and integrate over \( r \) from 0 to \( R \). On the left hand side you’ll get \( A_m R/2 \) since

\[ \int_0^R dr \sin(k_n r) \sin(k_m r) = (R/2)\delta_{mn}. \]

(This is the same as (15.3) in the textbook, with \( L \) replaced by \( R \) and with the range of integration cut in half.) On the right hand side you’ll have an integral that needs to be evaluated.