Electric equations: 

\[ S = \int -\frac{1}{4} F_{\mu \nu} F^{\mu \nu} \, d^4x + \int A_\mu j^\mu \, d^4x, \quad \text{Gauge invariant: } A_\mu \rightarrow A_\mu + \partial_\mu \theta \]

\[ F_{\mu \nu} \rightarrow F_{\mu \nu} \]

in flat space (Minkowski coods)

\[ S = \int -\frac{1}{2} F^2_{\mu \nu} F_{\mu \nu} - \frac{4}{3} F_{\mu \nu} F^{\mu \nu} \]

\[ \frac{i}{2} B_k B_k \]

\[ \text{So } A_\mu \text{ never appears } \rightarrow \]

\[ \Pi^0 \equiv 0 \left( \frac{\partial \theta}{\partial A_\mu} = 0 \right) \]

and \( \left( \frac{\partial}{\partial A^i} \right) : \)

\[ \Pi^i(x) = \frac{\partial}{\partial A^i(x)} = -F^{\mu \nu} \frac{\partial A_\nu(x)}{\partial A^i(x)} \]

The electric field.

Therefore:

\[ H = \int \left[ \frac{\partial}{\partial A^i} + \frac{1}{2} F_{\mu \nu} F^{\mu \nu} + \frac{1}{2} B^2 \right] d^4x = \]

\[ \Pi^i = 2\partial A_i - \partial A_0, \quad \dot{A}_i = \Pi^i + \partial_\mu A_\mu \]

\[ \int \left[ \frac{1}{2} (\Pi^2 + B^2) + \Pi \dot{A}_0 \right] d^4x = \int \frac{1}{2} (\Pi^2 + B^2) - A_0 \partial_\mu \Pi^i + A_\mu \partial_\nu A_\nu A^i \]

\[ \text{after imposing the constraints } \]

\[ A_0 \left( \partial_\mu \Pi^\mu + \partial_t \right) \]

\[ \text{Gauss' law } \Rightarrow \text{initial value constraint} \]

See:

\[ \partial_\mu \left( \nabla \cdot F - 4\pi j \right) = \nabla \cdot \partial_\mu E - 4\pi \partial_\mu j = \]

\[ = \nabla \cdot (\nabla \times B) - \nabla \cdot (4\pi j) - 4\pi \partial_\mu j = \]

\[ = -4\pi (\partial_\mu j + \nabla \cdot j) = 0 \quad \text{if charge is conserved.} \]

Canonical variables: \( A_i(x), \Pi^i(x) = -E^i(x) \).