Cyls: Defined for anti-symmetric covariant tensors (forms)

\[ F_{\alpha\beta} = \partial_{\alpha} A_{\beta} - \partial_{\beta} A_{\alpha} = V_{\alpha} A_{\beta} - V_{\beta} A_{\alpha} \]

if it has no torsion and basis vectors commute.

This generalizes:

\[ F_{\alpha\beta...s} = P_{p} A_{\alpha} B_{\beta...s} = (dB)_{\alpha...s} \]

\[ d^2 = 0 \]

\[ [L_x, d] = 0 \]

\[ L_x \omega = L_x (d^\alpha \epsilon_{\alpha...s}) = 0 \]

by the definition of the Lie derivative:

\[ L_x \omega_{\alpha...s} = \frac{\partial x^m}{\partial x^\alpha} \frac{\partial x^n}{\partial x^p} \frac{\partial \omega_{m...n}}{\partial s} \]

in a adapted coordinate system.

Therefore:

\[ dL_x \omega_{\mu p} = d \left( \frac{\partial x^m}{\partial x^\alpha} \frac{\partial x^n}{\partial x^p} \frac{\partial \omega_{m...n}}{\partial s} \right) = \frac{\partial}{\partial x^q} \left( \frac{\partial x^m}{\partial x^q} \frac{\partial x^n}{\partial x^p} \frac{\partial \omega_{m...n}}{\partial s} \right) \]

\[ = \frac{\partial x^q}{\partial x^q} \frac{\partial x^m}{\partial x^q} \frac{\partial x^n}{\partial x^p} \frac{\partial \omega_{m...n}}{\partial s} = \frac{\partial x^q}{\partial x^q} \frac{\partial x^m}{\partial x^q} \frac{\partial x^n}{\partial x^p} \frac{\partial \omega_{m...n}}{\partial s} \]

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so, \( dL_x = L_x d \rightarrow [L_x, d] = 0 \)