Killing vectors: \( \xi^a \rightarrow \partial^a \)

\( \xi^a u_a = \text{const.}, \text{along geod.} \)

\( \partial_a \xi^a = \text{const.} \)

The same as in Lagrange's eqn:

\[
L = \frac{1}{2} g_{\mu\nu} \dot{x}^\mu \dot{x}^\nu, \quad P^\mu = \frac{\partial L}{\partial \dot{x}^\mu}, \quad \frac{d}{dt} P^\mu = \frac{\partial L}{\partial x^\mu} = 0 \quad \text{if } x^\mu \text{ is ignorable.}
\]

Then used this to derive: \( V_a \xi^b + V_b \xi^a = 0 \).

But has an integrability condition: \( V_c V_b \xi^c = -R_{\xi^c \xi^d} \xi^d \)

\( \Rightarrow \xi \text{ is determined by } \xi^a \text{ and } V_a \xi^b \text{ at one point} \)

Maximally symmetric spacetime has maximal number of Killing Vectors.

Relation of \( V_a \xi^b = 0 \) and the fact that metric doesn't depend on \( x^4 \):

\[
V_a \xi^b + V_b \xi^a = g_{a(b} \nabla^{b)} \xi^c = g_{a\beta} \delta^c_\beta \xi^\beta + \frac{\partial g_{\beta \gamma}}{\partial x^\delta} \xi^\beta \xi^\gamma + (\alpha \rightarrow \beta) = 0, \quad \text{for } \xi^a = (0_4)^a \\
= 0, \quad \text{for } \xi^a = (0_4)^a \\
\frac{1}{2} (\partial_{\beta \delta} g_{\alpha \gamma} + \partial_{\alpha \delta} g_{\beta \gamma}, \alpha \rightarrow \beta) = 3 \delta^c_\beta + (\alpha \rightarrow \beta)
\]

So \( g_{\beta \delta} \text{ doesn't depend on } x^4 \).

Symmetry and conserved quantities.

- Momentum along geodesics:
  - Conserved current: \( V_a T^{ab} = 0 \) (a local statement)

  But if have a killing vector: \( J^a = T^{ab} \xi^b \), conserved current.

  \[
  V_a J^a = V_a (T^{ab} \xi^b) = V_a (\nabla^b T^{ab}) \xi^b + T^{ab} V_a \xi^b = 0
  \]

  In flat space \((t, x^i)\): \( V_a j^a = \partial_t j^i + \partial_i j^i = 0 \) - continuity eqn.