\[ \nabla_a \nabla_b \eta_c = -\nabla_b \nabla_c \eta_a - \nabla_c \nabla_a \eta_b \]

\[ = -\nabla_b \nabla_c \eta_a + \nabla_c \nabla_b \eta_a = -R_{bca^d} \eta_d \]

\[ \nabla_a \nabla_b \eta_c = -R_{bca^d} \eta_d \]

2nd derivatives of \( \eta \mu \) are defined by \( \frac{\partial \eta}{\partial x^i} \) and \( \frac{\partial^2 \eta}{\partial x^i \partial x^j} \), for a particular \( \eta \).

So, we only choose freely \( \eta_a \) and \( \nabla_a \eta_b \) at a given point. Then \( \eta_a \) is determined everywhere else. There's almost no solutions.

How much freedom? \( 4 + 6 = 10 \) (locally)

But if try to actually integrate for a general spacetime, likely will find no solution.

\[ \nabla_a \frac{\partial \eta}{\partial x^i} + \frac{\partial \eta}{\partial x^j} \nabla_j \frac{\partial \eta}{\partial x^i} = 0 \]

\[ \frac{\partial^2 \eta}{\partial x^i \partial x^j} = 0 \implies \frac{\partial \eta}{\partial x^i} = f(y) = -cy + e \] rotation

\[ \frac{\partial^2 \eta}{\partial y \partial y} = 0 \implies \frac{\partial \eta}{\partial y} = g(x) = cx + \circ \] translation

\[ \frac{\partial^2 \eta}{\partial x \partial y} + \frac{\partial^2 \eta}{\partial y \partial x} = 0 \implies \frac{dy}{dx} = -\frac{df}{dy} = c \]