A. Are the states that diagonalize $V$ in the degenerate subspace the same as the states that diagonalize $H_0 + \lambda V$?
Either prove that they are or give a counter-example.