I. REFLECTION WITH DISPERSION

A plane wave of frequency $\omega$ is incident normally from vacuum on a semi-infinite slab of material with a complex $n(\omega)$.

a) Show that the ratio of reflected power to incident power is

$$R = \left|\frac{1 - n(\omega)}{1 + n(\omega)}\right|^2.$$  

II. ANISOTROPIC MEDIUM

Plane waves propagate in a homogeneous, nonpermeable ($\mu = 1$) but anisotropic dielectric medium. The dielectric is characterized by a tensor $\epsilon_{ij}$ such that $D_i = \epsilon_{ij}E_j$. $\epsilon_{ij}$ is symmetric so it can be diagonalized by choosing an appropriate orthogonal coordinate system.

a) Show that plane waves with frequency $\omega$ and wave vector $k$ must satisfy

$$k \times (k \times E) + \frac{\omega^2}{c^2}D = 0.$$  

b) Show that for a given wave vector $k = kn$ there are two distinct modes of propagation with different phase velocities $v = \omega/k$ which satisfy the Fresnel equation:

$$\sum_{i=1}^{3} \frac{n_i^2}{v^2 - v_i^2} = 0,$$

where $v_i = c/\sqrt{\epsilon_i}$ and $n_i$ is the $i^{th}$ component of $k/|k|$.

III. ORDERS OF MAGNITUDE

Without looking at books/internet, try to guess the (order of magnitude of) the wavelengths of
a) visible light
b) radio waves
c) microwave
d) X-Ray

Add a very short (one line?) rationale of how you arrive at this estimate.
Find from some reliable source a rough estimate for the numbers above and compare them with your guesses.

IV. OPTICALLY ACTIVE MEDIUM

A dextrose solution is optically active and is characterized by a polarization vector satisfying $P = \gamma \nabla \times E$ for a real constant $\gamma$ which depends on the dextrose concentration. The solution is non-conducting and non-magnetic (that is, the magnetization vanishes). Consider a plane wave with frequency $\omega$ propagating in this solution. For definiteness, assume the propagation is in the $z$ direction. Also assume $\gamma \omega << c$ so that square roots can be approximated as $\sqrt{1 + A} \approx 1 + A/2$.

a) Find the two possible indices of refraction for such a wave.

b) Suppose linearly polarized light is incident on the dextrose solution. After traveling a distance $L$ through the solution, the light is still linearly polarized but the direction of polarization has been rotated by an angle $\phi$ (this is called Faraday rotation). Find $\phi$ in terms of $L, \gamma$ and $\omega$. 
