a) \[ L = \frac{1}{2} m [ \dot{r}^2 + (r\dot{\phi})^2 ] + mG/r \]  
\[ (A1) \]

b) \[ \ddot{r} = -\frac{mG}{r^2} + r\dddot{\phi} \]  
\[ (A2) \]

\[ r\dddot{\phi} + 2\dddot{r} = 0 \]  
\[ (A3) \]

c) \[ \omega^2 = \frac{mG}{R^3} \]  
\[ (A4) \]

d) \[ \dddot{\rho} = 3\omega^2 \rho + 2\omega R\dddot{\phi} \]  
\[ \dddot{\psi} + 2(\omega/R)\dddot{\rho} = 0 \]  
\[ (A5) \]

\[ (A6) \]
e) Integrating (A6) and using initial conditions gives

\[ \dddot{\psi} + 2(\omega/R)\dddot{\rho} = 0 . \]  
\[ (A7) \]

Putting this in (A5) gives

\[ \dddot{\rho} = -\omega^2 \rho . \]

Taking into account the initial condition, the solution for \( \rho \) is

\[ \rho = -\frac{V}{\omega} \sin \omega t . \]  
\[ (A8) \]

Correspondingly, \( \psi \) can be found by putting (A8) in (A7) and integrating. The result is

\[ \psi = \frac{2V}{\omega R} (1 - \cos \omega t) . \]  
\[ (A9) \]

Note that when \( t = \tau = 2\pi/\omega \) (one period later), then \( \rho(\tau) = \psi(\tau) = 0 \). Also

\[ \dddot{\psi}(\tau) = -2(\omega/R) \dddot{\rho}(\tau) = 0 . \]