Next examine non-circular orbits to see if I restrictions on a, b, c. Then
\[ \ddot{u} + u = -m^2L^{-2}\dot{G}'(u) \Rightarrow \ddot{u} + u = -m^2L^{-2}v a_0 u^{-1}. \]
Write
\[ u = u_0 (1 + \varepsilon) \Rightarrow u_0 \left[ \ddot{\varepsilon} + 1 + \varepsilon \right] = -m^2L^{-2}v a_0 u_0^{-1} (1 + \varepsilon)^{-1}. \]
Define \( u_0 \) so that
\[ u_0 = -m^2L^{-2} v a_0 u_0^{-1} \Rightarrow \left( \ddot{\varepsilon} + \varepsilon = (1 + \varepsilon)^{-1} \right) \]
Expand
\[ (1 + \varepsilon)^{-1} = 1 + (\varepsilon - 1) \varepsilon + \sum_{n=2}^{\infty} \lambda_n \varepsilon^n \Rightarrow \]
\[ \ddot{\varepsilon} + (2 - \nu) \varepsilon = \sum_{n=2}^{\infty} \lambda_n \varepsilon^n \]
Now, for nearly circular orbits, \( \varepsilon \approx 0 \) and we get
\[ \ddot{\varepsilon} + (2 - \nu) \varepsilon = 0 \Rightarrow O(\varepsilon^2) \]
We see that the "precession frequency" is independent of \( u_0 \) for small oscillations as expected. We now require that the oscillation frequency of the non-linear equation \( I \) be independent of amplitude, i.e. we require that \( \omega^2 = \omega_0^2 = 2 - \nu \) even when \( \varepsilon \neq 0 \). An esoteric result from non-linear oscillation theory states that a necessary condition is that
\[ \frac{5}{6} \lambda_2^2 \omega_0^{-2} + \frac{3}{4} \lambda_3 = 0 \]
[This problem should be with 15 points]
\[ \Rightarrow \frac{5}{6} \frac{(\nu-1)(\nu-2)^2}{(2-\nu)} + \frac{3}{4} \frac{(\nu-1)(\nu-2)(\nu-3)}{6} = 0 \Rightarrow (\nu-2) \left( 3(\nu-3) - 5(\nu-1) \right) = 0 \Rightarrow \nu = 1, 2, -2 \]
\[ \therefore V = \frac{1}{r}, \frac{1}{r^2}, + \text{r}^2. \] \( \frac{1}{r^2} \) is ruled out by stability \( \Rightarrow V(r) = -\frac{a}{r} \) and \( a \neq 0 \) remain and actually work as you saw in the movie.

Note \( \frac{1}{r^2} \) gives spirals, but it is ruled out.