\[ \theta'^2 - \theta^2 - \sin^2 \theta = -\frac{1}{K} \sqrt{\theta'^2 + \sin^2 \theta} \]

or

\[ K \sin^2 \theta = \sqrt{\theta'^2 + \sin^2 \theta} \]

Now it should be clear that we can rotate coordinates \((\theta, \phi)\) on the sphere such that at a given point \(P\) on a curve \(\gamma\), \(\theta = \frac{\pi}{2}\) is tangent to the curve \(\gamma\), i.e. at the point \(P\), \(\theta = \frac{\pi}{2}\) and \(\theta' = 0\).

This fixes the constant \(K\) to have value 1, so

\[ \sin^4 \theta = \theta'^2 + \sin^2 \theta \]

\[ \theta'^2 = \sin^4 \theta - \sin^2 \theta = -\sin^2 \theta \cos^2 \theta \]

\[ \theta'^2 = -\frac{1}{\theta} \sin^2 (2\theta) \]

But \(LHS \geq 0\), \(RHS \leq 0\) so both must be 0.

\[ \theta' = 0 \quad \& \quad 2\theta = \pi \]

So

\[ \theta = \frac{\pi}{2} = \text{const wrt } \phi \]

which is clearly a great circle.

See the next few sheets for a previous graders' quite different solution to the same problem.