Also note that \[ \left[ n_0^2 - 2K \right]^{-1/2} = -\frac{1}{H} \] and
\[ H = \text{const} \] since \[ \frac{\partial H}{\partial z} = 0. \]

Let \[ \zeta = \left[ n_0^2 - 2K \right]^{-1/2} z \] be a new independent variable. Then we get
\[ X = \frac{dx}{dz} = \frac{dx}{dz} \frac{dz}{d\zeta} = \left[ n_0^2 - 2K \right]^{-1/2} \left[ n_0^2 - 2K \right]^{1/2} \frac{d\zeta}{d\zeta} \]

Thus,
\[ X = \frac{\partial k}{\partial \zeta}, \quad \hat{p}_x = -\frac{\partial k}{\partial x} \text{ etc.} \]

But \[ K = \frac{p_x^2 + p_y^2}{2} + \frac{w^2}{2} (x^2 + y^2), \] a simple Harmonic Oscillator

where \[ w = n_0 \alpha = 1 \]

\[ x = p_x, \quad \hat{p}_y = -\omega^2 x = 7 x + \omega^2 x = 0. \]

\[ X = A_x \sin (\omega \zeta + \phi_x), \quad Y = A_y \sin (\omega \zeta + \phi_y) \]